1. Let $C$ be a clause and $l$ be a literal. Then both the clauses $C$ and $C \cup \{l\}$ are $l$-extensions of $C$.

Let $S$ be a set of clauses and $l$ be a literal. Then we say that $S'$ is a $l$-extension of $S$ if the following condition holds:

- If $C \in S$ then there is a $C' \in S'$ where $C'$ is a $l$-extension of $C$.

a. Suppose $C_1$ and $C_2$ are two clauses and $l$ is a literal. Let $C'_1$ and $C'_2$ be some $l$-extensions of $C_1$ and $C_2$ respectively. Show that if $R$ is obtained by resolving $C_1$ and $C_2$ then there is an $l$-extension of $R$ which can be obtained by resolving $C'_1$ and $C'_2$.

b. Suppose $S$ is a set of clauses and $l$ is a literal. Let $S'$ be an $l$-extension of $S$. Show that if there is a resolution derivation of a clause $C$ from $S$ then there is an $l$-extension of $C$ which can be derived by resolution from $S'$.

c. Let $S$ be a set of clauses and $A$ be a statement letter. Assume that there is no clause in $S$ which contains both $A$ and $\neg A$. Define $S^A_0$ as follows: for each $C \in S$, if $A \in C$ then $C - \{A\}$ is included in $S^A_0$; if $\neg A \in C$ then no counterpart of $C$ is included in $S^A_0$; and if neither $A$ nor $\neg A$ is in $C$ then $C$ is included in $S^A_0$. For example, if $S = \{[A, B, \neg C], [A, B], [\neg A, B, D], [\neg B, \neg C]\}$ then $S^A_0 = \{[B, \neg C], [B], [\neg B, \neg C]\}$.

Let $S$ be a set of clauses. Show that if $S^A_0$ is satisfied by a valuation then there is a valuation that satisfies $S$.

2. Show

a. $\vdash_{res} ((A \Rightarrow B) \land (B \Rightarrow C)) \Rightarrow ((\neg C \land A) \Rightarrow B)$.

b. $\{((A \land B) \lor (C \Rightarrow D))\} \vdash_{res} ((A \lor (C \Rightarrow D)) \land (B \lor (C \Rightarrow D)))$

c. $(B \Rightarrow C)$ is a consequence of the set $\{\neg((\neg C \Rightarrow B) \Rightarrow B), (\neg C \Rightarrow \neg A), ((B \land \neg C) \Rightarrow A)\}$ using resolution.