CISC 304

Handout: Conversion to CNF/Clausal Form

A literal is either an atomic formula (positive literal) or the negation of an atomic formula (negative literal). A clause contains literals and is interpreted as their disjunction. If a clause contains the literals $l_1, \ldots, l_k$ then we will write it as $[l_1, \ldots, l_k]$, which is *different* from the notation used in the textbook. Thus, $[l_1, \ldots, l_k]$ is interpreted the same way as the formula $(l_1 \vee \ldots \vee l_k)$.

Here is a (*non-deterministic*) algorithm to convert a formula, A, into a set of clauses. Start with $\{[A]\}$.

At any point, we will have a set that has the form, $\{C_1, \ldots, C_i, \ldots, C_k\}$, where the $C's$ will be in the form $[A_1, \ldots A_n]$, and the $A's$ are formulae. We are finished with a C if it contains only literals. (i.e., C is already a clause)

While some C has a non-literal

Let C_i include a non-literal.

WLOG, we can express C_i as $[A_1, \ldots A_{n-1}, A_n]$ where A_n is a non-literal.

Case 1: (double negation case)

$A_n = \neg\neg B$, for some formula B.

Replace C_i by $[A_1, \ldots A_{n-1}, B]$.

Case 2: (disjunctive case).

A_n is a "disjunctive" formula, say β.

Replace C_i by $[A_1, \ldots A_{n-1}, \beta_1, \beta_2]$, where

β_1 and β_2 can be determined from β by using the table in Page 18 of the textbook.

Case 3: (conjunctive case).

A_n is a "conjunctive" formula, say α.

Replace C_i by C_i^1 and C_i^2 where

$C_i^1 = [A_1, \ldots A_{n-1}, \alpha_1]$,

$C_i^2 = [A_1, \ldots A_{n-1}, \alpha_2]$, and

α_1 and α_2 can be determined from α by using the table in Page 18 of the textbook.

Example: Converting $A = (\neg p \vee q) \rightarrow (r \vee p)$ into a set of clauses:

$\{[(\neg p \vee q) \rightarrow (r \vee p)]\}$ (starting with $\{[A]\}$)

$\{[\neg(\neg p \vee q), (r \vee p)]\}$ (applying Rule 2)

$\{[\neg(\neg p \vee q), r, p]\}$ (applying Rule 2)

$\{[\neg\neg p, r, p], [\neg q, r, p]\}$ (applying Rule 3)

$\{[p, r], [p \neg q, r]\}$ (applying Rule 1)

Note we write $[p, r]$ rather than $[p, r, p]$ and that $[\neg q, r, p]$ can also be written as $[p, \neg q, r]$.