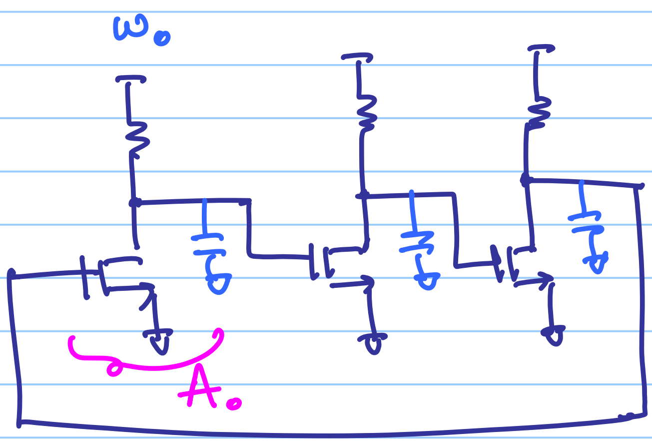
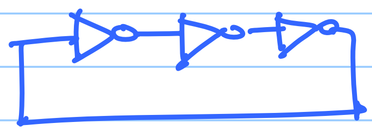
Note Title 10/12/2016

ECE 504 - Lecture 15

Oscillators:



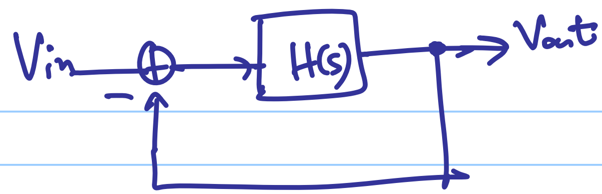
3-stage oscillator



for oscillation $A_0 = 2$

$\omega_{osc} = \omega_0 \sqrt{3}$

$L(s) = H(s)$



$$\frac{V_{out}}{V_{in}}(s) = \frac{H(s)}{1 + L(s)}$$

$$= \frac{\dfrac{-A_0^3}{(1+s/\omega_0)^3}}{1 - \dfrac{A_0^3}{(1+s/\omega_0)^3}}$$

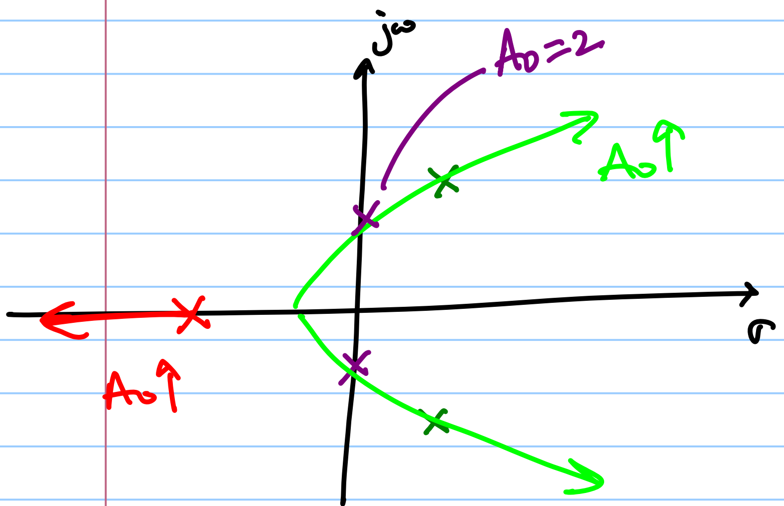
$$= \frac{-A_0^3}{(1+s/\omega_0)^3 - A_0^3}$$

closed loop poles :

$$D(s) = 0$$

$$\Rightarrow \quad \left(1 + \frac{s}{\omega_o}\right)^3 - A_o^3 = 0$$

$$\Rightarrow \quad \left(1 + \frac{s}{\omega_o}\right)^3 = A_o^3$$

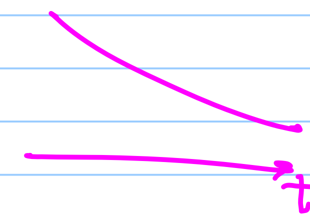


$$s = \begin{cases} -(A_o+1)\,\omega_o \\ \left(\dfrac{A_o(1 \pm j\sqrt{3})}{2} - 1\right)\omega_o \end{cases}$$

$\rightarrow \pm j\sqrt{3}\,\omega_o$ for $A_o = 2$

* first pole leads to decaying exponential term

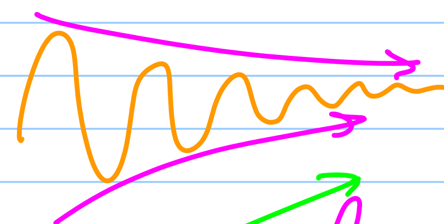
$$e^{-(A_0+1)\omega_0 t}$$



* the complex conjugate poles are on the $j\omega$-axis

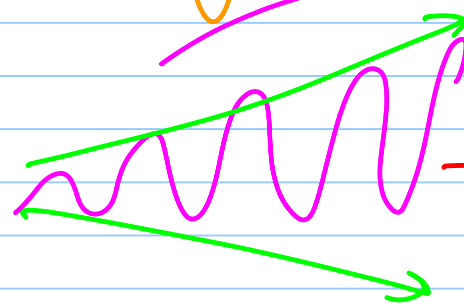
$\Rightarrow$ for $A_0 = 2$.

for $A_0 < 2$



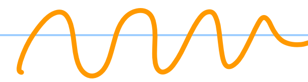
$\rightarrow$ No oscillation

for $A_0 > 2$

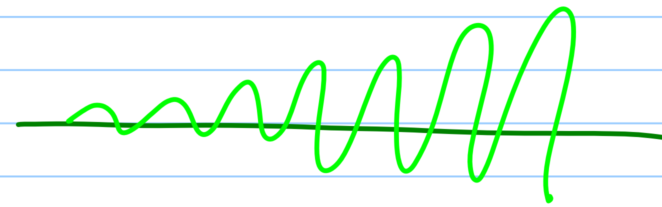


$\rightarrow$ exponentially growing sinusoidal waveform

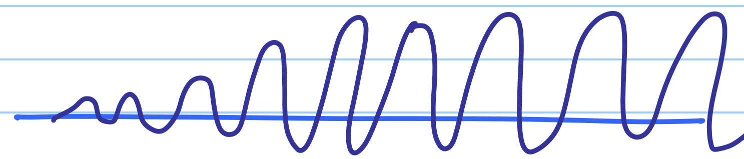
$$V_{out}(t) = a\left[e^{-(A_0+1)\omega_s t} + \cos\left(\frac{A_0\sqrt{3}\,\omega_0 t}{2}\right)\right]$$

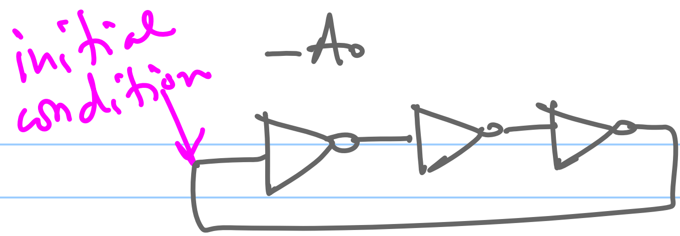


In theory
for
$A_0 > 2$

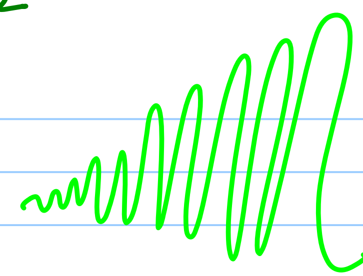


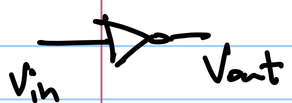
In practice

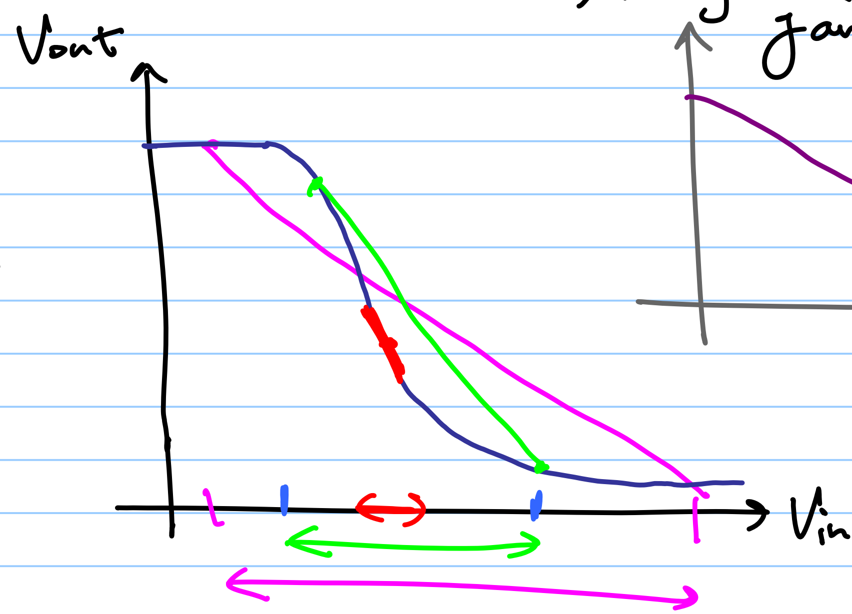


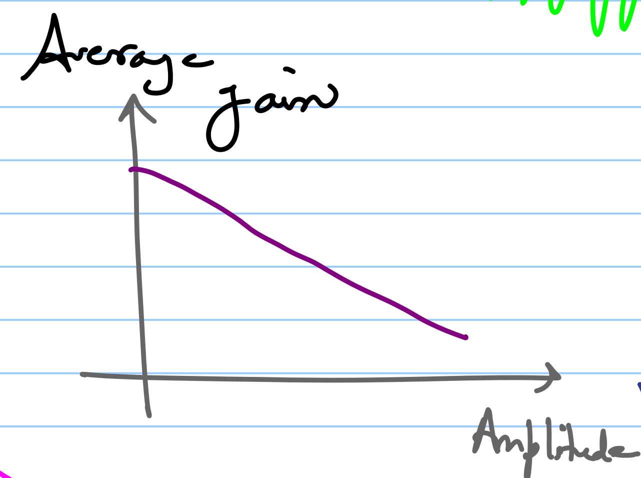

initial condition
$-A_0$

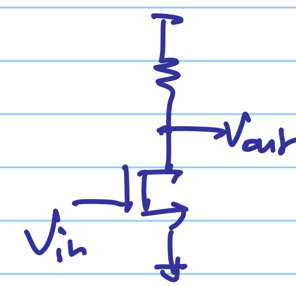
$A_0 > 2$

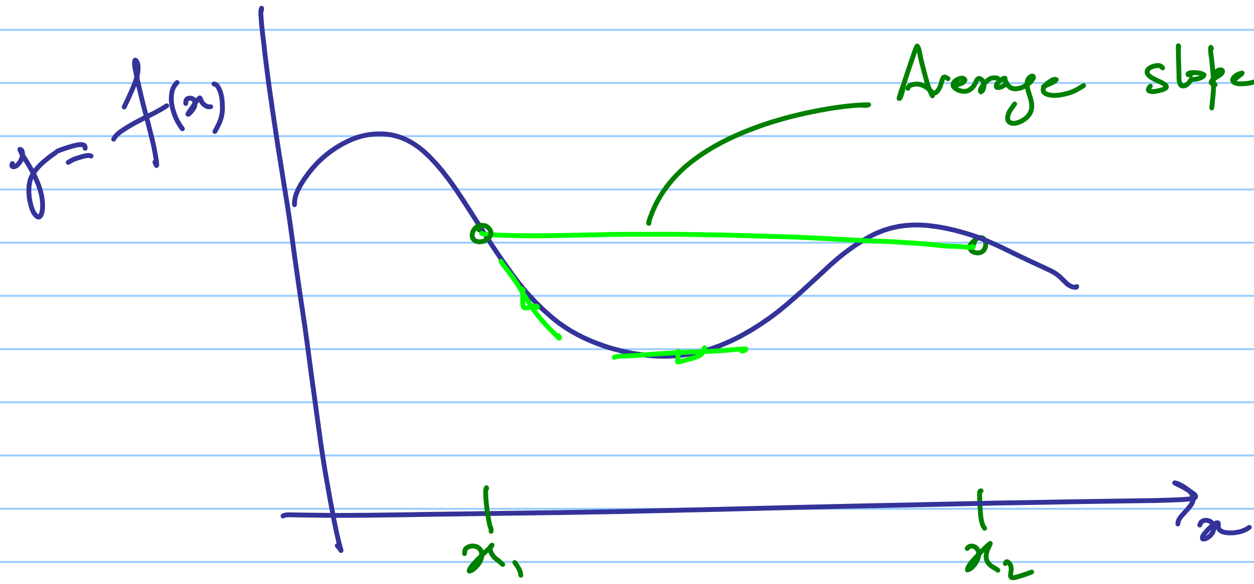



V_{in}
V_{out}


V_{out}
V_{in}


Average gain
Amplitude


V_{out}
V_{in}



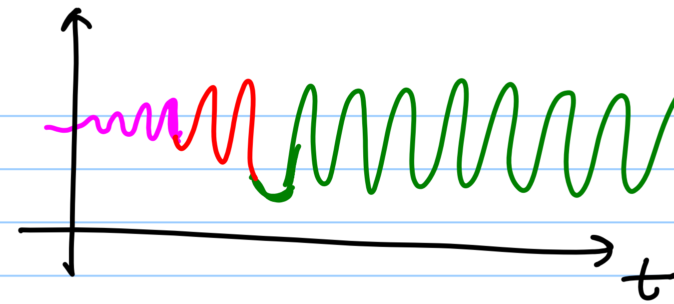
$$\frac{f(b) - f(a)}{b-a}$$

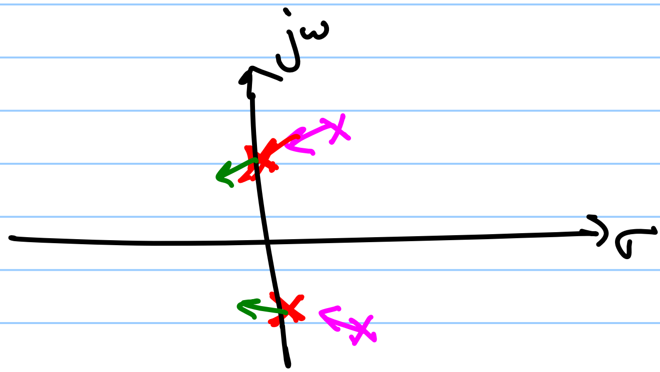
* Oscillation amplitude increases

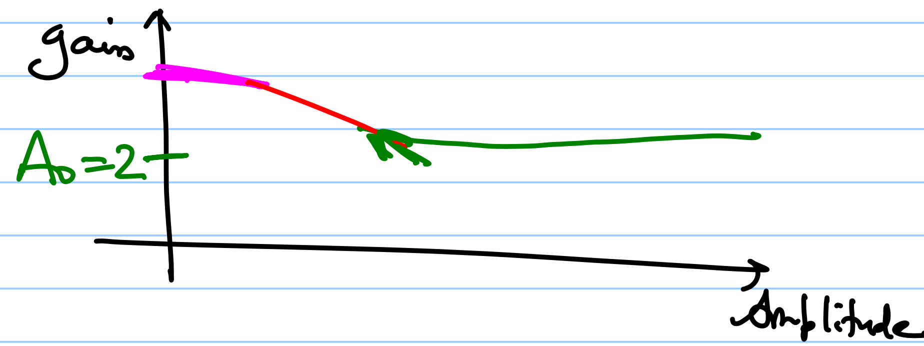
* gain stage experiences non-linearity

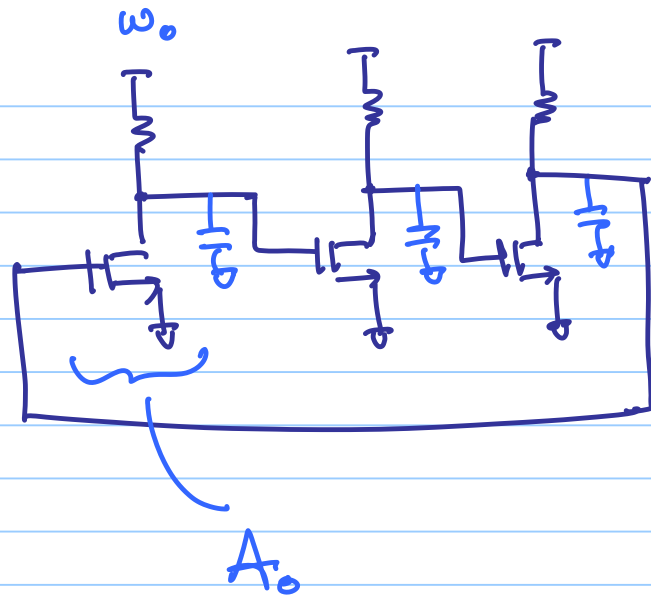
  ↳ average gain drops

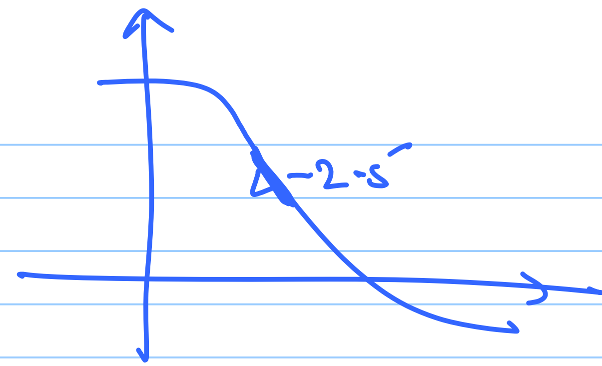
    ↳ conjugate poles move towards $j\omega$-axis





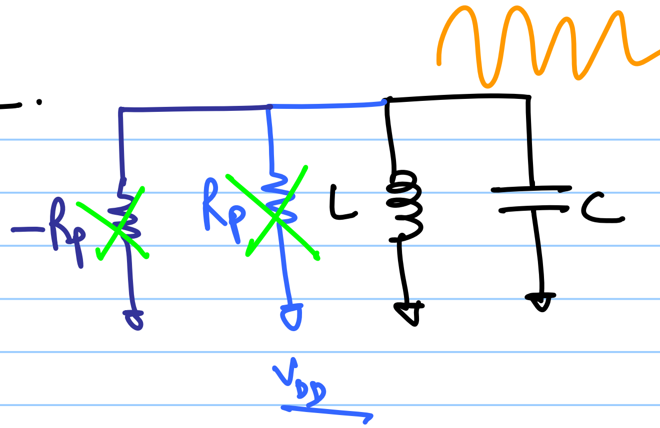



ω₀
A₀

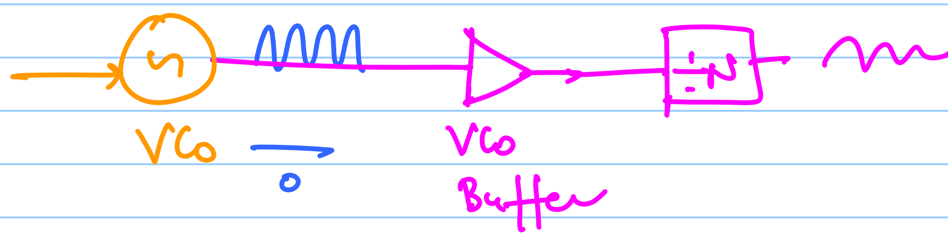

-2.5

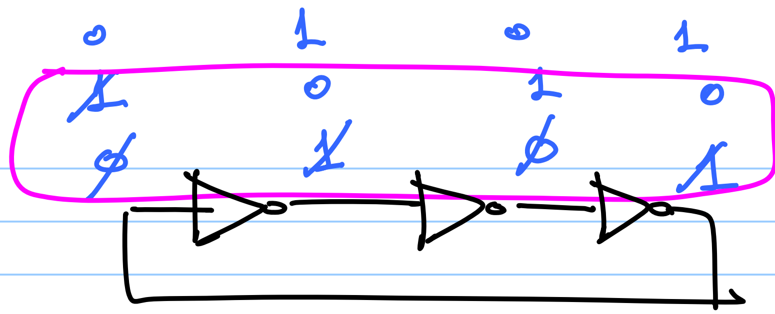
Aside:

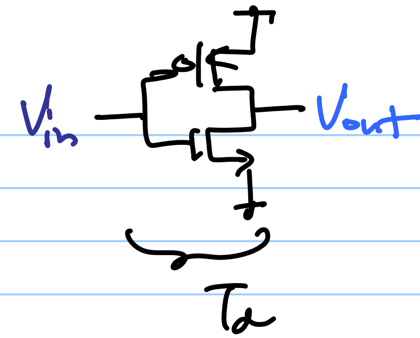
LC-Tank

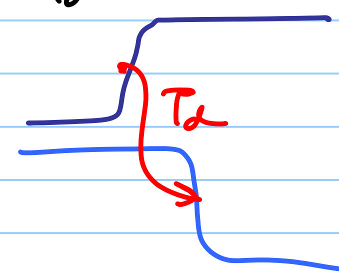


$$\omega_0 = \frac{1}{\sqrt{LC}}$$









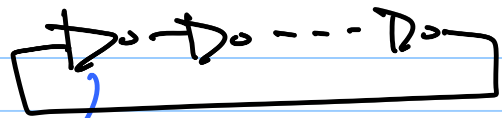
$$T_{CK} = 2nT_d = 6T_d$$

$$\underbrace{\omega_{osc} = \frac{2\pi}{6T_d} = \frac{\pi}{3T_d}}_{\text{Large-signal}}$$

But from small signal analysis

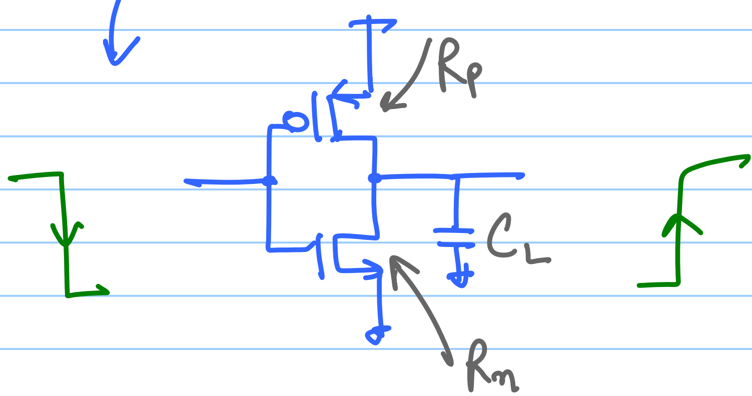
$$\underbrace{\omega_{osc} = \omega_0\sqrt{3}}_{\text{small-signal}}$$

Reconciliation : the small-signal oscillation starts with $\omega_0\sqrt{3}$ and as the amplitude builds up, the circuit gets more non-linear and the frequency of oscillation drops to $\frac{2\pi}{6T_d}$

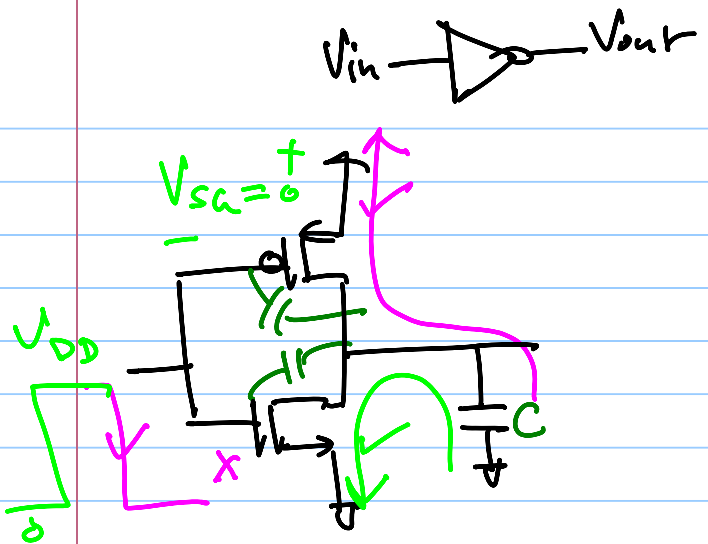


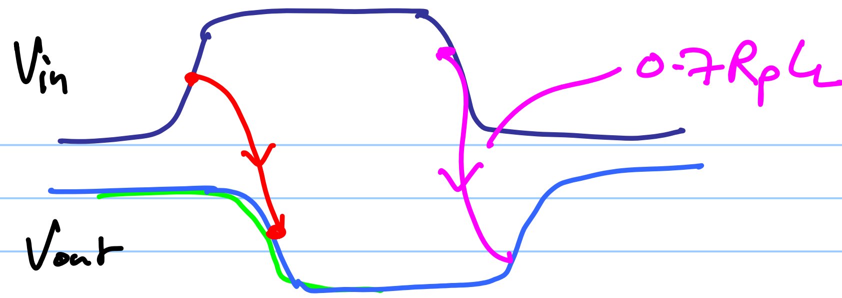
$$f_{osc} = \frac{1}{2n T_d}$$

↳ T_d = inverter delay



$t_d = 0.7 R_p C_L$





$t_{pHL} = 0.7 R_n C_L$

$$R_n = \frac{1}{KP_n \left(\frac{W}{L}\right)_n (V_{DD} - V_{THN})}$$

$$R_p = \frac{1}{KP_p \left(\frac{W}{L}\right)_p (V_{DD} - |V_{THP}|)}$$

$$f_{osc} = \frac{1}{2n\,(0.7 R_n C)}$$

$n = 3;$

$$C = \frac{5}{2}\left[C_{oxn} + C_{oxp}\right]$$

$C_{oxn,p} = WL\, C'_{ox}$

Scribe lines → Ring oscillator

$$R = \frac{1}{KP\left(\frac{W}{L}\right)\left(V_{DD}-V_{THN}\right)}$$

Need to make a VCO from oscillator

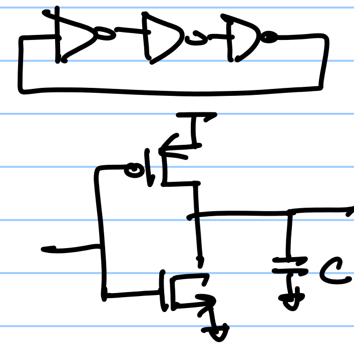
Control variable

↳ $V_{DD}\uparrow \Rightarrow R\downarrow \Rightarrow f_o\uparrow$

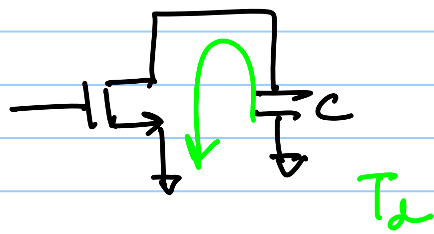
↳ vary R

↳ vary C

Vary charging/discharging current

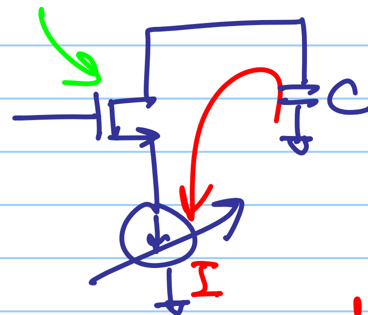


$$f_{osc} = \frac{1}{2n\times 0.7RC}, \quad R = \frac{1}{KP\left(\frac{W}{L}\right)\left(V_{DD}-V_{THN}\right)}$$



discharge rate $\propto \frac{1}{RC}$

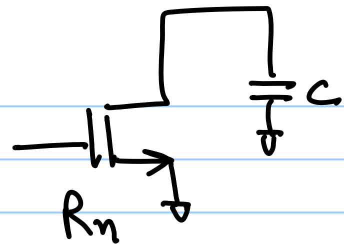
$\Rightarrow$

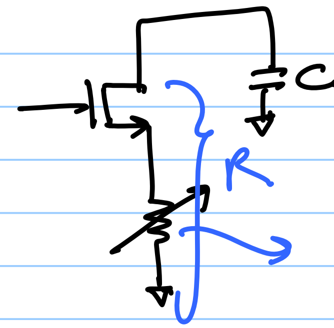


discharge rate

$\frac{dV_{out}}{dt} \propto \frac{I}{C}$

$f(V_c)$





Voltage controlled resistor

↳ trioded transistor