Homework 5 solution

3. Show $L_5 = \{ M_x \# M_y \mid L(M_x) \subseteq L(M_y) \}$ is co-re but not re. (actually, it turns out that it’s not co-re...)

To show that $L_5$ is not recursive:

Assume that $L_5$ is recursive, we can generate a machine $K$ that takes $M_x \# M_y$ as input and returns ‘Y’ if $L(M_x) \subseteq L(M_y)$ and ‘N’ otherwise.

Define a machine $M_0$ be such that $L(M_0) = \phi$

Now use $K$ to generate machine $M_1$ that appears below:

![Diagram](image)

The $M'$ component simply appends $M_0$ to input $M$ for input to $K$.

We know that $L(M) \subseteq L(M_0) = \phi$ iff $L(M) = \phi$

So $M_1$ accepts $\{ M \mid L(M) \neq \phi \}$

(note that the final ‘Y’ and ‘N’ are reversed from the output from $K$)

However, we know that the language $L = \{ M \mid L(M) \neq \phi \}$ is not recursive, so this is a contradiction.

Therefore, $L_5$ is not recursive.

It turns out that $L_5$ is NOT co-re either because that would require showing that $M_x$ accepts a string that $M_y$ does not, and $M_y$ could 'loop forever' on a particular string accepted by $M_x$, and it is not possible to determine this in finite time.