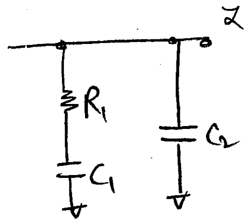
From H. Rategh's JSSC 2000 paper



$I_o \rightarrow$ CP current

$N \rightarrow$ divider ratio

$$Z(s) = \left(\frac{b}{b+1}\right) \cdot \frac{(s/\omega_z + 1)}{sC_1\left(\frac{s}{(b+1)\omega_z} + 1\right)}$$

, where $b = \frac{C_1}{C_2}$ & $\omega_z = \frac{1}{R_1 C_1}$

$$\Rightarrow \omega_{p2} = \frac{1}{R_1 C_1 \frac{C_2}{C_1+C_2}} = \frac{1}{R_1 C_1} \cdot \left(\frac{C_1+C_2}{C_2}\right)$$

$$= \omega_z \cdot (b+1)$$

* for a 3rd-order Type-II PLL

$$L(s) = \frac{K_{VCO} \cdot I_o}{N} \cdot \left(\frac{b}{b+1}\right) \cdot \frac{(s/\omega_z + 1)}{s^2 C_1\left(\frac{s}{\omega_z(b+1)} + 1\right)} \rightarrow (1)$$

$\Rightarrow$ Phase Margin of the loop:

$$PM = \tan^{-1}\left(\frac{\omega_{u,loop}}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega_{u,loop}}{\omega_{p2}}\right)$$

$$= \tan^{-1}\left(\frac{\omega_{u,loop}}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega_{u,loop}}{(b+1)\omega_z}\right)$$

$\Rightarrow$ $\frac{\partial PM}{\partial \omega_{u,loop}} = 0$ for PM maxima

$\Rightarrow$ $PM = PM_{max}$ for $\omega_{u,loop} = (\sqrt{b+1})\,\omega_z$

$$\Rightarrow \boxed{PM_{max} = \tan^{-1}(\sqrt{b+1}) - \tan^{-1}\left(\frac{1}{\sqrt{b+1}}\right)} = f(b) \longrightarrow (2)$$

PM_{max} is a function of $b = \frac{C_1}{C_2}$

for $b < 1 \Rightarrow PM < 20^\circ$

$\Rightarrow$ unstable loop.

force $\boxed{\omega_{u,loop} = \sqrt{b+1} \cdot \omega_z}$ for maximum PM $\longrightarrow$ (3)

From (1), we have

$$\omega_{u,loop} \simeq \frac{K_{VCO} \cdot I_o}{N} \cdot \left(\frac{b}{b+1}\right) \cdot \frac{1}{C_1} \longrightarrow (4)$$

from (3) & (4)

$$\Rightarrow \omega_{u,loop} = \frac{K_{VCO} \cdot I_o}{N} \left(\frac{b}{b+1}\right) \frac{1}{C_1} \cdot \frac{1}{\omega_z} = \sqrt{b+1} \cdot \omega_z$$

$$\Rightarrow \boxed{\frac{K_{VCO} \cdot I_o}{N} = \frac{(b+1)^{3/2}}{b} C_1 \cdot \omega_z^2} \longrightarrow (4)$$

from (2)

$$\Rightarrow PM_{max} = \boxed{\phi_M = \tan^{-1}\left(\frac{b}{2\sqrt{1+b}}\right)} \longrightarrow (5)$$

using $\tan^{-1}A - \tan^{-1}B = \tan^{-1}\left(\frac{A-B}{1-AB}\right)$

or $\boxed{b = 2\left(\tan^2\phi_M + \tan\phi_M \sqrt{1+\tan^2\phi_M}\right)} \longrightarrow (6)$

Also $\zeta = \frac{1}{2}\left(\frac{C_1}{C_1 \| C_2}\right)^{1/4} \longrightarrow (7)$

$\zeta \simeq 0.783$ for $b = \frac{C_1}{C_2} = 5$

Design Procedure:

① find K_{vco} from simulation (units $\frac{Hz}{V}$)

② choose a desired phase margin (ϕ_M) and find b from Eq.6

③ choose the loop bandwidth and find ω_z from Eq.3

④ Select C_1 & I_o such that Eq. 4 is satisfied

⑤ Calculate noise contribution from R_1. If design is "noisy" then goto ④ and increase C_1.

* see the paper for a 4^{th}-order loop analysis.