CISC301
Examples Pumping Lemma

1. To show $A_1 = \{a^n b^n \mid n \geq 0\}$ is not regular.

Let k be the constant of the pumping lemma (PL). Consider $x = \epsilon$, $y = a^k$ and $z = b^k$. This meets the requirements of PL since $xyz = a^k b^k \in A_1$ and $\mid y \mid \geq k$. No matter how y is split into the 3 substrings, u, v, and w, we can say $u = a^p$, $v = a^q$, and $w = a^{k-(p+q)}$ for some p, q where $q \geq 1$ (since $\mid v \mid \geq 1$). Now consider $xuv^2wz = \epsilon a^p a^q a^q a^{k-(p+q)} b^k = a^{k+q} b^k$. This is not in A_1 since $q \geq 1$. Thus, since $xuv^i wz \notin A_1$ for some i, using PL, we can conclude that A_1 is not regular.

2. To show $A_2 = \{w \mid$ #a's in w = #b's in w$\}$ is not regular.

Note $A_1 = A_2 \bigcap L(a^* b^*)$. If A_2 were regular then A_1 is regular (we have shown in class that class of regular sets is closed under intersection). Since we have shown A_1 is not regular, we can now conclude A_2 is also not regular.

3. To show $A_3 = \{a^i b^j \mid i < j\}$ is not regular.

Let k be the constant of the PL. Consider $x = \epsilon$, $y = a^k$ and $z = b^{k+1}$. This meets the requirements of PL since $xyz = a^k b^{k+1} \in A_3$ and $\mid y \mid \geq k$. Regardless of how y is split into the 3 substrings, u, v, and w, we can say $u = a^p$, $v = a^q$, and $w = a^{k-(p+q)}$ for some p, q where $q \geq 1$ (since $\mid v \mid \geq 1$). Now consider $xuv^2wz = \epsilon a^p a^q a^q a^{k-(p+q)} b^k = a^{k+q} b^k$. This is not in A_1 since $q \geq 1$ and hence $k + q \geq k + 1$. Thus, since $xuv^i wz \notin A_1$ for some i, using PL, we can conclude that A_1 is not regular.

4. To show that $A_4 = \{a^{n^2} \mid n \geq 0\}$ is not regular.

Let k be the constant of the PL. Consider $x = \epsilon$, $y = a^k$ and $z = a^{k^2-k}$. Hence $xyz = a^{k^2} \in A_4$ and $\mid y \mid \geq k$ as required. We can write $y = uvw$ where $u = a^p$, $v = a^q$, and $w = a^{k-(p+q)}$ for some p, q where $q \geq 1$ (since $\mid v \mid \geq 1$). We can also note that since $uvw = a^k$, we must have $q = \mid v \mid \leq k$. Consider $i = 2$. $xuv^i wz = a^{k^2+q}$. But $k^2 + q$ is not a perfect square as $k^2 < k^2 + q < (k+1)^2 = k^2 + 2k + 1$. Thus, since $xuv^i wz \notin A_1$ for some i, using PL, we can conclude that A_1 is not regular.