CISC 304

Prenex Normal Form & Skolemization

In the following, I will use Q to stand for a quantifier, i.e., $Q = \forall$ or $Q = \exists$. And when $Q = \forall$ then $\overline{Q} = \exists$, and when $Q = \exists$ then $\overline{Q} = \forall$.

Also, if A is a formula, and x, x_1 are variables, then $A[x \mid x_1]$ stands for the formula obtained from A by replacing all free occurrences of x in A by x_1.

The rules for conversion to prenex normal form then are as follows:

- If you have a subformula of the form $\neg(Qx\ A)$ then replace it by $\overline{Q}x\ \neg A$.
- If you have a subformula of the form $((Qx\ A) \wedge B)$ then replace it by $Qx_1(A_1 \wedge B)$, where x_1 is a new variable not occurring in the given formula and $A_1 = A[x \mid x_1]$. Note that the subformulae A and B may contain quantifiers.
- If you have a subformula of the form $(B \wedge (Qx\ A))$ then replace it by $Qx_1(B \wedge A_1)$, where x_1 is a new variable not occurring in the given formula and $A_1 = A[x \mid x_1]$. Note that the subformulae A and B may contain quantifiers.
- If you have a subformula of the form $((Qx\ A) \vee B)$ then replace it by $Qx_1(A_1 \vee B)$, where x_1 is a new variable not occurring in the given formula and $A_1 = A[x \mid x_1]$. Note that the subformulae A and B may contain quantifiers.
- If you have a subformula of the form $(B \vee (Qx\ A))$ then replace it by $Qx_1(B \vee A_1)$, where x_1 is a new variable not occurring in the given formula and $A_1 = A[x \mid x_1]$.Note that the subformulae A and B may contain quantifiers.
- If you have a subformula of the form $((Qx\ A) \rightarrow B)$ then replace it by $\overline{Q}x_1(A_1 \rightarrow B)$, where x_1 is a new variable not occurring in the given formula and $A_1 = A[x \mid x_1]$. Note that the subformulae A and B may contain quantifiers.
- If you have a subformula of the form $(B \rightarrow (Qx\ A))$ then replace it by $Qx_1(B \rightarrow A_1)$, where x_1 is a new variable not occurring in the given formula and $A_1 = A[x \mid x_1]$. Note that the subformulae A and B may contain quantifiers.

Skolemization: Let A be a formula of the form $\forall x_1 \ldots \forall x_n \exists y B$, where B may or may not have additional quantifiers. Then after skolemization (of A), we obtain the formula $\forall x_1 \ldots \forall x_n B_1$, where B_1 is obtained by replacing every free occurrence of y in B by the term $f(x_1, \ldots x_n)$. Here f is a new function letter.