A literal is either an atomic formula (positive literal) or the negation of an atomic formula (negative literal). A clause contains literals and is interpreted as their disjunction. If a clause contains the literals $l_1, \ldots, l_k$ then we will write it as $[l_1, \ldots, l_k]$ rather than the notation $\{l_1, \ldots, l_k\}$ used in the textbook. Thus, $[l_1, \ldots, l_k]$ is interpreted the same way as the formula $(l_1 \lor \cdots \lor l_k)$.

Here is a (non-deterministic) algorithm to convert a formula, $F$, (assumed to not include $\leftrightarrow$) into a set of clauses. Start with $\{[F]\}$.

At any point, we will have a set that has the form, $\{C_1, \ldots, C_i, \ldots, C_k\}$, where the $C$’s have the form $[F_1, \ldots, F_n]$, and the $F$’s are formulae. We are finished with a $C$ if every formula in $C$ is already a literal.

While not done do (i.e., some $C$ has a non-literal)

Let $C_i$ include a non-literal.

WLOG, we can express $C_i$ as $[F_1, \ldots, F_{n-1}, F_n]$ where $F_n$ is a non-literal.

Case 1: $F_n = \neg G$, for some formula $G$.

Replace $C_i$ by $[F_1, \ldots, F_{n-1}, G]$.

Case 2: (disjunctive case).

Case 2a: $F_n = (F \lor G)$ for some $F, G$.

Replace $C_i$ by $[F_1, \ldots, F_{n-1}, F, G]$.

Case 2b: $F_n = (F \rightarrow G)$ for some $F, G$.

Replace $C_i$ by $[F_1, \ldots, F_{n-1}, \neg F, G]$.

Case 2c: $F_n = \neg (F \land G)$ for some $F, G$.

Replace $C_i$ by $[F_1, \ldots, F_{n-1}, \neg F, \neg G]$.

Case 3: (conjunctive case).

Case 3a: $F_n = (F \land G)$ for some $F, G$.

Replace $C_i$ by $C_i^1$ and $C_i^2$, where

$C_i^1 = [F_1, \ldots, F_{n-1}, F]$, and

$C_i^2 = [F_1, \ldots, F_{n-1}, G]$

Case 3b: $F_n = \neg (F \rightarrow G)$ for some $F, G$.

Replace $C_i$ by $C_i^1$ and $C_i^2$, where

$C_i^1 = [F_1, \ldots, F_{n-1}, F]$, and

$C_i^2 = [F_1, \ldots, F_{n-1}, \neg G]$

Case 3c: $F_n = \neg (F \lor G)$ for some $F, G$.

Replace $C_i$ by $C_i^1$ and $C_i^2$, where

$C_i^1 = [F_1, \ldots, F_{n-1}, \neg F]$, and

$C_i^2 = [F_1, \ldots, F_{n-1}, \neg G]$

Example: Converting $F = (\neg A \lor B) \rightarrow (C \lor A)$ into a set of clauses:

$\{[\neg A \lor B) \rightarrow (C \lor A)\}$ (starting with $\{[F]\}$)

$\{[\neg (\neg A \lor B), (C \lor A)]\}$ (applying rule 2b)

$\{[\neg (\neg A \lor B), C, A]\}$ (applying rule 2a)

$\{[\neg A, C, A], [\neg B, C, A]\}$ (applying rule 3c)

$\{[A, C], [A \neg B, C]\}$ (applying rule 1) Note we write $[A, C]$ rather than $[A, C, A]$ and that $[\neg B, C, A]$ can also be written as $[A, \neg B, C]$. 