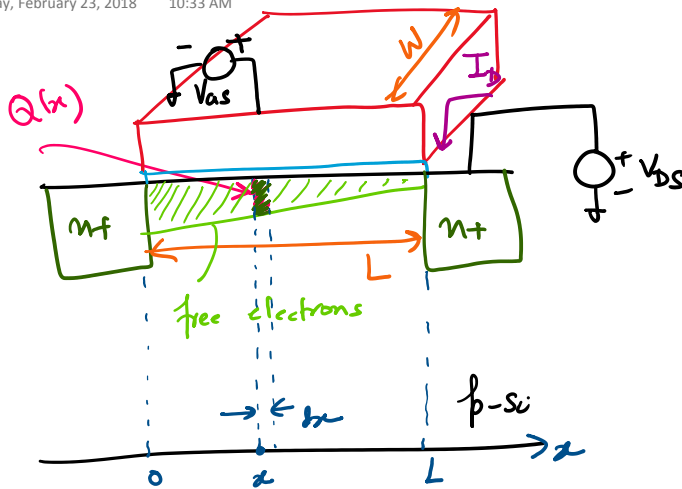


# Lecture 17

Friday, February 23, 2018 10:33 AM



$$V_{DS,sat} = V_{GS} - V_{THN}$$

↑ pinch-off occurs  
onset of saturation region

1-D model

channel charge density  $\Rightarrow$  charge due to free electrons per unit length

$$Q_{total} = \underbrace{\text{Area}}_{W \cdot L} \underbrace{C_{ox}' (V_{GS} - V_{THN})}_{\text{Cap for unit area}}$$

$C_{ox}' = \frac{\epsilon \epsilon_{ox}}{t_{ox}}$   
↓  
ox thickness

charge density (per unit length)

$$Q = \frac{Q_{total}}{L} = W C_{ox}' (V_{GS} - V_{THN}) \longrightarrow \textcircled{1}$$

$$\Rightarrow Q(x) = W C_{ox}' [V_{GS} - V_{THN} - V(x)] \longrightarrow \textcircled{2}$$

charge is flowing due to Drift  
↓  
 $v$

$$i = \frac{dQ}{dt}$$

$v = \mu E$

$$I = Q v$$

$\textcircled{3}$

If the carriers move with velocity ' $v$ ' then the enclosed charge moves with

$$= \mu_n \left[ \frac{dV(x)}{dx} \right] \longrightarrow \textcircled{4}$$

$$I = \frac{d}{dt} (Q(x) dx)$$

$$= \mu_n \left[ \frac{dV(x)}{dx} \right] \longrightarrow (4)$$

$$\begin{aligned} I &= \frac{d}{dt} (Q(x) dx) \\ &= Q(x) \cdot \frac{dx}{dt} \\ &= Q(x) v \end{aligned}$$

from (2), (3) & (4) we get

$$I_D = W C_{ox} [V_{GS} - V(x) - V_{THN}] \mu_n \frac{dV(x)}{dx} \longrightarrow (5)$$

$$I_D \int_0^L dx = \mu_n W C_{ox} \int_{V(x)=0}^{V(x)=V_{DS}} [V_{GS} - V(x) - V_{THN}] dV(x)$$

$$I_D = \mu_n C_{ox} \frac{W}{L} \left[ (V_{GS} - V_{THN}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

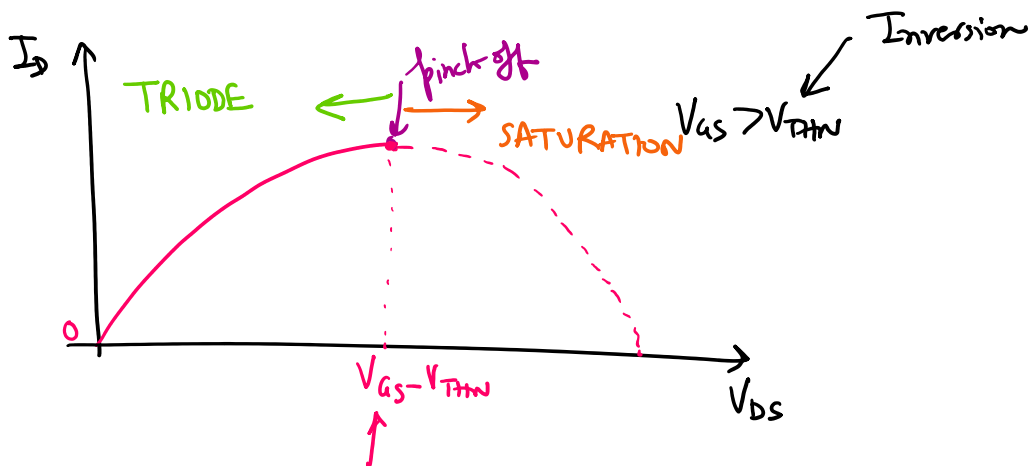
$C_{ox} \in \begin{matrix} \text{Gox} \\ \text{Cap} \\ \text{p.u. Area} \end{matrix}$   
 $\hookrightarrow C_{ox}$

TRIODE

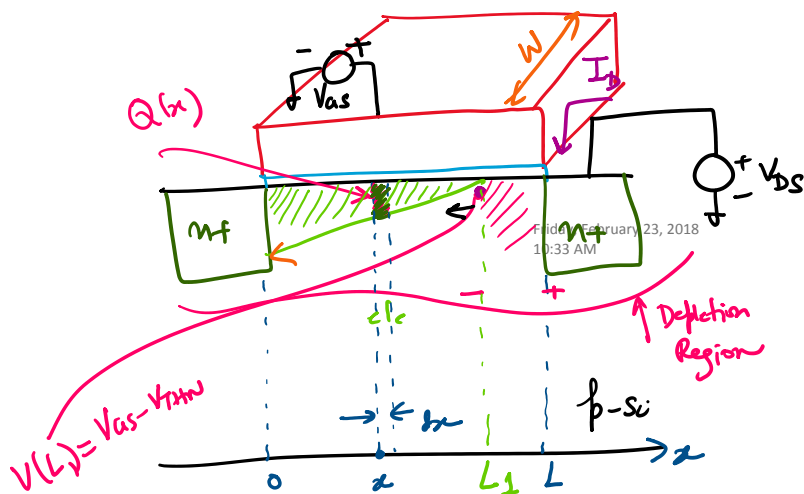
$I_D \propto \mu_n$  (mobility)

$\propto C_{ox}$

$\propto \frac{W}{L} \Leftarrow$  Aspect ratio or "sizing" of the MOSFET



Recall that pinch-off occurred at  $V_{DS} = V_{GS} - V_{THN}$



for  $V_{DS} > V_{DS,sat}$   
 $\downarrow$   
 $= V_{GS} - V_{THN}$

further increase in  $V_{DS}$  simply shifts the pinch-off point towards the source

$\Rightarrow$  integration must be done from  $x=0$  to  $x=L$

$\downarrow x=L$   $V(x) = V_{GS} - V_{THN}$  ← beyond pinch-off, voltage drop across channel is  $(V_{GS} - V_{THN})$  even if we increase  $V_{DS}$ !

$$\int_{x=0}^{x=L} I_D dx = \int_{V(x)=0}^{V(x)=V_{GS}-V_{THN}} \mu n C_{ox} W [V_{GS} - V_{THN} - V(x)] \cdot dV(x)$$

$$I_D = \frac{1}{2} \mu n C_{ox} \frac{W}{L} (V_{GS} - V_{THN})^2$$

→ SATURATION for long-L devices  $L_1 \approx L$

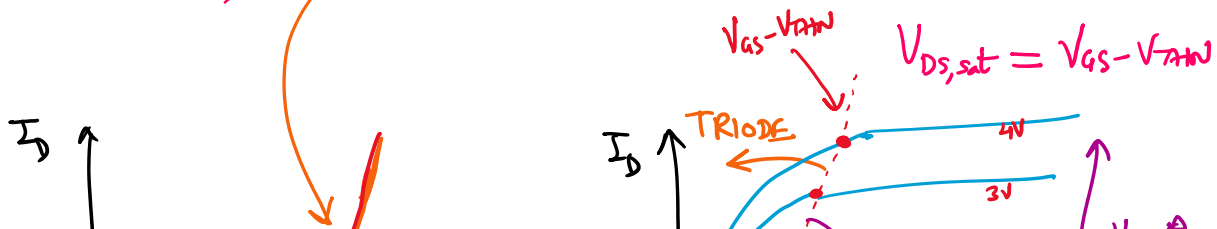
Long-channel Equation for Saturation region

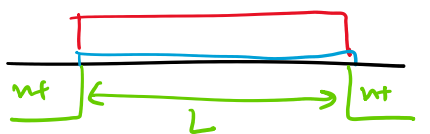
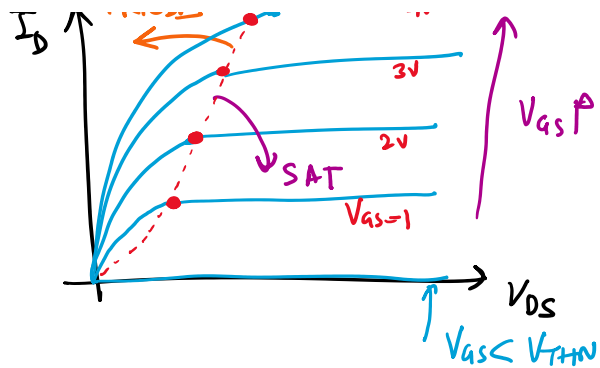
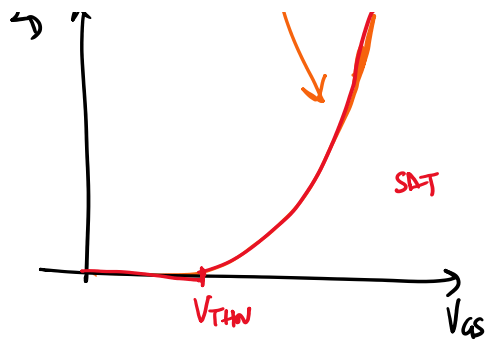
$V_{GS} < V_{THN}$  Cutoff

$$I_D = \begin{cases} 0, & V_{GS} < V_{THN} \\ \mu n C_{ox} \frac{W}{L} \left[ (V_{GS} - V_{THN}) V_{DS} - \frac{V_{DS}^2}{2} \right], & V_{DS} < V_{DS,sat} \\ \frac{1}{2} \mu n C_{ox} \frac{W}{L} (V_{GS} - V_{THN})^2, & V_{DS} \geq V_{DS,sat} \end{cases}$$

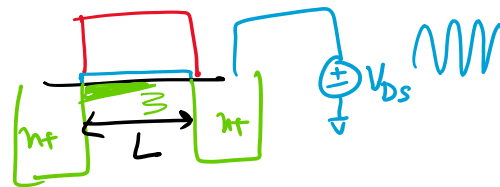
$V_{DS} < V_{DS,sat}$  → TRIODE  
 $V_{DS} \geq V_{DS,sat}$  → SATURATION

$V_{GS} > V_{THN}$



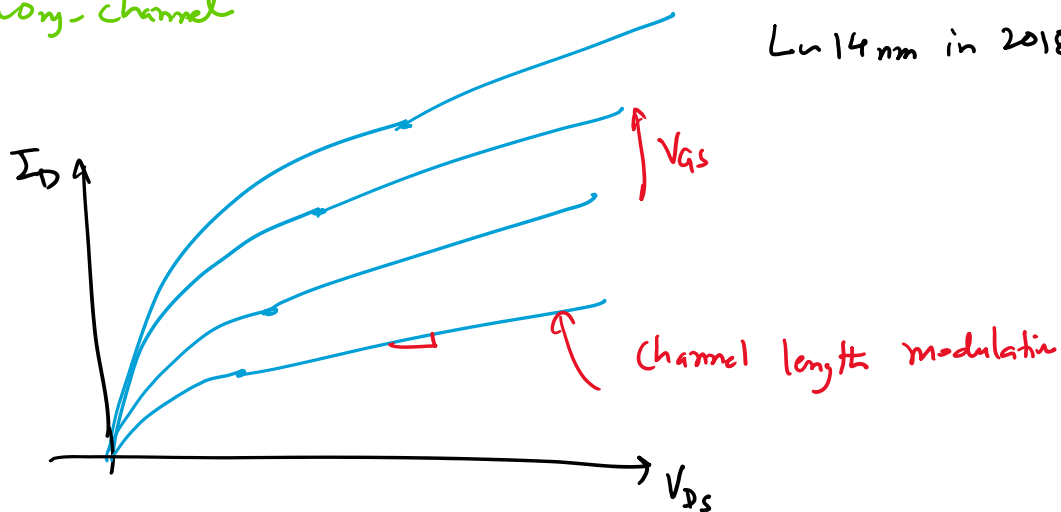


$L \geq 1 \mu m$   
long-channel



short-channel

$L \sim 14 nm$  in 2018



In saturation:

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{THN})^2 (1 + \lambda V_{DS})$$