CISC301
HW2 Solutions Sketch

1. a) We will prove this by contradiction. Suppose (bwoc) $\{A \rightarrow (B \rightarrow \neg A)\} \not\models (B \rightarrow \neg A)$. Then there is an assignment $\mathcal{A}$ such that $\mathcal{A}(A \rightarrow (B \rightarrow \neg A)) = 1$ and $\mathcal{A}(B \rightarrow \neg A) = 0$. Therefore, (from the latter) $\mathcal{A}(B) = 1$ and $\mathcal{A}(A) = 1$. But then $\mathcal{A}(A \rightarrow (B \rightarrow \neg A)) = 0$, a contradiction.

b) Counterexample: Consider $\mathcal{A}$ where $\mathcal{A}(A) = 1$, $\mathcal{A}(B) = 0$, and $\mathcal{A}(C) = 0$.

c) BWOC, suppose $\{A \rightarrow C, B \rightarrow C, A \vee B\} \not\models C$. Then there is an assignment $\mathcal{A}$ such that $\mathcal{A}(A \rightarrow C) = 1$, $\mathcal{A}(B \rightarrow C) = 1$, $\mathcal{A}(A \vee B) = 1$, and $\mathcal{A}(C) = 0$. Since $\mathcal{A}(A \rightarrow C) = 1$ and $\mathcal{A}(C) = 0$ it must be the case that $\mathcal{A}(A) = 0$. Similarly, we can infer that $\mathcal{A}(B) = 0$. But then, $\mathcal{A}(A \vee B) = 0$, a contradiction.

d) Consider $\mathcal{A}$ where $\mathcal{A}(A_i) = 1$ if i is odd and $\mathcal{A}(A_i) = 0$ if i is even.

2. BWOC, suppose $S \models F$ and $S \bigcup \{\neg F\}$ is satisfiable. Then there is an assignment $\mathcal{A}$ such that $\mathcal{A}$ makes every element of S true and $\mathcal{A}$ makes $\neg F$ true. Then $\mathcal{A}$ makes F false and satisfies all members of S, a contradiction with $S \models F$.