Let $L$ be a set of literals that we wish to unify.

Let $\text{sub} \leftarrow \epsilon$.

While $|L_{\text{sub}}| \neq 1$

- Scan left to right till a disagreement is found
- If neither term (at place of disagreement) is a variable then return(“not unifiable”)
  - else
    - Let $x$ be the ”variable” term and let the other term be $t$
    - If $x$ occurs in $t$ then return(“not unifiable”)
    - else
      - $\text{sub} \leftarrow \text{sub} \cdot [x \mid t]$
  - return $\text{sub}$

We will now apply this algorithm from the example from Uwe Schoening’s book that was discussed in class.

$L = \{P(f(z,g(a,y)),h(z)), P(f(f(u,v),w),h(f(a,b)))\}$

$\text{sub} \leftarrow [z \mid f(u,v)]$

$L_{\text{sub}} = \{P(f(f(u,v),g(a,y)),h(f(u,v))), P(f(f(u,v),w),h(f(a,b)))\}$

$\text{sub} \leftarrow [z \mid f(u,v)] \cdot [w \mid g(a,y)]$

$L_{\text{sub}} = \{P(f(f(u,v),g(a,y)),h(f(u,v))), P(f(f(u,v),g(a,y)),h(f(a,b)))\}$

$\text{sub} \leftarrow [z \mid f(u,v)] \cdot [w \mid g(a,y)] \cdot [u \mid a]$,

$L_{\text{sub}} = \{P(f(f(a,v),g(a,y)),h(f(a,v))), P(f(f(a,v),g(a,y)),h(f(a,b)))\}$

$\text{sub} \leftarrow [z \mid f(u,v)] \cdot [w \mid g(a,y)] \cdot [u \mid a] \cdot [v \mid b]$,

$L_{\text{sub}} = \{P(f(f(a,b),g(a,y)),h(f(a,b)))\}$

The unifying substitution can also be rewritten as $[z \mid f(a,b), w \mid g(a,y), u \mid a, v \mid b]$
A clause $R$ is obtained by resolving clauses $C_1$ and $C_2$ if

1. Let $\sigma_1$ and $\sigma_2$ be substitutions such that the clauses $C_1\sigma_1$ and $C_2\sigma_2$ have no variables in common,

2. Let $L_1, \ldots, L_n \in C_1\sigma_1(n \geq 1)$ and $L'_1, \ldots, L'_m \in C_2\sigma_2(m \geq 1)$. Let $\sigma$ be the unifier of $\{L_1, \ldots, L_n, L'_1, \ldots, L'_m\}$, and

3. $R$ is $((C_1\sigma_1 - \{L_1, \ldots, L_n\}) \cup (C_2\sigma_2 - \{L'_1, \ldots, L'_m\}))\sigma$

Notice in step 2, the fact that the set $\{L_1, \ldots, L_n, L'_1, \ldots, L'_m\}$ is unifiable means that all the $m+n$ literals involved the same predicates. If $L_1$ is a positive literal then so are $L_2, \ldots, L_n$ and furthermore $L'_1, \ldots, L'_m$ must all be negative (so that $L'_1, \ldots, L'_m$ are positive). On the other hand, if $L_2$ is a negative literal then $L_2, \ldots, L_n$ must be negative literals and $L'_1, \ldots, L'_m$ must all be positive.

Suppose $C_1 = [P(x, y), P(a, b), P(z, f(y)), Q(x)]$ and $C_2 = [\neg P(x, f(z)), \neg P(x, y), \neg P(z, a)]$. How many ways are there to resolve $C_1$ and $C_2$?