CISC 404/604
Homework 3
Due on Thursday, April 22, 2010

1. (35 points) For each of the following wfs, give a structure that satisfies the wf and another structure that falsifies it.
   a. \((\forall x)P(x) \Rightarrow (\forall x)Q(x)) \Rightarrow (\forall x)(P(x) \Rightarrow Q(x)).\)
   b. \((\forall x)(P(x) \Rightarrow P(a)).\)
   c. \((\exists x)(\exists y)[P(x, y) \land (\forall z)(\neg P(x, z) \land P(z, y))].\)
   d. \((\forall x)(\forall y)(\exists z)(P(x, y, z) \Rightarrow (\forall x)(\forall y)(P(x, y, f(z, y))).\)
   e. \((\forall x)(\forall y)(\exists z)(P(x, y, z) \Rightarrow (\forall x)(\forall y)(P(x, y, f(x, y))).\)

2. (35 points)
   a. Consider the following sentences.
      i. Any student who takes CISC301 will also take MATH210.
      ii. Some instructor of a CISC course has taught a LING course.
      iii. Some CISC major has taken all CISC courses.
      iv. No instructor who teaches CISC301 has taught a course taken by a LING student.
      v. Every LING major has taken a course taught by some instructor who has taught CISC301.

      Translate the sentences above using the following constants, 3, m2, c, and l (for CISC301, MATH210, CISC, and LING respectively); unary predicate symbols, S and I (for is-student, and is-instructor respectively); and binary predicate symbols, C, T1, T2, and M where
      C(x, y) stands for course x is offered by department y,
      T1(x, y) corresponds to course x is taught by y,
      T2(x, y) corresponds to course x is taken by y, and
      M(x, y) corresponds to x is the major of y

      Important: Do not use any other function, constant or predicate symbols.
      Clearly parenthesize your formulae, especially to ensure that there is no ambiguity about the scope of the quantifiers.

   b. (Exercise 86 from Schoening’s book) Translate the following two sentences into wfs:
      i. Every barber shaves all persons who do not shave themselves.
      ii. No barber shaves any any person who shaves himself.

      For this part, use S(x, y) to stand for x shaves y and B(x) to correspond to x is a barber.