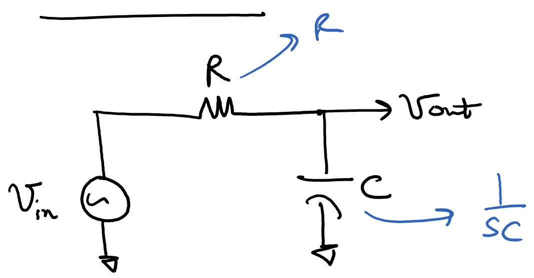
ECE 445 - Lecture 2

Monday, January 14, 2019 8:01 AM

RC Circuits:



$$\frac{V_{out}}{V_{in}}(s) = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{1+sRC}$$

1st-order Transfer function

$\tau = RC$

$\omega_{3dB} = \frac{1}{\tau}$ ← 3dB Bandwidth

Frequency Response:

$$H(s) = \frac{1}{1+sRC} = \frac{1}{1+\frac{s}{\omega_{3dB}}}$$

Single-pole

$s \leftarrow j\omega$

$$H(j\omega) = \frac{1}{1+j\frac{\omega}{\omega_{3dB}}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_{3dB}}\right)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_{3dB}}\right)$$

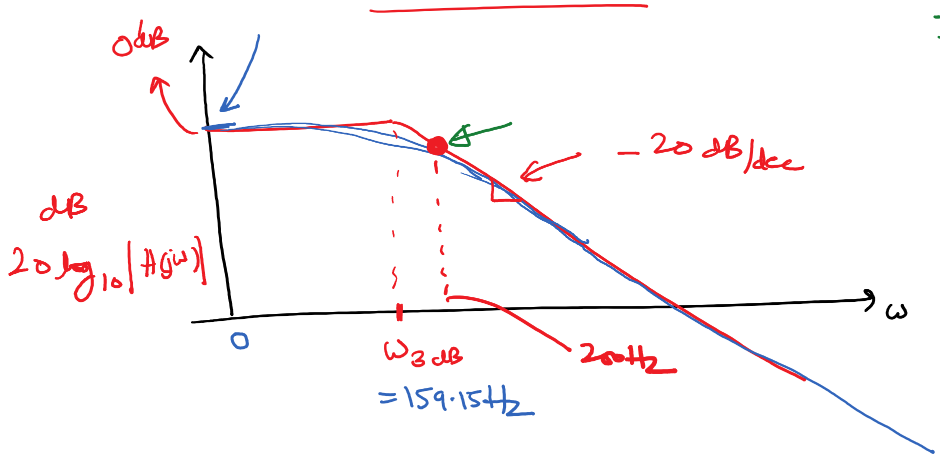
$$H(j\omega) = \frac{1}{\left(1+j\frac{\omega}{\omega_{3dB}}\right)} \cdot \frac{\left(1-j\frac{\omega}{\omega_{3dB}}\right)}{\left(1-j\frac{\omega}{\omega_{3dB}}\right)}$$

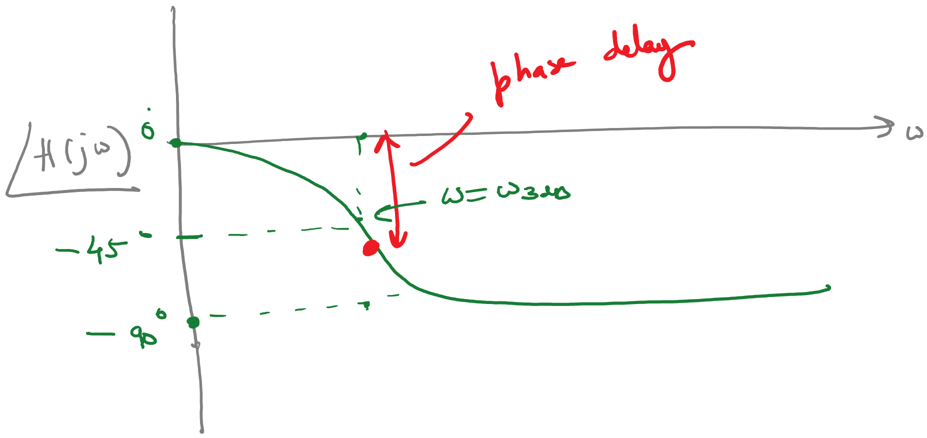
$$= \frac{\left(1-j\frac{\omega}{\omega_{3dB}}\right)}{1+\left(\frac{\omega^2}{\omega_{3dB}}\right)^2}$$

$$\angle = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-\frac{\omega}{\omega_{3dB}}}{1}\right)$$

$$= \tan^{-1}\left(\frac{-\omega}{\omega_{3dB}}\right) = -\tan^{-1}\left(\frac{\omega}{\omega_{3dB}}\right)$$

Bode Plots





$$-\tan^{-1}\left(\frac{\omega}{\omega_{3dB}}\right)$$

Let $R = 1k\Omega$, $C = 1\mu F$

$$\omega_{3dB} = \frac{1}{RC} \quad \leftarrow \quad \text{rad/s}$$

$$f_{3dB} = \frac{\omega_{3dB}}{2\pi} = \frac{1}{2\pi RC} \quad \leftarrow \quad \text{Hz}$$

$$= 159.155 \text{ Hz}$$

$\omega \to 2\pi f$

$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{\omega_{3dB}}}$$

$$H(f) = \frac{1}{1 + j\frac{f}{f_{3dB}}}$$

$f_{in} = 200$ Hz

Amplitude $= 1V$

$$V_{in} = 1 \cdot \sin(2\pi f_{in} t) \longrightarrow \boxed{H(s)} \longrightarrow V_{out}$$

$$= |H(f_{in})| \cdot \sin\left(2\pi f_{in} t + \angle H(f_{in})\right)$$

$$|H(f_{in})| = \frac{1}{\sqrt{1 + \left(\frac{f_{in}}{f_{3dB}}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{200}{159.15}\right)^2}} = \underline{0.623}$$

$-$ve sign shows lag

$$\angle H(f_{in}) = -\tan^{-1}\left(\frac{200}{159.15}\right) = \underline{-0.898 \text{ rad}}$$

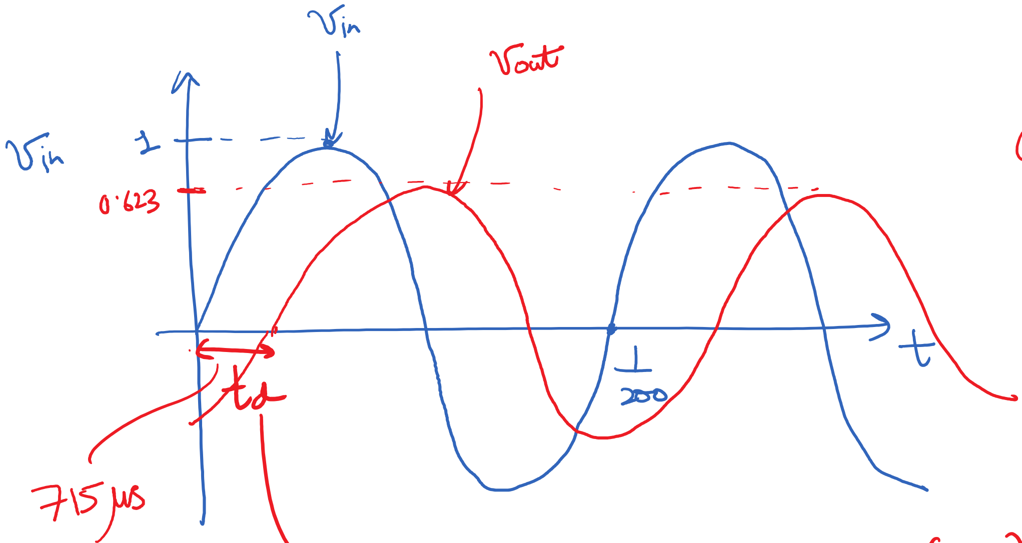
$$\angle H(f_{in}) = -\tan^{-1}\left(\frac{200}{159.15}\right) = -0.898 \text{ rad}$$

$\Rightarrow$ output is lagging the input by 0.898 rad

$$\hookrightarrow \left(\frac{0.898}{\pi}\right)180^\circ = \underline{\underline{51.5^\circ}}$$

lag

$$\left(\frac{\text{radians}}{\pi}\right) \times 180^\circ = \text{degrees}$$

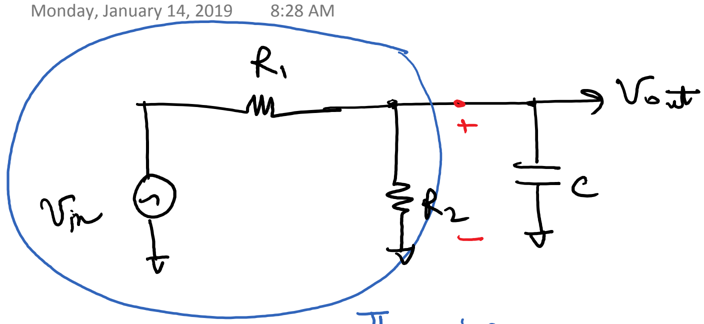


output lag the input

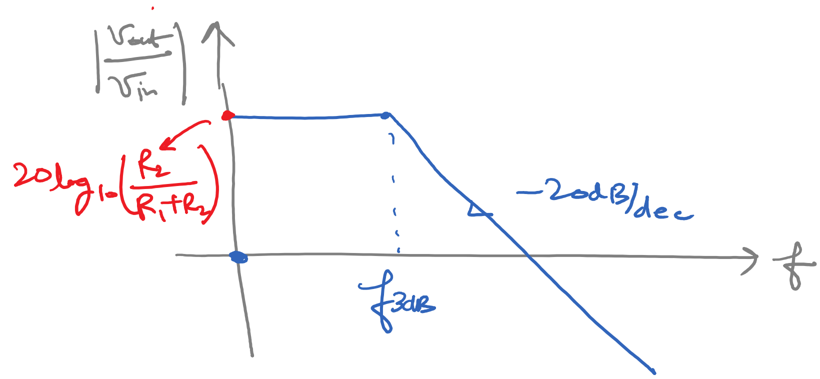
$$\text{time delay, } t_d = \left(\frac{\Delta\theta}{360^\circ}\right) T$$

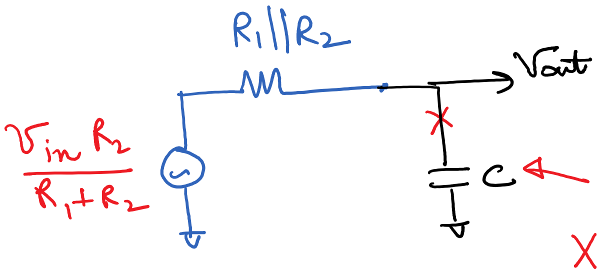
$$= \frac{-51.5^\circ}{360^\circ} \times \left(\frac{1}{200 \text{ Hz}}\right)$$

$$= -715 \, \mu s$$



Thevenize

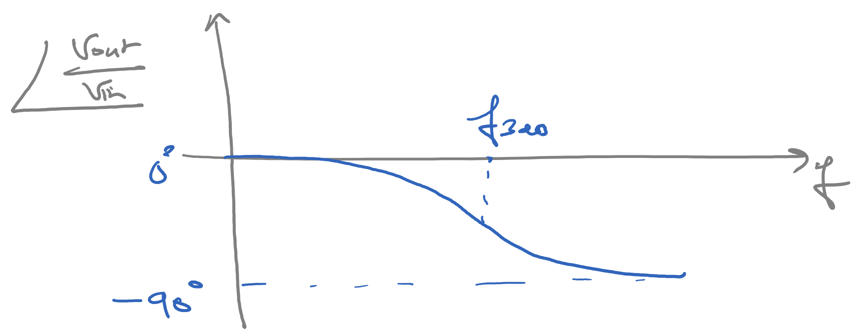




$X_c = \frac{1}{j\omega C} \longrightarrow \infty$ at DC

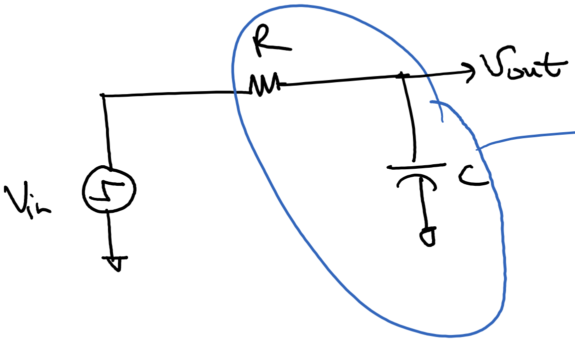
Capacitor is open at DC

$$f_{3dB} = \frac{1}{2\pi (R_1 || R_2) C}$$

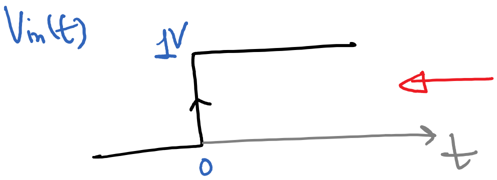


Transient Response of an RC Circuit:

plot V_{out}



$$H(s) = \frac{1}{1+\frac{s}{\omega_{3dB}}} = \frac{1}{1+s\tau}$$



$$v_{in}(t) = 1 \cdot u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}$$

Step function

$$V_{out}(s) = H(s) \cdot V_{in}(s)$$

$$= \frac{1}{\left(1+\frac{s}{\omega_{3dB}}\right)} \cdot \frac{1}{s}$$

partial fraction

$$= \frac{1}{s} - \frac{1/\omega_{3dB}}{\left(1+s/\omega_{3dB}\right)}$$

$$= \frac{1}{s} - \frac{1}{s+\omega_{3dB}}$$

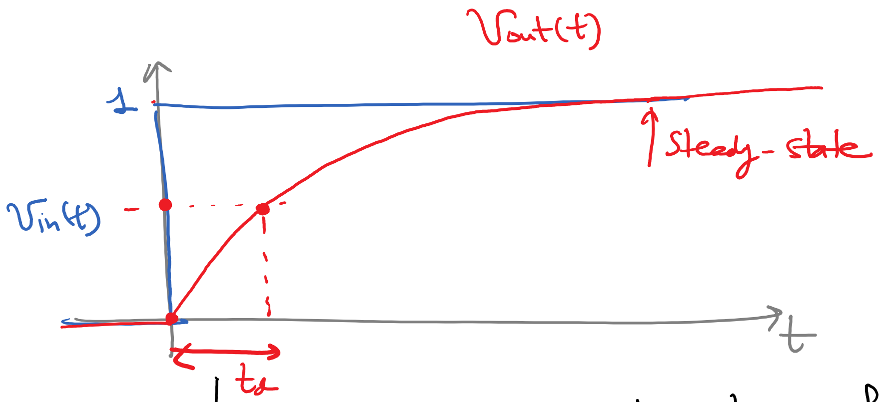
go back to the time domain

$$v_{out}(t) = u(t) - e^{-\omega_{3dB}t} \cdot u(t)$$

$$V_{out}(t) = u(t) - e^{-\omega_{3dB} t} \cdot u(t)$$

$$= \left(1 - e^{-\omega_{3dB} t}\right) u(t) \triangleq \left(1 - e^{-t/\tau}\right) \cdot u(t)$$

$$V_{out}(t) = 1 \cdot \left(1 - e^{-t/\tau}\right) u(t) \leftarrow$$



$\hookrightarrow$ 50% delay $\rightarrow$ time taken for the output to reach 50% of its final value.

$\hookrightarrow$ propagation delay, t_d

$$V_{out}(t) = V_o \left(1 - e^{-t/\tau}\right) u(t)$$

$$@\ t = t_d \Rightarrow \quad V_{out}(t_d) = \frac{V_o}{2}$$

$$\frac{\cancel{V_o}}{2} = \cancel{V_o}\left(1 - e^{-t_d/\tau}\right)$$

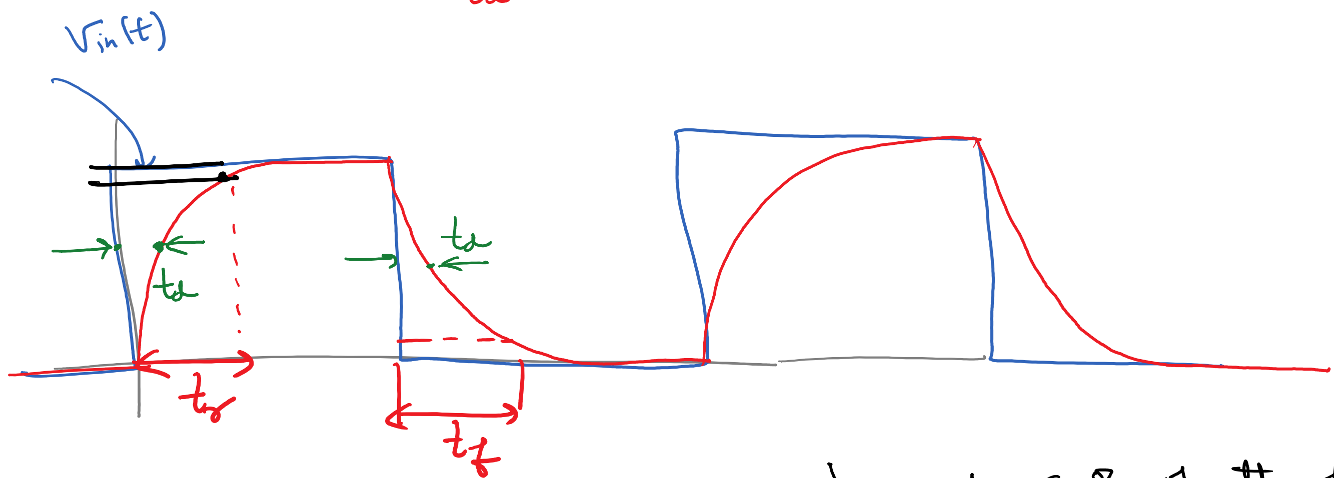
$$\Rightarrow \quad e^{-t_d/\tau} = \frac{1}{2}$$

$$\Rightarrow \quad -\frac{t_d}{\tau} = \ln\left(\frac{1}{2}\right) = -\ln(2)$$

$$\frac{t_d}{\tau} = \ln(2)$$

$$\Rightarrow \quad \boxed{t_d = \tau \ln(2)} \qquad \ln(2) = 0.7$$

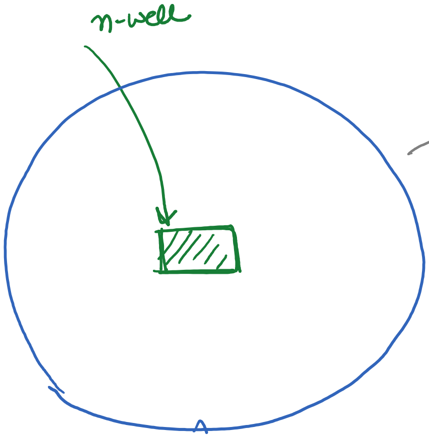
$$t_d = 0.7\,\tau = \boxed{0.7RC}$$



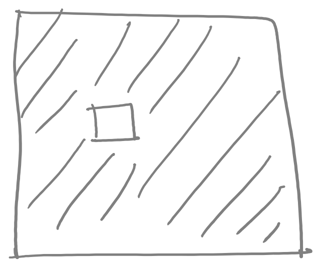
90 % rise time $\Rightarrow$ time taken to reach 90% of the final output

$$t_r = 2.2\,\tau = 2.2RC$$

Chapter 2: N-well:



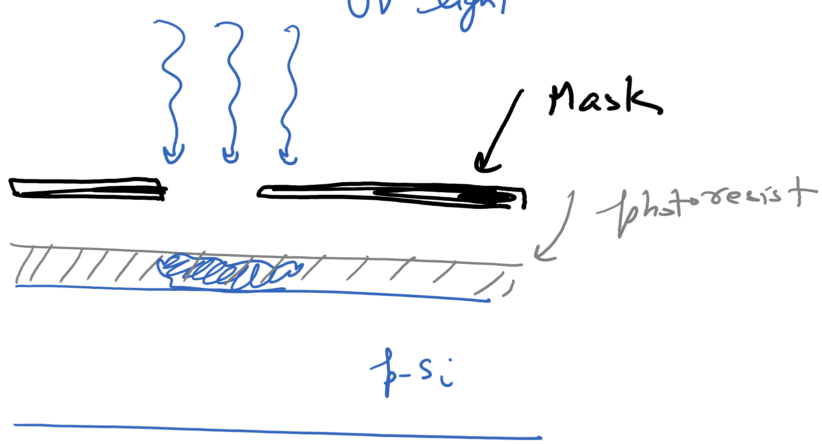
photomask



dark field

p- doped Silicon (p-Si)

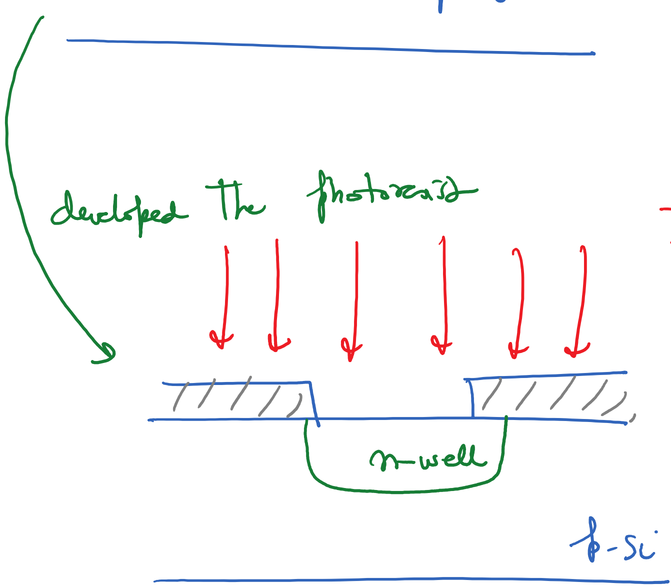
Photolithography



photoresist

developed the photoresist

Ion Implant



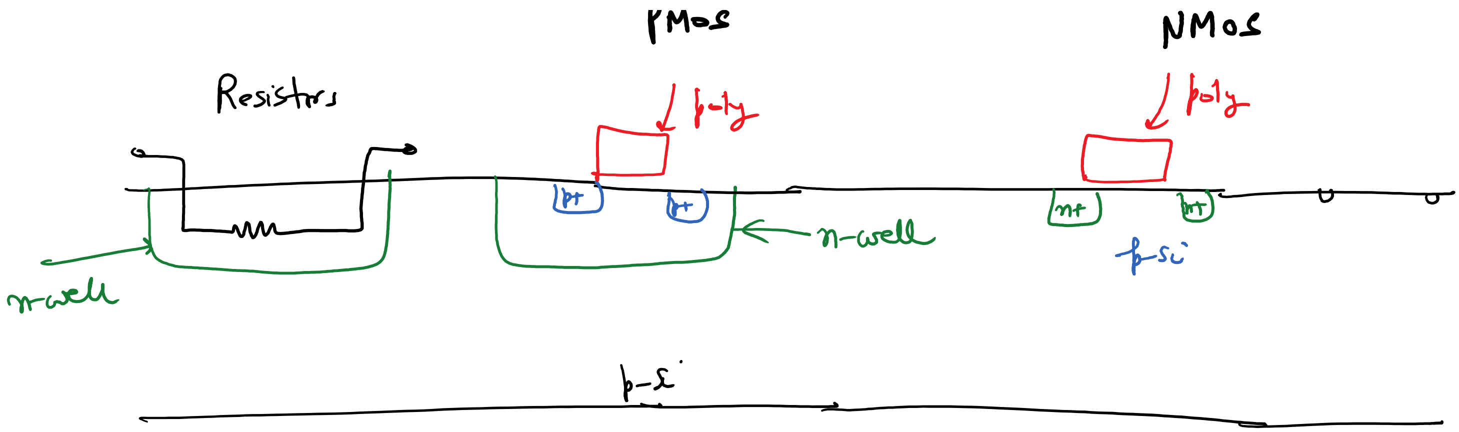
Implant with n-type dopants

$\Rightarrow$ P, As

PMOS

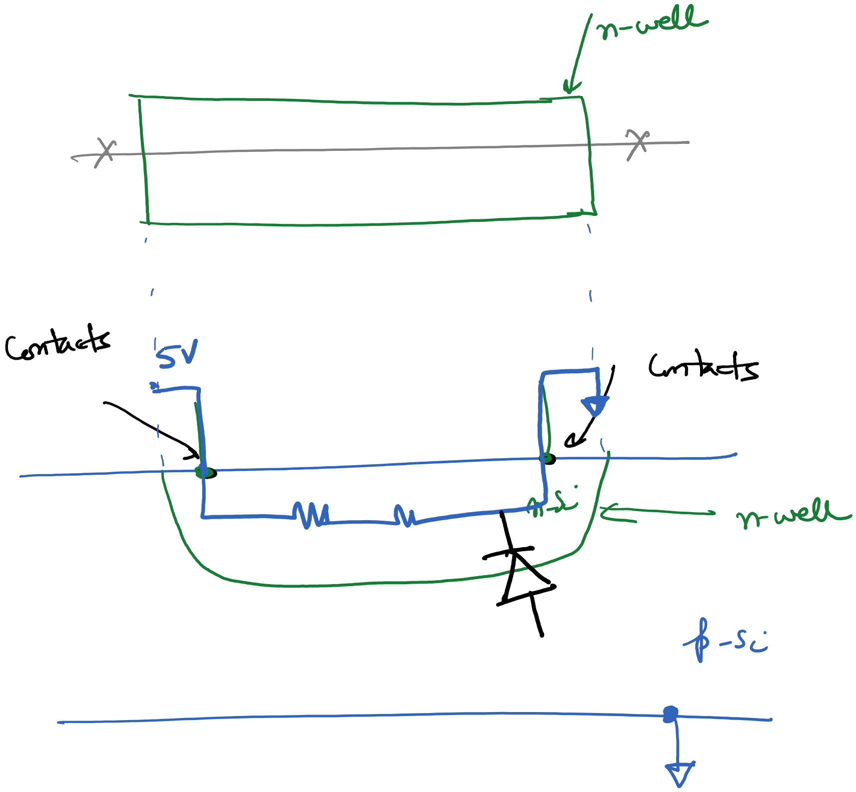
NMOS

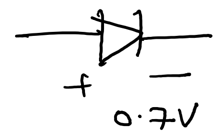
/ poly



N-well $\longrightarrow$ fabrication of p-channel MOSFETs (PMOS)

n-well resistors

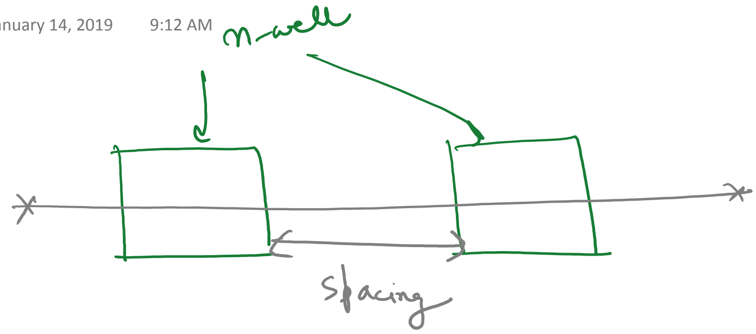


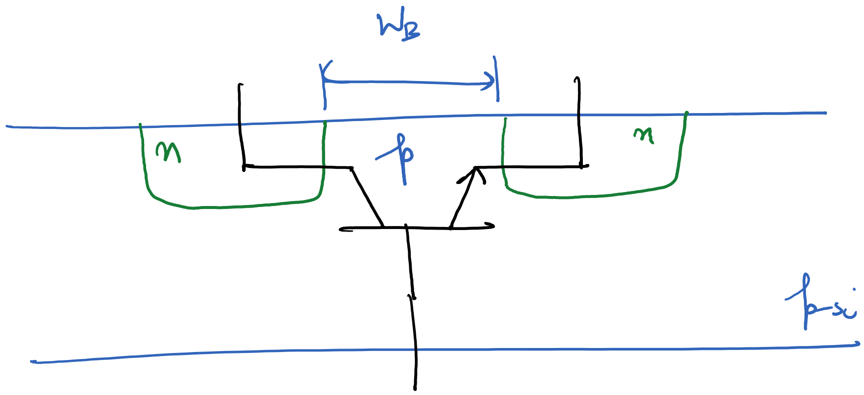


n-well diode turns on only for $V_A < -0.7V$

In CMOS technology ⇒ "typically" positive voltages are employed and the p-substrate is connected to the lowest potential on the chip i.e. ground.

Monday, January 14, 2019 9:12 AM





* parasitic npn BJT is formed