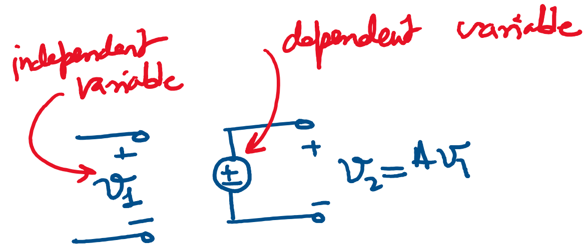
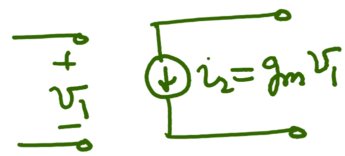
# Dependent Sources

voltage controlled voltage source
VCVS

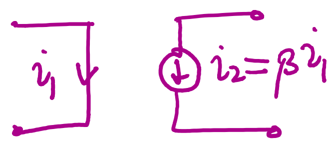



VCCS

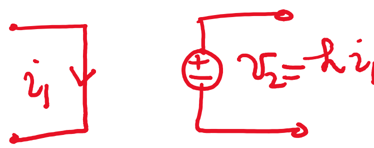



two-port components

CCCS




CCVS



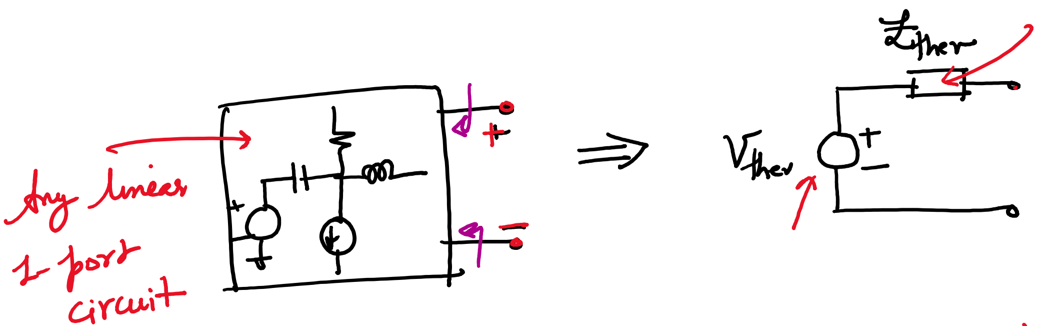

Dependent sources can model the microelectronic devices.

# Thevenin and Norton Equivalents

* Thevenin & Norton theorems greatly simplify algebra and provide insight into the circuit

## Thevenin's Theorem:

A linear one-port network and replace by an equivalent circuit consisting of one voltage source in series with an impedance




* $V_{thev}$ is obtained by leaving the output open and calculating the open circuit voltage at this port.

* $Z_{thev}$ is determined by setting ALL independent voltage and current sources to zero and then calculate the impedance between the two nodes.
  - → Short ALL independent voltage sources
  - → Open ALL independent current sources

* Be careful with dependent sources

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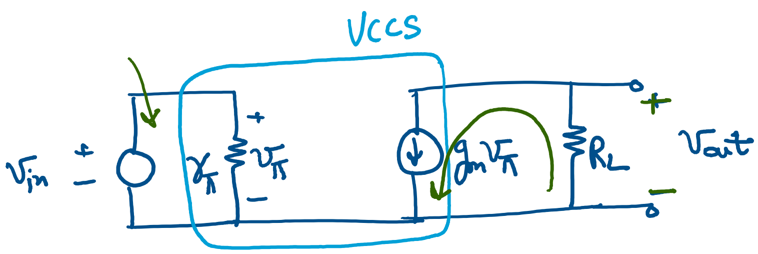
Ex.



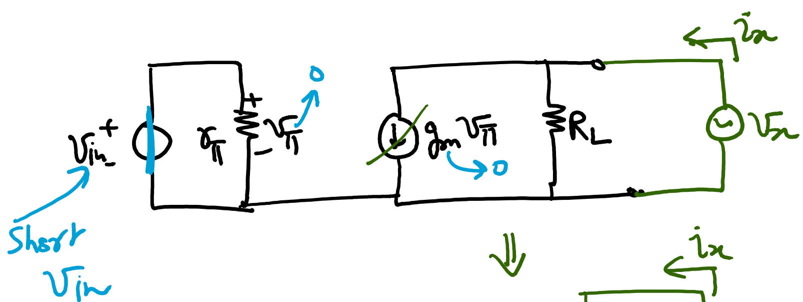

* $V_{thev}$ ⇒ open circuit voltage at the output

$$v_{out} = -g_m v_\pi R_L$$

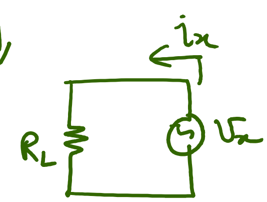
$$v_\pi = v_{in}$$

$$\Rightarrow \boxed{V_{thev} = -g_m v_{in} R_L}$$

* for $Z_{thev}$ calculation we set $v_{in} = 0$ and apply a test voltage source $v_x$ at the output port
  ↳ determine the current, $i_x$ ⇒ $Z_{thev} = \dfrac{v_x}{i_x}$

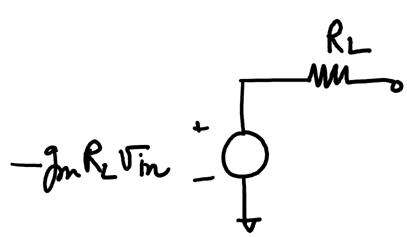



⇓




$$\Rightarrow \frac{v_x}{i_x} = R_L$$

$$Z_{thev} = R_{thev} = R_L$$




Thevenin Equivalent Circuit

Exercise ⇒ Repeat Ex 1.9 & 1.10

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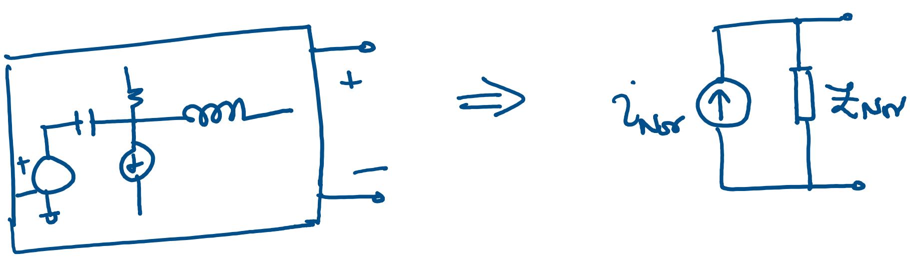
# Norton's Theorem :

Norton Equivalent Circuit




* $Z_{Nor} \equiv R_{Nor}$ is calculated as the same way as the $Z_{thev}$

↳ set ALL indep sources to zero and calculate the impedance at the output port.

* The equivalent current, $i_{Nor}$, is obtained by shorting the output port and computing the "short circuit" current that flows through it.

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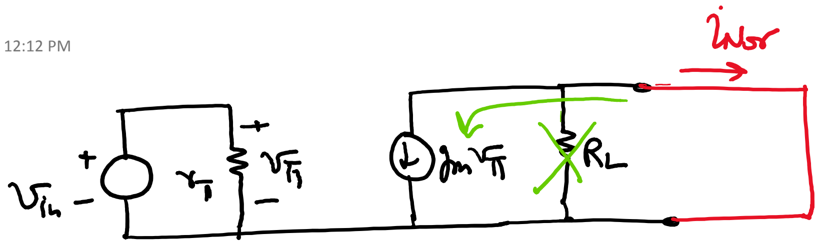
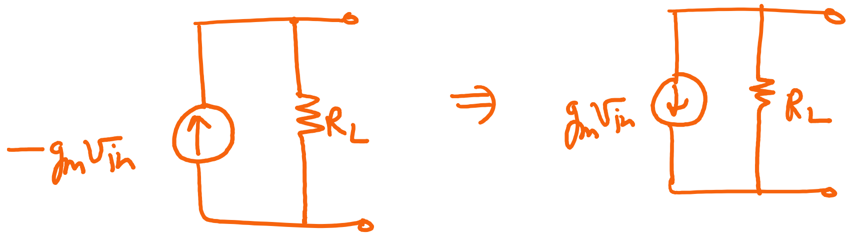
Ex.




short circuit

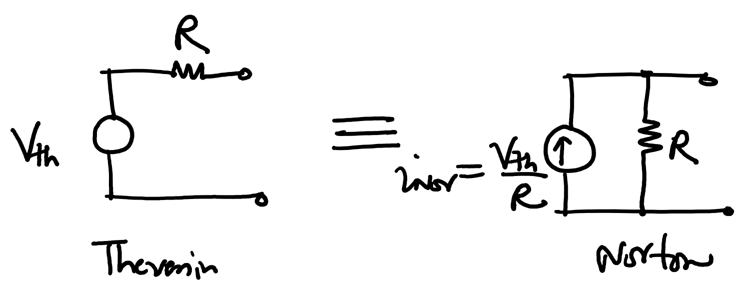
$$i_{Nor} = -g_m v_\pi = -g_m v_{in}$$

$$Z_{Nor} = Z_{Thev} = R_L$$



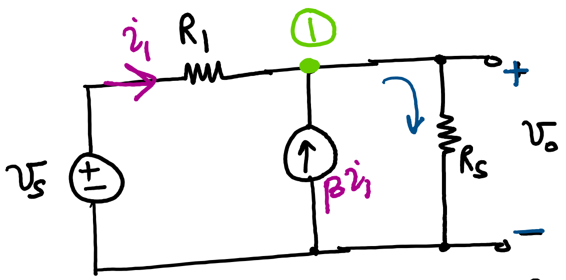

← Norton Equivalent Circuit.

$$V_{thev} = i_{Nor} \cdot Z_{thev} = i_{Nor} \cdot Z_{Nor}$$

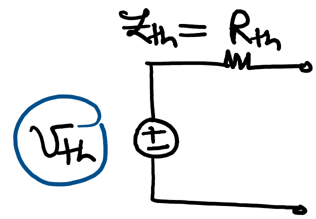



See Ex. 1.12

Ex.




**Find Thevenin Equivalent circuit**




$\beta = 50$, $R_1 = 20\,k\Omega$, $R_s = 1\,k\Omega$

$V_{th} \Rightarrow$ open circuit voltage at the output terminals

Apply KCL at node 1

$$\beta i_1 + i_1 - \frac{v_o}{R_s} = 0 \longrightarrow (1)$$

$$i_1 = \frac{v_s - v_o}{R_1} \longrightarrow (2)$$

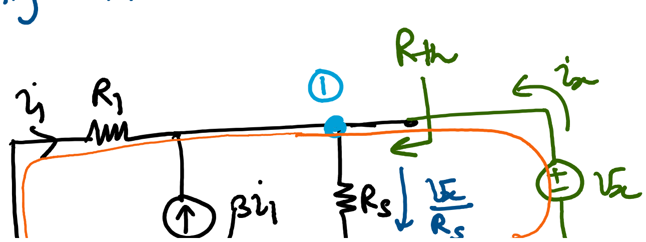
Eliminate $i_1$ in Eq. (1) and (2):

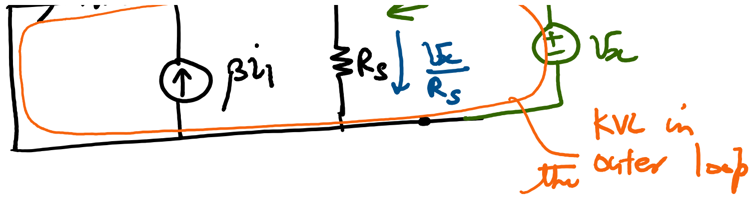
$$(\beta+1)\left(\frac{v_s - v_o}{R_1}\right) - \frac{v_o}{R_s} = 0$$

$$\Rightarrow \frac{(\beta+1)v_s}{R_1} = \left[\frac{\beta+1}{R_1} + \frac{1}{R_s}\right] v_o$$

$$\Rightarrow \boxed{v_{th} = v_o = \frac{(\beta+1)R_s}{(\beta+1)R_s + R_1} \cdot v_s} = 0.718\, v_s$$

for calculating $R_{th} \Rightarrow$ short indep voltage sources






Apply KCL at ①:

$$i_x + i_1 + \beta i_1 - \frac{V_x}{R_s} = 0$$

$$\Rightarrow \quad i_x + (\beta+1)i_1 - \frac{V_x}{R_s} = 0 \longrightarrow ①$$

Need to eliminate $i_1$

Taking KVL in the outer loop we get

$$-V_x - i_1 R_1 = 0$$

$$\Rightarrow \quad i_1 = -\frac{V_x}{R_1} \longrightarrow ②$$

from Eq. ①&② we get,

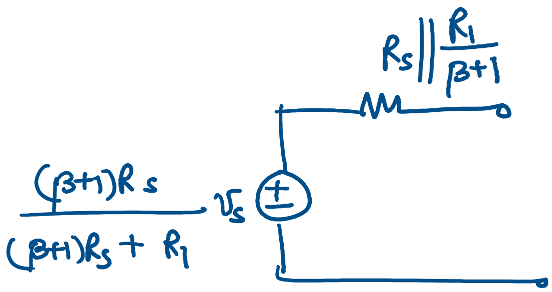
$$i_x - (\beta+1)\frac{V_x}{R_1} - \frac{V_x}{R_1} = 0$$

$$\Rightarrow \quad i_x = V_x\left[\frac{\beta+1}{R_1} + \frac{1}{R_s}\right]$$

$$\Rightarrow \quad R_{th} = \frac{V_x}{i_x} = \frac{1}{\frac{\beta+1}{R_1} + \frac{1}{R_s}} = \boxed{\frac{R_1 R_s}{(\beta+1)R_s + R_1}}$$

$$= \frac{\frac{R_1}{\beta+1} \times R_s}{R_s + \frac{R_1}{\beta+1}} = R_s \,\Big|\Big|\, \frac{R_1}{\beta+1}$$

$$\boxed{R_{th} = R_s \,\Big|\Big|\, \frac{R_1}{\beta+1}} = 282\Omega$$


$R_s \| \frac{R_1}{\beta+1}$
$\frac{(\beta+1)R_s}{(\beta+1)R_s + R_1} v_s$
±

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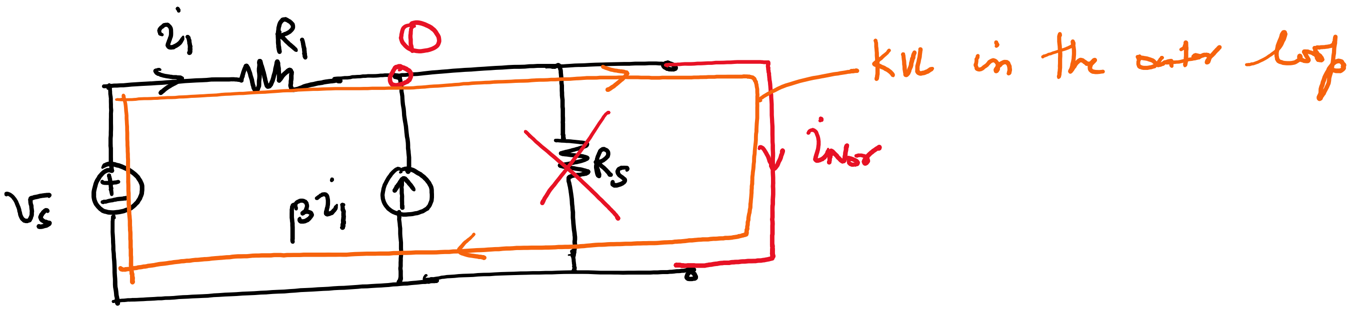
# Norton Method

$$R_{nor} \equiv R_{thev} = R_S \,\|\, \frac{R_1}{(\beta+1)}$$

Find $i_{Nor}$

$\Rightarrow$ Short-circuit output current




KCL at node 1

$$-i_{Nor} + \beta i_1 + i_1 = 0$$

$$\Rightarrow \quad i_{Nor} = (\beta+1)\, i_1 \longrightarrow (1)$$

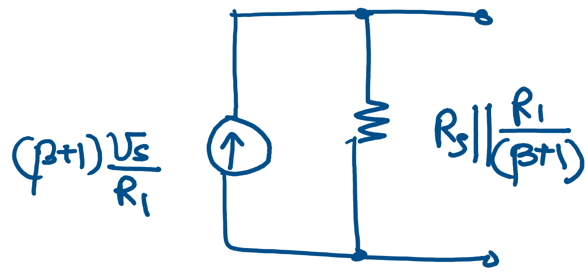
Need to find $i_1$

KVL in the outer loop

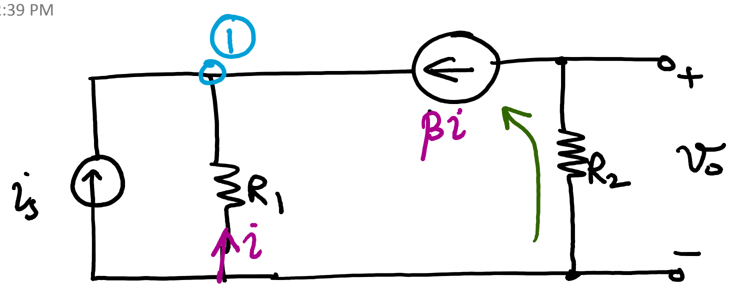
$$-v_S + i_1 R_1 = 0$$

$$\Rightarrow \quad i_1 = \frac{v_S}{R_1} \longrightarrow (2)$$

from (1) & (2) $\Rightarrow$ $\boxed{i_{Nor} = (\beta+1)\frac{v_S}{R_1}} = 2.55\,mS$


(β+1) Vs / R1
Rs || R1 / (β+1)

Ex.3




Open circuit voltage at the output

$$v_o = -\beta i R_2 \longrightarrow (1)$$

KCL at node (1)

$$i_s + i + \beta i = 0$$

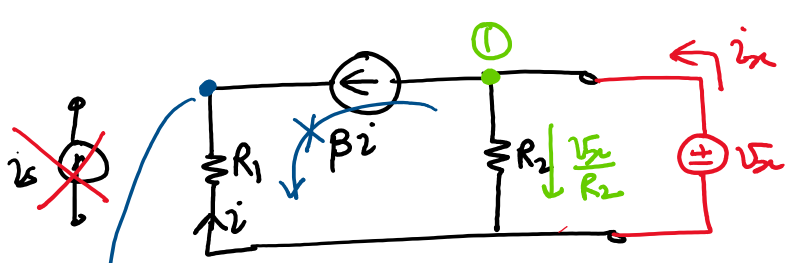
$$\Rightarrow \quad i = -\frac{i_s}{(\beta+1)} \longrightarrow (2)$$

from (1) & (2) we get

$$v_o = \frac{\beta R_2}{\beta+1} \cdot i_s \quad \Rightarrow \quad \boxed{V_{th} = \left(\frac{\beta}{\beta+1}\right) i_s R_2}$$

for $R_{th}$, open "$i_s$" $\Rightarrow$ independent current source

$$R_{th} = \frac{v_x}{i_x}$$




We have $\quad i = \beta i$

$$\Rightarrow \quad i = 0$$

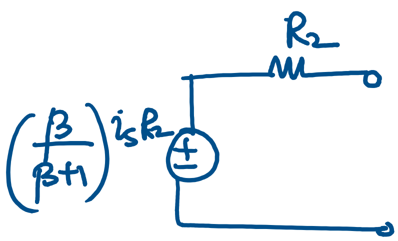
we have

$\Rightarrow i = 0$

KCL at ① $\Rightarrow$ $i_x - \dfrac{v_x}{R_2} - \cancel{\beta i}^{\,0} = 0$

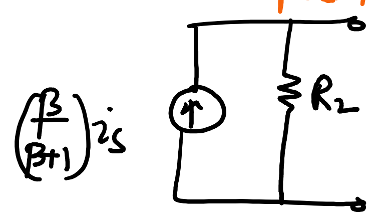
$\Rightarrow \dfrac{v_x}{i_x} = R_2$

$\Rightarrow \boxed{R_{th} = R_2}$




$\equiv$

Norton Equiv. Circuit.




Thevenin Equiv. Circuit

$i_{nor} = \dfrac{v_{th}}{R_{th}} = \left(\dfrac{\beta}{\beta+1}\right) i_s$