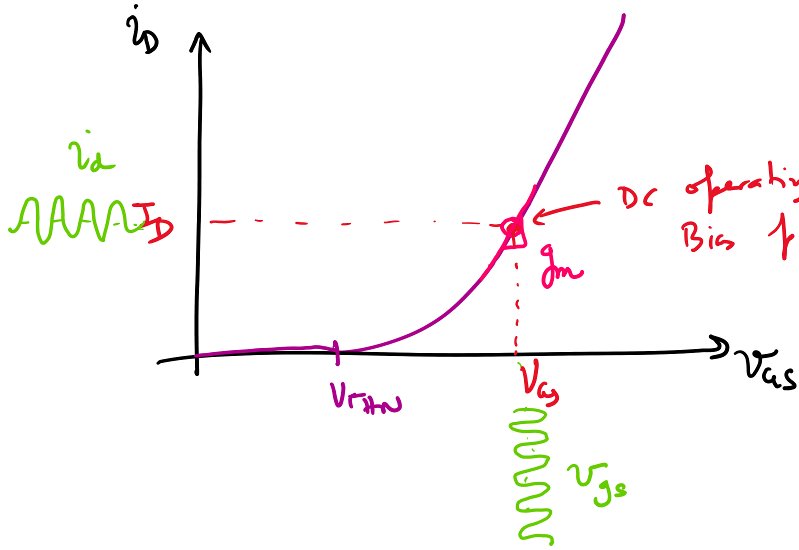
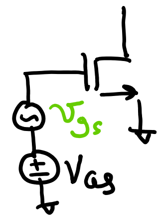
ECE 310 - Lecture 21

Monday, March 5, 2018 10:28 AM

MOS Transconductance





AC current
↓
$i_d = g_m v_{gs}$ ← AC voltage → small-signal

$g_m \Rightarrow$ slope of $I_D - V_{GS}$ curve at the DC bias point
↳ valid in SATURATION Region

ignore CLM
$\lambda = 0$

$$i_D = \frac{1}{2} KP_n \frac{W}{L} (v_{GS} - V_{THN})^2$$

$$g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{v_{GS} = V_{GS}} = \frac{1}{2} KP_n \frac{W}{L} \cdot 2 (V_{GS} - V_{THN})$$

$$\boxed{g_m = KP_n \frac{W}{L} (V_{GS} - V_{THN})} \rightarrow ①$$

$= \beta V_{ov}$

* $g_m \propto \frac{W}{L}$
* $g_m \propto (V_{GS} - V_{THN})$ for fixed W/L

$V_{ov} = V_{GS} - V_{THN}$ overdrive voltage

↓
$g_m \propto KP_n \frac{W}{L} V_{ov}$

square law equation
$I_D = \frac{1}{2} KP_n \frac{W}{L} (V_{GS} - V_{THN})^2$

$$g_m = Kp_n \frac{W}{L}(V_{GS} - V_{THN})$$

$$= Kp_n \frac{W}{L} \cdot \sqrt{\frac{2I_D}{Kp_n W/L}}$$

$$g_m = \sqrt{2 Kp_n \frac{W}{L} \cdot I_D} \rightarrow (2)$$

$$= \sqrt{2\beta_n I_D}$$

$$I_D = \frac{1}{2} Kp_n \frac{W}{L}(V_{GS} - V_{THN})^2$$

Recast

$$V_{GS} - V_{THN} = \sqrt{\frac{2I_D}{Kp_n W/L}} \rightarrow (A)$$

Beta $\beta_n = Kp_n \cdot \frac{W}{L}$

$$g_m = Kp_n \cdot \frac{W}{L} \cdot (V_{GS} - V_{THN})$$

$$\Rightarrow g_m = \frac{2I_D}{V_{GS} - V_{THN}} \rightarrow (3)$$

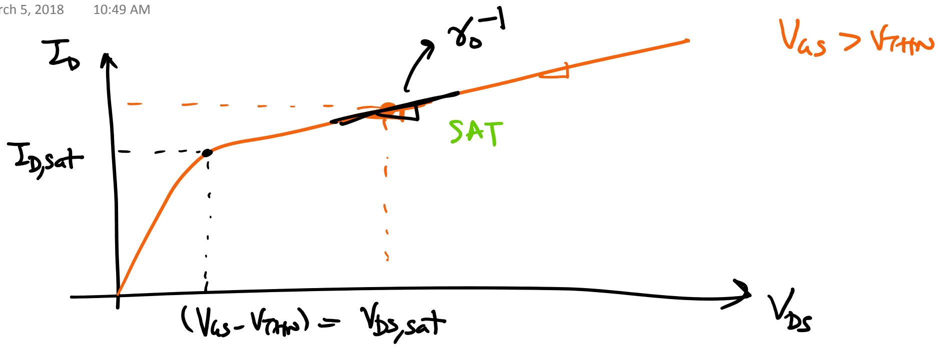
$$g_m = \frac{2I_D}{V_{ov}}$$

$$I_D = \frac{1}{2} Kp_n \frac{W}{L}(V_{GS} - V_{THN})^2$$

$$\Rightarrow Kp_n \frac{W}{L} = \frac{2I_D}{(V_{GS} - V_{THN})^2} \rightarrow (B)$$

Various forms of $g_m \leftarrow \{ \frac{W}{L}, V_{GS}-V_{THN}, I_D \}$

I_D is variable, W/L is constant, $V_{GS}-V_{THN}$ is variable	I_D is variable, W/L is variable, $(V_{GS}-V_{THN})$ is constant	I_D is constant, W/L & $V_{GS}-V_{THN}$ are variable
$g_m = \sqrt{2KP_n \frac{W}{L} I_D}$ $g_m = KP_n \frac{W}{L}(V_{GS}-V_{THN})$ $\rightarrow g_m \propto (V_{GS}-V_{THN})$	$g_m = \frac{2I_D \nearrow}{V_{GS}-V_{THN}}$	$g_m = \sqrt{2KP_n \frac{W}{L} I_D}$ $g_m \propto \sqrt{W/L}$ $g_m = \frac{2I_D}{V_{GS}-V_{THN}}$ $g_m \propto \frac{1}{(V_{GS}-V_{THN})}$



In SAT:

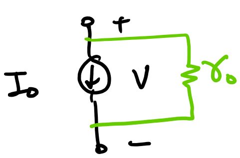
$$I_D = \underbrace{\frac{1}{2} k'_n \frac{W}{L} (V_{GS}-V_{THN})^2}_{I_{D,sat}} \cdot [1+\lambda V_{DS}]$$

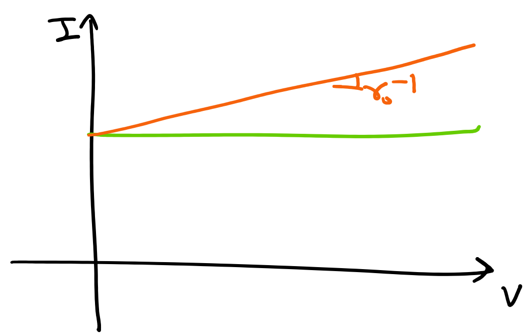
$[1+\lambda V_{DS}]$ → channel length modulation

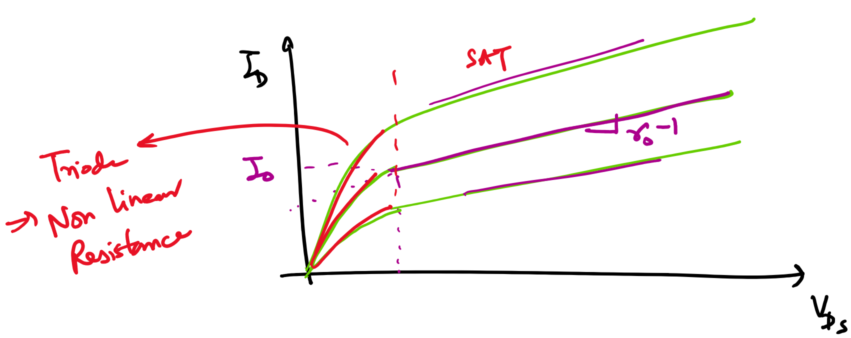
$$g_o = g_{ds} = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{v_{DS}=V_{DS}} = I_{D,sat}\cdot\lambda.$$

g_o: output conductance

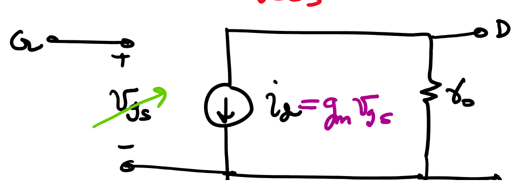
output resistance ⇒ $$r_o = \frac{1}{\lambda I_{D,sat}}$$



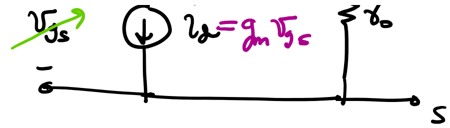




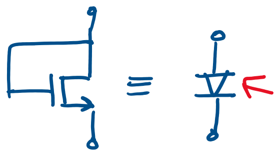
VCCS ✓

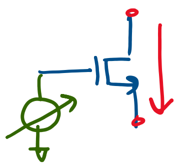


Voltage Controlled Current Source (VCCS) with an output resistance



(VCCS) with an output resistance





$$g_m = \begin{cases} \beta V_{ov} \\ \sqrt{2\beta I_D} \\ \dfrac{2 I_D}{V_{ov}} \end{cases}$$

$$r_o = \frac{1}{\lambda I_{D,sat}}$$

$$A_v = g_m r_o$$

← maximum small-signal gain we can get from the transistor (NMOS)

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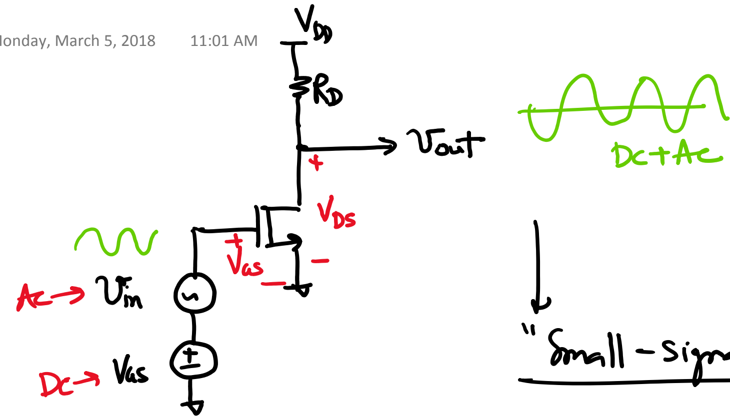

"Small-signal Analysis"

Procedure:

1) Look at the "DC picture (equivalent circuit)"
   - ↳ Solve for the DC operating (or bias) points
   - ↳ all AC voltages are shorted, all AC currents sources are opened

2) from DC analysis ⇒ small-signal parameters ⇒ g_m, r_o, ...

3) Analyze small-signal equivalent circuit (AC picture) of the circuit (linear circuit analyze).
   - ↳ All devices are replaced by the their small signal models
   - ↳ All indep. DC voltage sources are shorted & DC current sources are opened
   - ↳ obtain small signal gains, input / output resistance for the circuit

4) final voltages & currents are superposition of DC & AC quantities.

... disturb the DC bias conditions

* Small signal shouldn't disturb the DC bias conditions
  ↳ 10% variation could be a rule of thumb.

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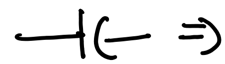
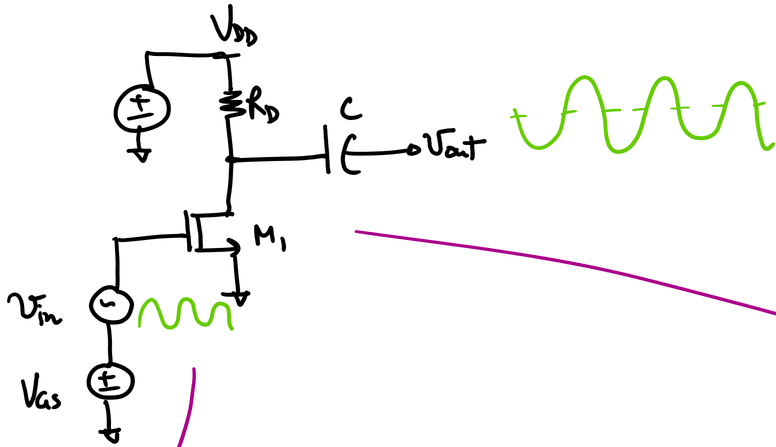
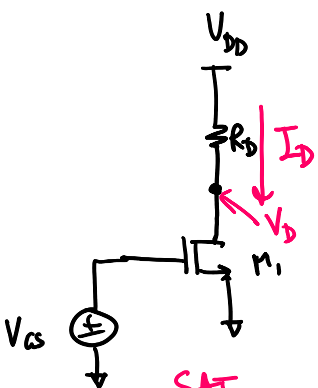


⇒ DC open

AC short



DC picture



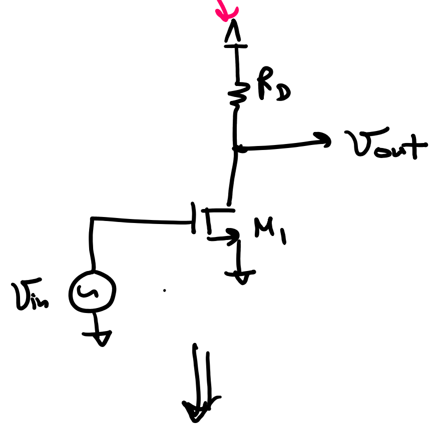
SAT

find g_m, r_o for (V_{DS}, I_D, V_{GS})

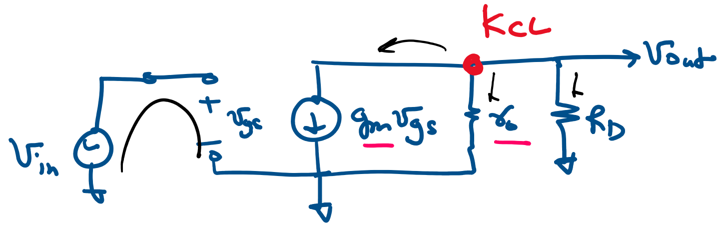
AC picture

Voltage is fixed

AC ground



⇓



$v_{gs} = v_{in}$

KCL: $$g_m v_{in} + \frac{v_{out}}{r_o} + \frac{v_{out}}{R_D} = 0$$

$$\Rightarrow v_{out}\left(\frac{1}{r_o} + \frac{1}{R_D}\right) = -g_m v_{in}$$

$\left(\frac{1}{r_o} + \frac{1}{R_D}\right) = \frac{1}{r_o \| R_D}$

$$\Rightarrow \frac{v_{out}}{v_{in}} = -g_m \cdot (r_o \| R_D)$$

$$\frac{v_{out}}{v_{in}} = -g_m \cdot (r_o \| R_D)$$

Small signal voltage gain

$$A_v = \frac{v_{out}}{v_{in}} = -g_m (r_o \| R_D)$$

Lets say $R_D \to \infty$

$$A_v = -g_m r_o$$ → max gain from NMOS

Lets say $R_D \ll r_o$

$$\boxed{A_v = -g_m R_D}$$