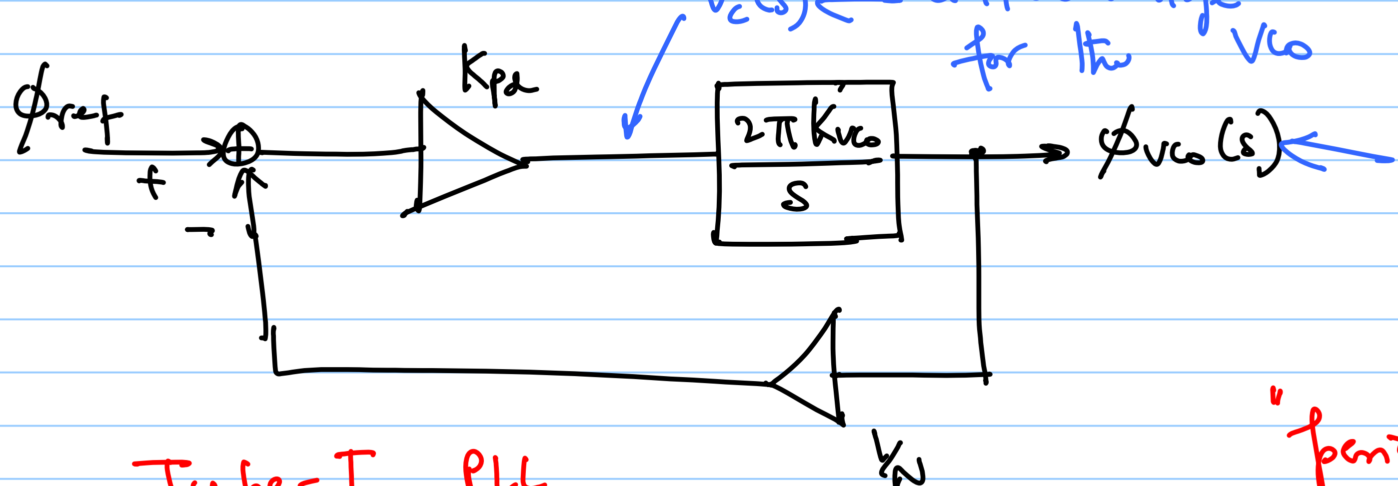
# ECE 504 - Lecture 5

Note Title 9/1/2016

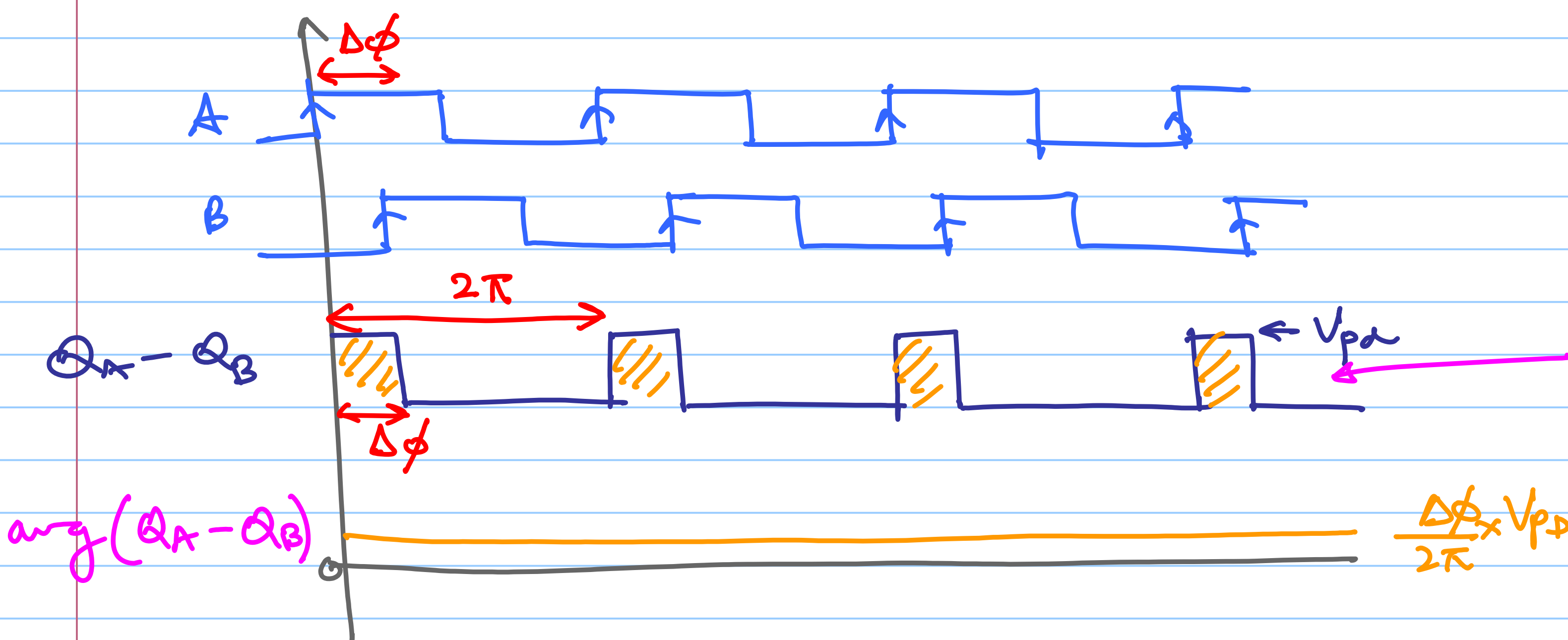
Incremental model for the PLL (variables in phase domain).

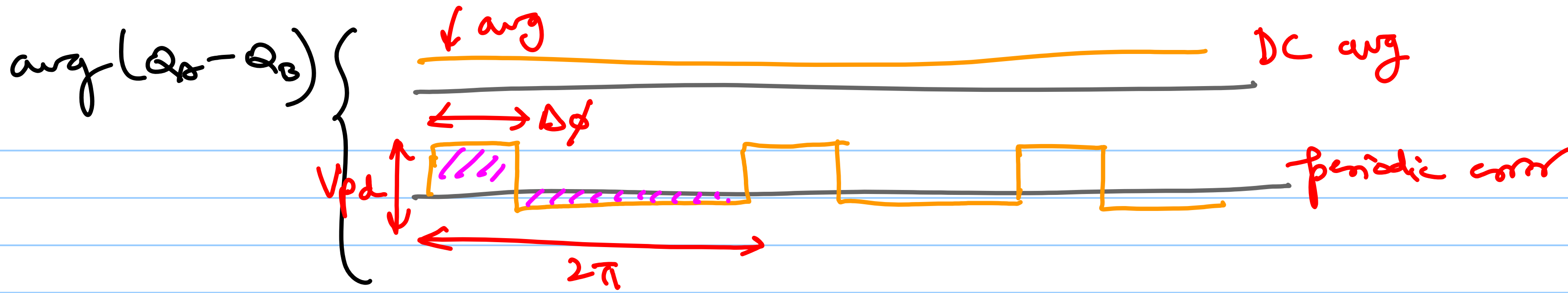



Type-I PLL

"periodic error from the PFD"

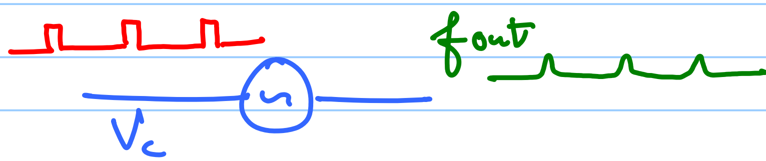
## Phase detector Output :





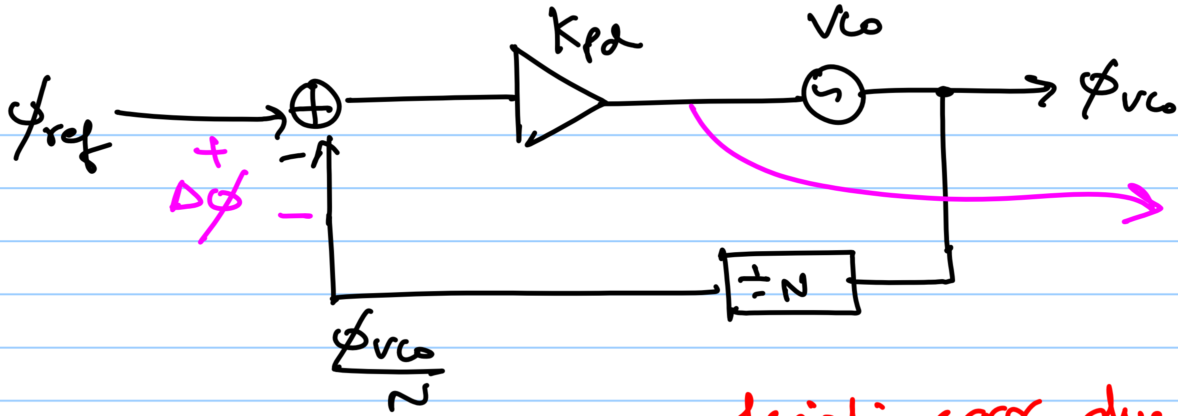

* periodic disturbance ⇒ periodic output of the PFD

↳ phase of the VCO is periodically disturbed



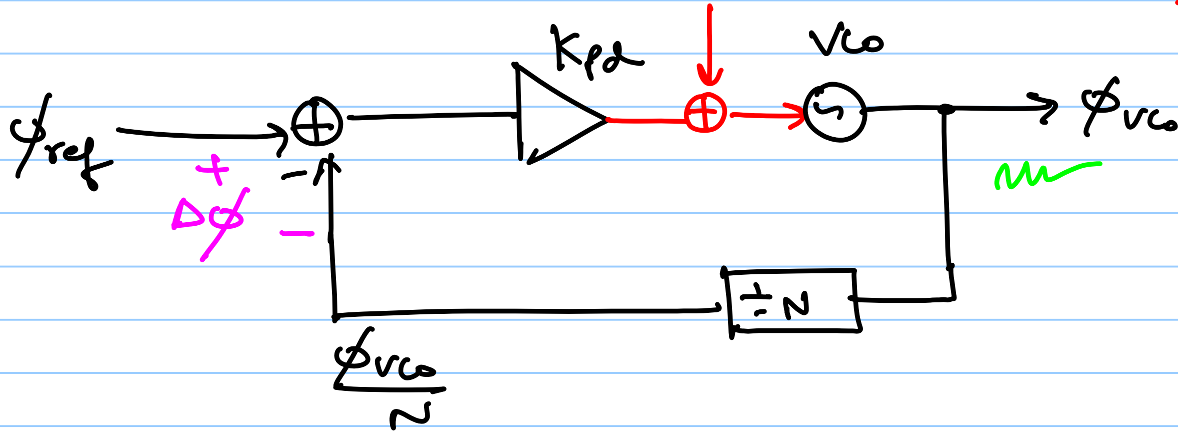

⇒ VCO's output will not be periodic

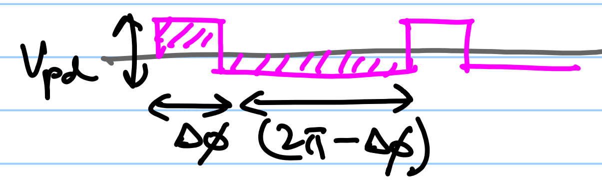
↳ frequency is not constant



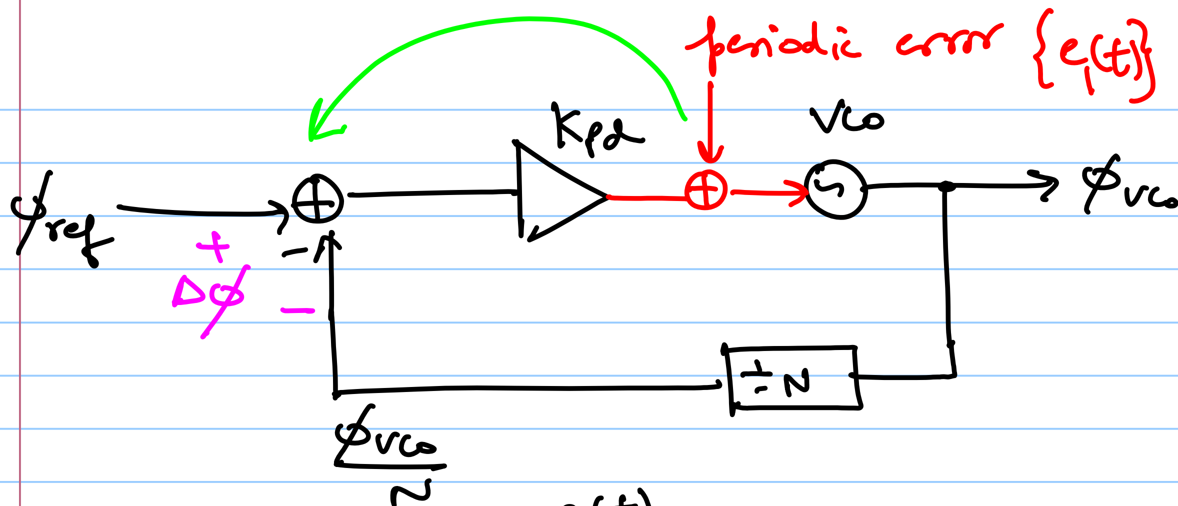

$$v_c = V_{PD}\frac{\Delta\phi}{2\pi} \leftarrow \text{avg value}$$

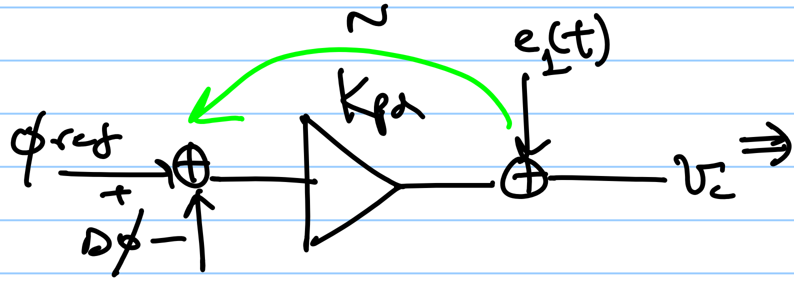
periodic error due to $\Delta\phi$

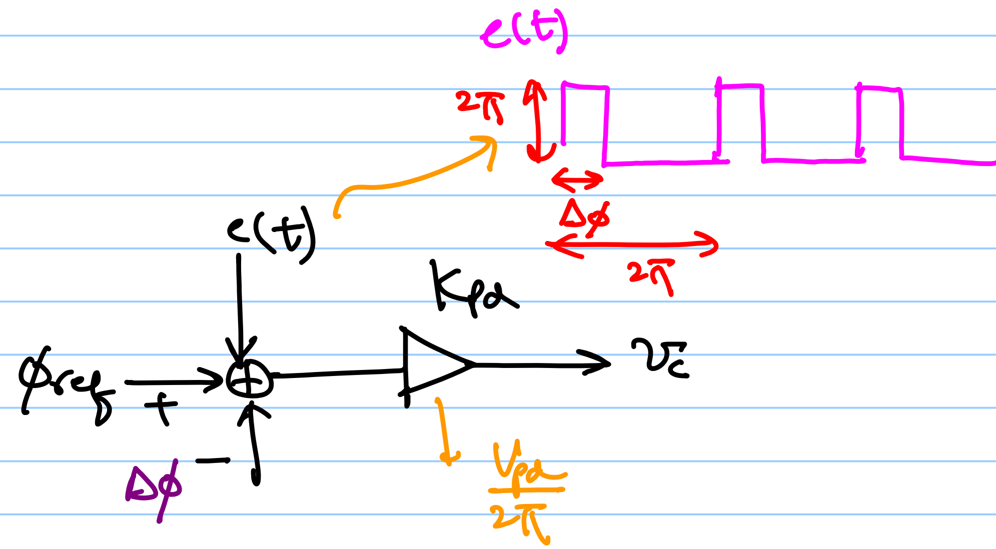


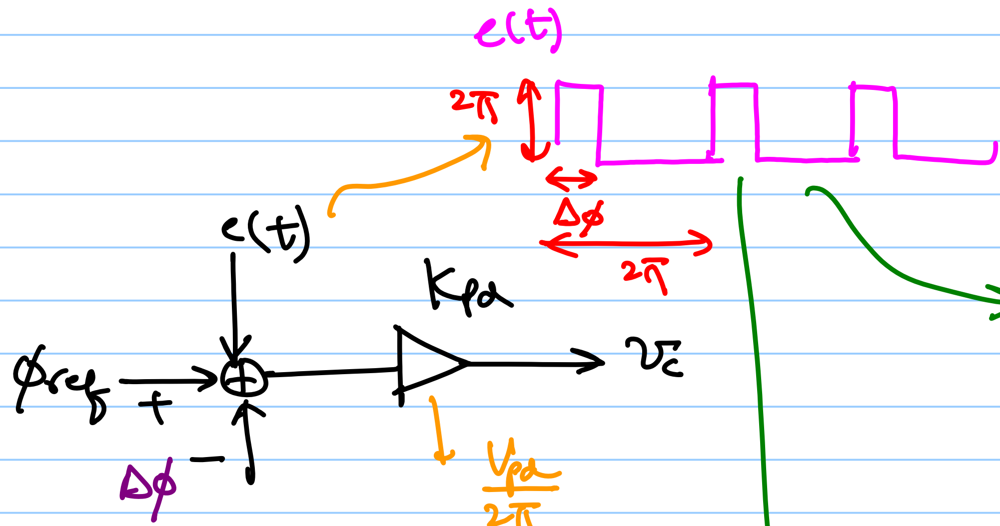
Refer the periodic error to the input



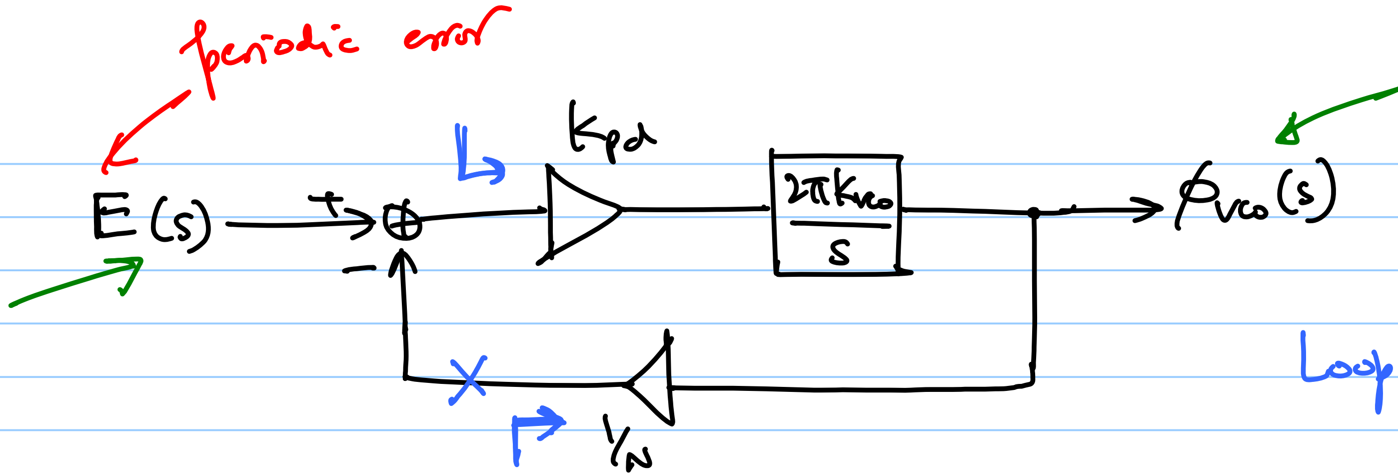



Equivalent Model for the periodic error due to finite $|\Delta\phi| > 0$




Time period of this disturbance $\Rightarrow$ reference period $\Rightarrow 1/f_{ref}$

frequency of disturbance $= f_{ref}$




$$\text{Loop gain} = \frac{2\pi K_{pd} K_{vco}}{sN}$$

$$= \frac{2\pi K_{pd} K_{vco}}{N} \cdot \frac{1}{s}$$

$$\text{Closed-loop gain} = \frac{K_{pd} \cdot \frac{2\pi K_{vco}}{s}}{1 + \frac{2\pi K_{pd} K_{vco}}{N} \cdot s}$$

$$\Rightarrow \quad \frac{\phi_{vco}(s)}{E(s)} = \frac{N}{1 + \dfrac{s \cdot N}{2\pi K_{vco} K_{PD}}}$$

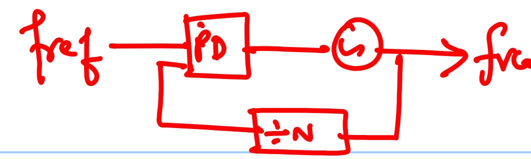
Ideal loop gain

first-order filtering in the loop.

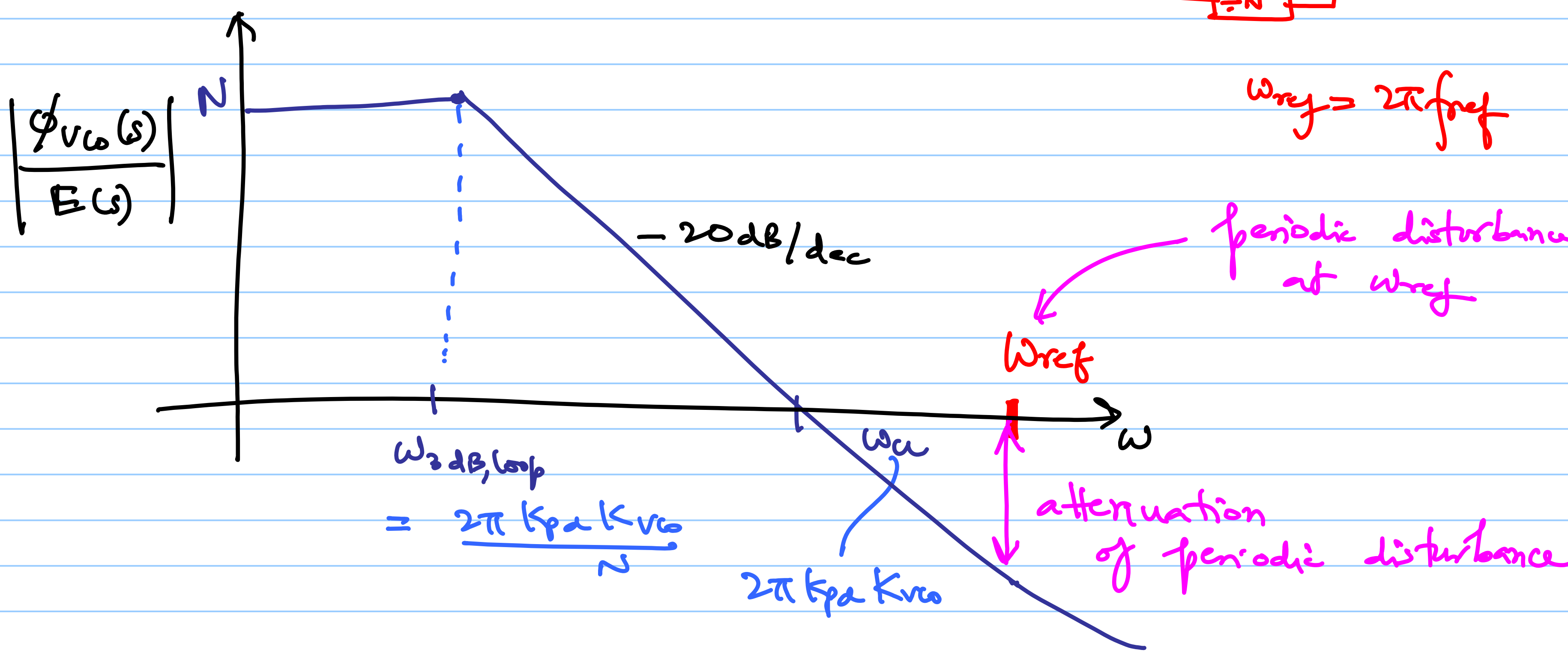
$$= \frac{N}{1 + \dfrac{s}{\omega_{3dB}}},$$

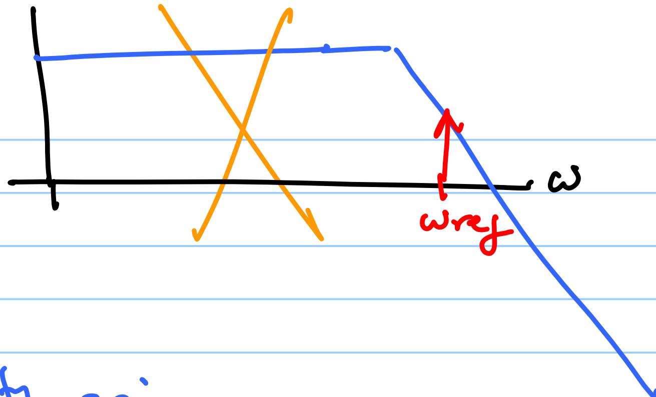
$$\omega_{3dB,\,loop} = \frac{2\pi K_{vco} K_{PD}}{N}$$

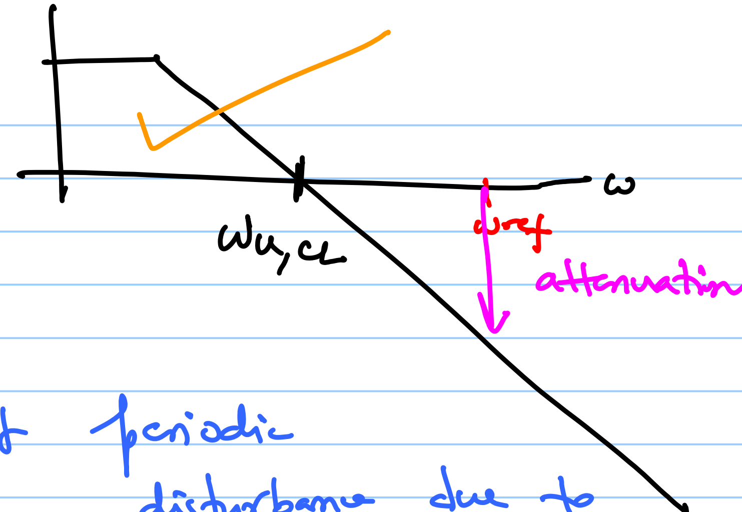
$$\omega_u = 2\pi K_{vco} \cdot K_{PD}$$




$\omega_{ref} = 2\pi f_{ref}$






unity gain frequency $\omega_{u,CL} \ll \omega_{ref}$ to reject periodic disturbance due to the AC content in the PD output

$$\omega_{u,c} \ll \omega_{ref} \qquad 2\pi f_{ref}$$

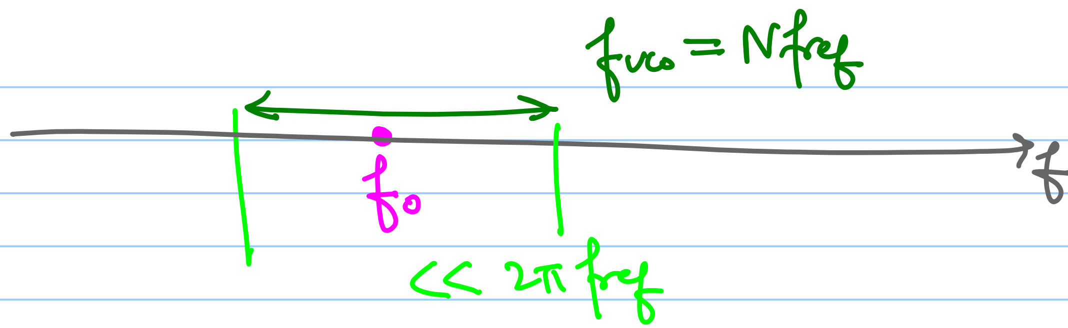
$$\Rightarrow \quad \omega_{ref} \gg 2\pi K_{pd} K_{vco} \longrightarrow (1)$$

contraint on the range

$$|Nf_{ref} - f_o| < 2\pi K_{PD} K_{vco} \longrightarrow (2)$$

$$\Rightarrow \quad \overbrace{|Nf_{ref} - f_o|}^{\Delta f_{vco}} < 2\pi K_{PD} K_{vco} \ll 2\pi f_{ref}$$

$$\Delta f_{vco} \ll 2\pi f_{ref}$$




$\therefore$ range of the PLL o/p frequency is severely restricted

very limited tuning range for the PLL $\sim \dfrac{f_{ref}}{100}$ $\longrightarrow$ just $\dfrac{f_{ref}}{100}$ around $f_{vco} = N f_{ref}$

↳ severely restricts the application

↳ we can't even tune from

$f_{VCO} = N f_{ref}$ to $(N+1) f_{ref}$

loop-gain ⟷ loop-bandwidth

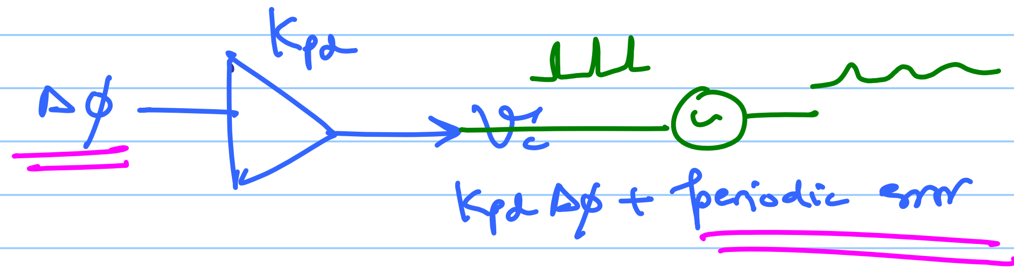
large for lower $\Delta\phi$

↳ lower periodic noise

for large lock-range

static phase offset

* We want $\Delta\phi$ to be zero to have a useful design.



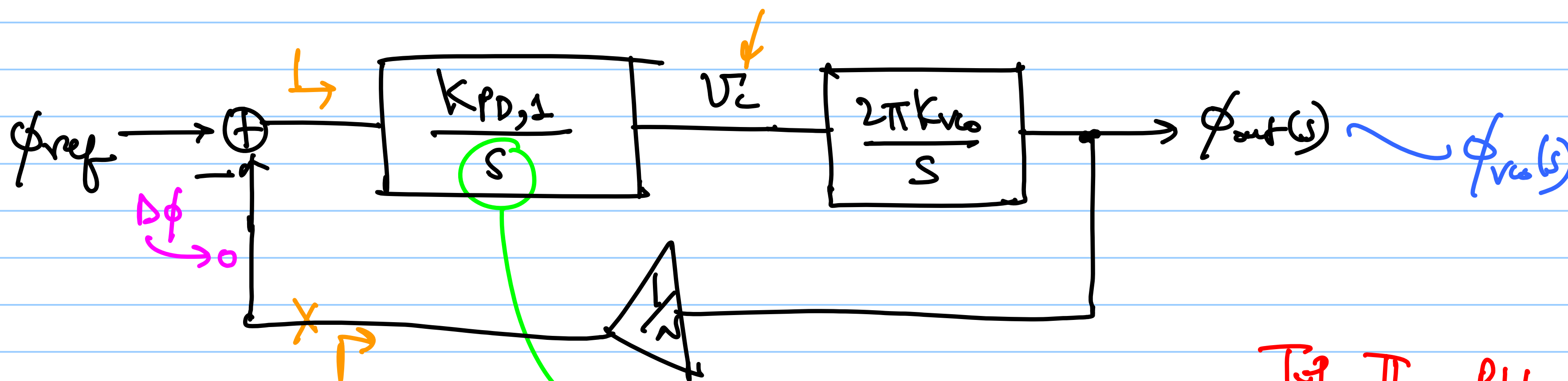

↳ We need an integrator in the loop to support non-zero VCO input ($V_c$) while having $\Delta\phi = 0$.

↳ We need zero DC phase error

↳ increase the loop gain to $\infty$ at DC

↳ integrator!

↳ The PD has to be an integrator, not a proportional block ($K_{pd}$)




new addition
↳ integrator in the PD

Type-II PLL

$$L(s) = \frac{2\pi K_{PD1} K_{vco}}{N \cdot S^2}$$

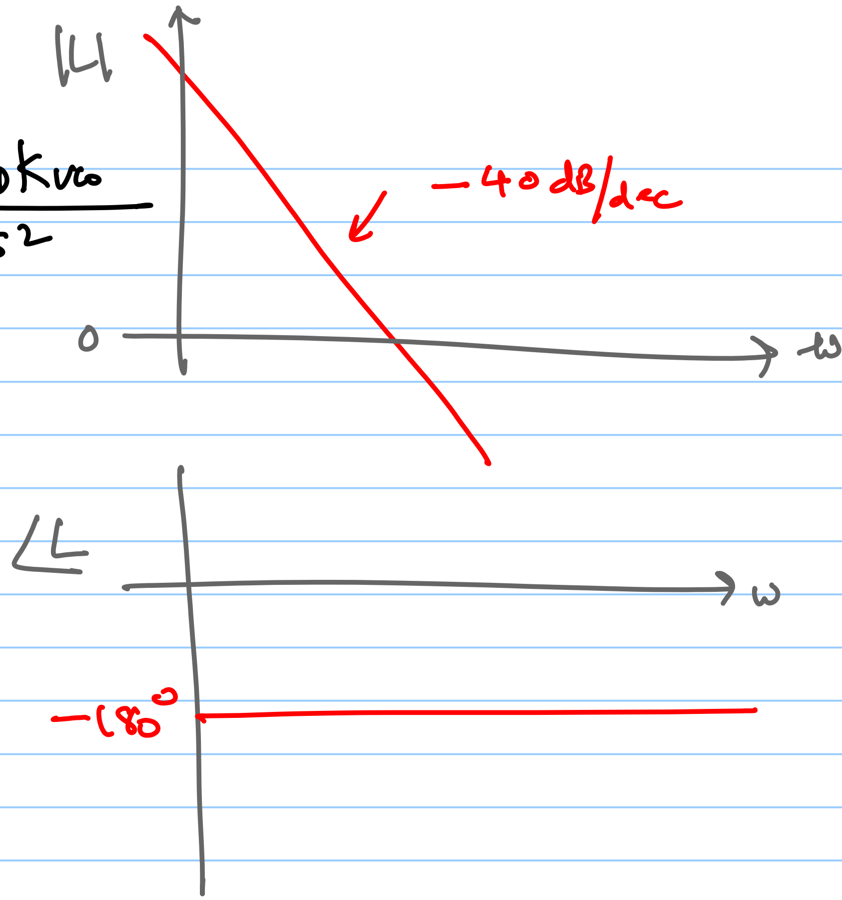
Type-II system

Type $\Rightarrow$ # of poles at DC in the open-loop response

Problem $\Rightarrow$ Two poles at DC $\Rightarrow$ instability!

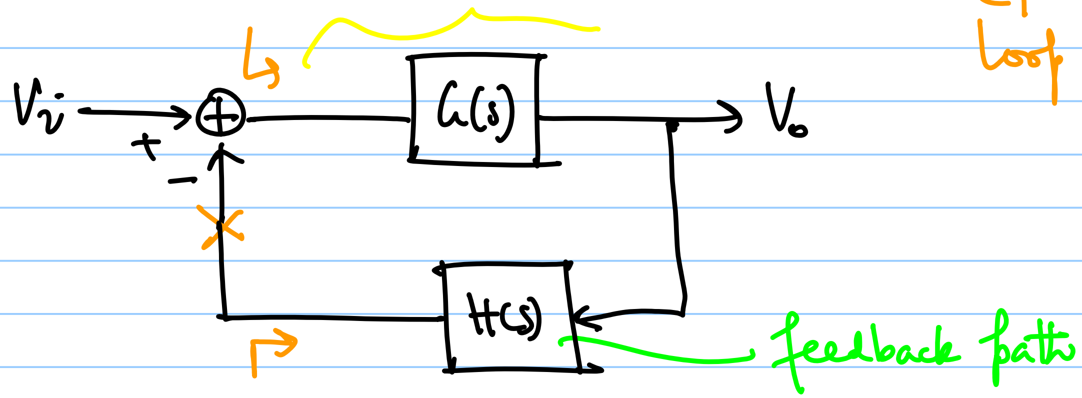
$$L(s) = \frac{2\pi K_{PD} K_{VCO}}{N \cdot s^2}$$

$$L(s) = \frac{2\pi K_{PD} K_{VCO}}{N \cdot s^2}$$




Need to stabilize Type-II PLL

# Feedback System Review




(open)
loop gain $= G(s)H(s)$

$$\text{closed-loop gain} = \frac{G(s)}{1 + \text{loop gain}} = \frac{G(s)}{1 + G(s)H(s)} = \frac{1}{H(s)} \cdot \frac{1}{1 + \frac{1}{G(s)H(s)}}$$

$$= \frac{1}{H(s)} \times \frac{1}{1 + \frac{1}{GH}}$$ ← loop-gain

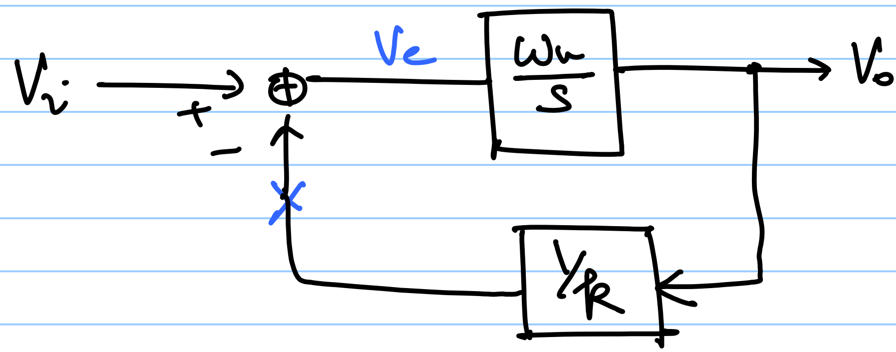
$$\simeq \begin{cases} \frac{1}{H(s)} & \text{for } |GH| \gg 1 \\ G(s) & \text{for } |GH| \ll 1 \end{cases}$$

for large loop-gain $\Rightarrow$ cl gain $\simeq \frac{1}{H}$

for $|L| \ll 1$, the feedback is effectively broken

$$\frac{V_o}{V_i}(s) \simeq G(s)$$

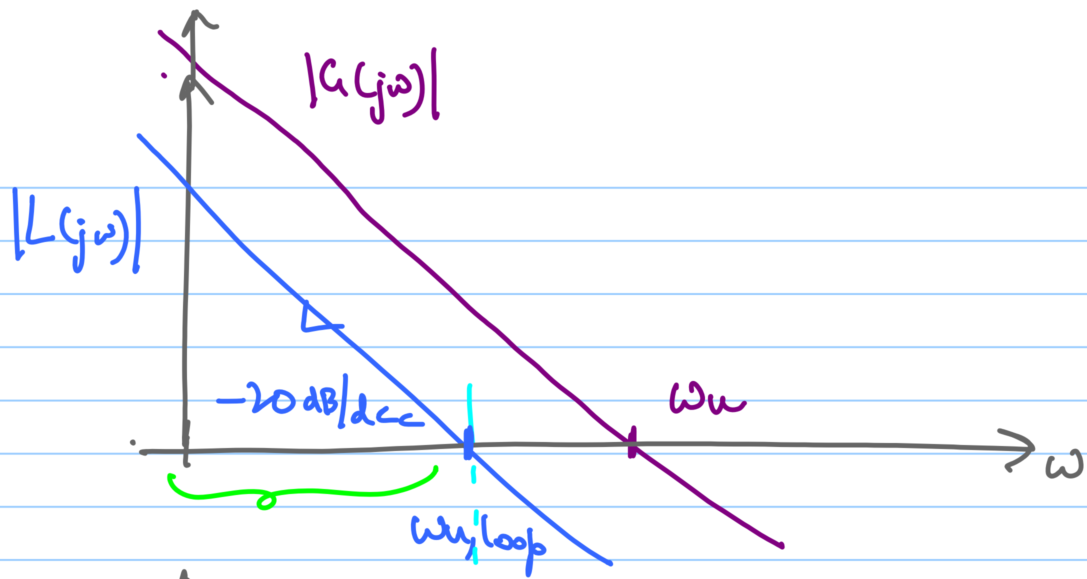
We now have,




$\omega_u$: unity gain frequency of the integrator controller

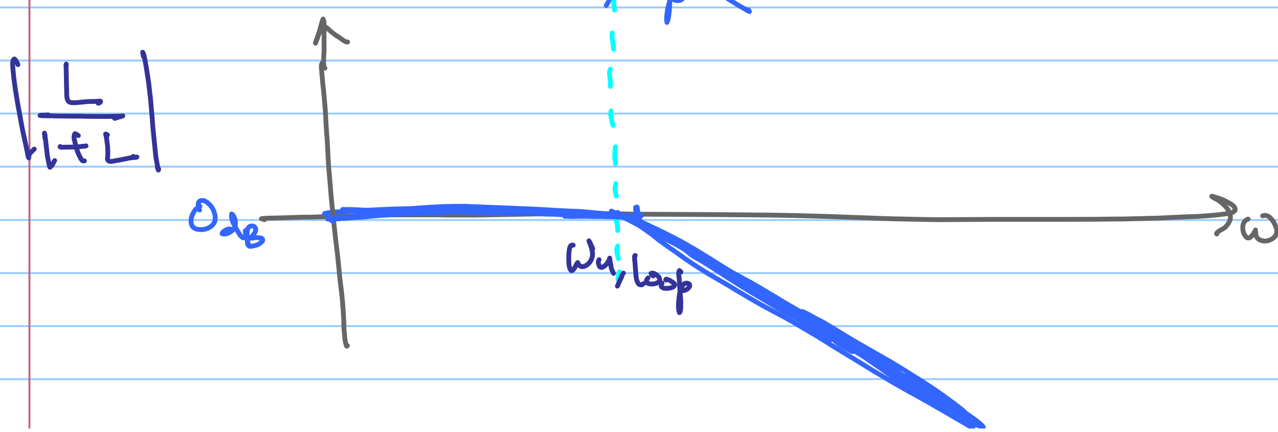
$$L(s) = \frac{\omega_u}{k \cdot s}$$ , loop gain

unity gain frequency for the loop $\quad \omega_{u,\,loop} = \frac{\omega_u}{k}$ , unity loop gain frequency




$L(j\omega) \Rightarrow$ loop-gain frequency response

$$G(j\omega) = \frac{\omega_u}{j\omega}$$




$$\left|\frac{L}{1+L}\right| \simeq \begin{cases} L, & |L| \ll 1 \\ 1, & |L| \gg 1 \end{cases}$$

$$\frac{G}{1+GH} = k \cdot \frac{L}{1+L}$$

$$\left|\frac{G}{1+GH}\right|$$

closed-loop gain