CISC301 HW1 Solutions Sketch

Fall 2005

18 Sep 2005

1. a) We will disprove it by providing a counterexample: $F = A \vee B$, $G = A$, $H = B$.

b) Since F is satisfiable, there is an assignment $\mathcal{A}$ such that $\mathcal{A}(F) = 1$. Now, we also now that for this assignment, $\mathcal{A}$, $\mathcal{A}(G) = 1$ (because G is valid). Hence $\mathcal{A}(F \wedge G) = 1$. $\mathcal{A}((F \wedge G) \rightarrow H) = 1$ because $(F \wedge G) \rightarrow H$ valid. Hence $\mathcal{A}(H) = 1$. Thus we have shown H is satisfiable.

2. a)

Sample False Assignment: $\mathcal{A}(A) = 0$, $\mathcal{A}(B) = 1$, $\mathcal{A}(C) = 1$.
Sample True Assignment: $\mathcal{A}(A) = 1, \mathcal{A}(B) = 1, \mathcal{A}(C) = 0$.
b)
Sample False Assignment: $\mathcal{A}(A) = 0, \mathcal{A}(B) = 0, \mathcal{A}(C) = 0$.
Sample True Assignment: $\mathcal{A}(A) = 1, \mathcal{A}(B) = 1, \mathcal{A}(C) = 1$.
c)
Sample False Assignment: $\mathcal{A}(A) = 1, \mathcal{A}(B) = 0, \mathcal{A}(C) = 0$.
Sample True Assignment: $\mathcal{A}(A) = 0, \mathcal{A}(B) = 1, \mathcal{A}(C) = 1$.

3) a) Since $\mathcal{A}((A \leftrightarrow B) \leftrightarrow (C \implies \neg A)) = 1$ and $\mathcal{A}(A \leftrightarrow B) = 0$, we have $\mathcal{A}(C \rightarrow \neg A) = 0$. Therefore, $\mathcal{A}(C) = 1$ and $\mathcal{A}(\neg A) = 0$, i.e., $\mathcal{A}(A) = 1$. Since $\mathcal{A}(A \leftrightarrow B) = 0$ and $\mathcal{A}(A) = 1$, we can infer $\mathcal{A}(B) = 0$.

b) Since $\mathcal{A}(\neg(A \wedge B)) = 1$, we have $\mathcal{A}(A) = 0$ or $\mathcal{A}(B) = 0$. In either case, $\mathcal{A}(A \rightarrow \neg B) = 1$. Since we already know $\mathcal{A}(\neg(A \wedge B)) = 1$, we can infer so $\mathcal{A}((\neg(A \wedge B)) \leftrightarrow (A \rightarrow \neg B)) = 1$.