Homework 1

ECE 4/504 − PLL and High-Speed Link Design

Note: *These problems are meant to refresh your signals and circuits concepts. These concepts will be repeatedly used in phase-locked loop design. If you have difficulty with this material please contact the instructor.*

Problem 1- Signals and Systems Review:

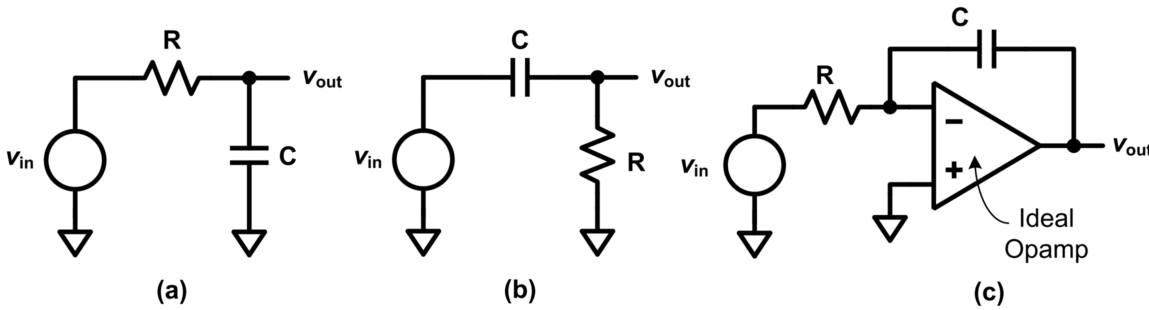
1. Write the Laplace transforms for the following signals. Here, V_p is a constant voltage, $u(t)$ is the step function, and $x(t)$ is an LTI signal (or system) with its Laplace transform given by $X(s)$. S*tandard notations apply.*

Time domain	**Laplace domain**
$x(t)$	$X(s)$
$V_p u(t)$	
$V_p e^{-\alpha t} u(t)$	
$t \cdot u(t)$	
$t^2 \cdot u(t)$	
$\int_0^t x(\tau) d\tau$	
$\frac{dx(t)}{dt}$	
$e^{-\alpha t} x(t)$	
$sin(\alpha t) \cdot u(t)$	
$cos(\alpha t) \cdot u(t)$	
$x_1(t) + x_2(t)$	
$x_1(t) \otimes x_2(t)$	
Initial value: $x(0^+)$	
Final value: $x(\infty)$	

2. Sketch Bode plots (magnitude and phase) for the following transfer functions. Verify your plots using Matlab (you can use *bode* function). Also, find the pole and zero locations, and the phase at $0\,dB$ gain.
   (a) $H(s) = \frac{10(s+1)}{(s+10)(s+100)}$
   (b) $H(s) = \frac{10(s+10)}{(s+1)(s+100)}$
   (c) $H(s) = \frac{10(s+10)}{s^2(s+100)}$

3. A second-order system can be expressed by the transfer function, $H(s) = \frac{k_1 \omega_0^2}{(s^2 + s\frac{\omega_0}{Q} + \omega_0^2)}$. Recall that Q is called the quality factor and ω_0 is the natural frequency.
   (a) Show that the two poles are real if $Q < 0.5$ and complex conjugate when $Q > 0.5$.
   (b) Show that the low frequency magnitude, $|H(j0)| = k_1$ and the magnitude at $\omega = \omega_0$, $|H(j\omega_0)| = k_1 Q$.
   (c) Derive the peak frequency, ω_{pk}, where the magnitude peaks (i.e. $\frac{d|H(j\omega)|^2}{d\omega}$=0).
   (d) Find the Q, pole locations, and ω_{pk} for the following transfer functions. Hand sketch and generate Bode plots using Matlab.
      i. $H(s) = \frac{10^4}{(s^2+1001s+1000)}$
      ii. $H(s) = \frac{100}{(s^2+11s+10)}$
      iii. $H(s) = \frac{100}{(s^2+5s+100)}$

Problem 2- RC Circuits Review:



Here $R = 1k\Omega$ and $C = 10nF$.

1. Find Laplace transfer functions $H(s) = \frac{v_{out}(s)}{v_{in}(s)}$ for the above circuits and sketch the Bode plots (mag and phase).
2. An input $v_{in} = V_p u(t)$ is applied to the above circuits. Express the output $V_{out}(s)$ in Laplace domain.
3. Using the partial fraction and inverse Laplace transform method (refresh from your signals/circuits textbook), find the time-domain output, $v_{out}(t)$. Show the steps.
4. An input $v_{in}(t) = sin(2\pi \cdot 10^5 t)$ is applied to the RC circuit in **(a)**. Using Laplace methods, find and plot the transient, steady-state and total outputs.