## CISC 301 Homework 1 Due on Thursday, September 15, 2004 **No Late Submissions**

1.  $(3 \ge 10)$  = 30 points) Prove or disprove the following statements. (If a statement is false, you can disprove it by giving a counter-example.)

a. Let F, G, and H be formulae such that  $F \to (G \lor H)$  is valid. Then  $F \to G$  is valid or  $F \to H$  is valid.

b. Let F, G, and H be formulae such that  $(F \wedge G) \to H$  is valid, F is satisfiable and G is valid. Then H is satisfiable.

c. Let F, G, and H be formulae such that  $F \to (G \lor H)$  is not satisfiable. Then  $G \to F$  is valid.

2 (3 x 6 = 18 points) Prove that the following are satisfiable but not valid. For each of the three formulae below, you will need to give one assignment that makes the formula true and one that makes it false.

a. 
$$(((A \to B) \land (B \to C)) \to (\neg C \land A))$$
  
b.  $((A \to \neg C) \to ((\neg A) \lor B)) \to (A \land C).$   
c.  $(B \to A) \to ((A \land \neg C) \to ((\neg C) \to B))$ 

3.  $(2 \ge 6 = 12 \text{ points})$ 

a. Suppose  $\mathcal{A}$  is an assignment such that  $\mathcal{A}((A \leftrightarrow B) \leftrightarrow (C \rightarrow \neg A)) = 1$  and  $\mathcal{A}(A \leftrightarrow B) = 0$ . Then what can you surmise about  $\mathcal{A}(A)$ ,  $\mathcal{A}(B)$ , and  $\mathcal{A}(C)$ . Give a brief but clear explanation.

b. Suppose  $\mathcal{A}$  is an assignment such that  $\mathcal{A}(\neg(A \land B)) = 1$ . Then what is the value of  $\mathcal{A}((\neg(A \land B)) \leftrightarrow (A \rightarrow \neg B))$ ? Give a brief but clear explanation.