CISC301 Example Questions Set I

1. For each of the following formulae, determine if it is valid. If so, prove it is validy? If it is not valid, is it satisfiable? Why?

a. $(((A \to B) \land (B \to C)) \to (\neg C \land A))$

b. $(A \to (B \lor C)) \to (A \to B)$.

c. $(B \iff (B \to A)) \to A$.

d. $((A \lor B) \land \neg A) \to B$.

e. $(B \to A) \to ((A \to \neg C) \to (\neg C \to B))$.

2. a. If $\mathcal{A}(F \to G) = 1$ then what can you say about $\mathcal{A}((F \lor H) \to (G \lor H))$, $\mathcal{A}((F \land H) \to (G \land H))$.

b. What further truth values can you deduce from

i. $\mathcal{A}((\neg A \lor B) \to (A \to \neg C)) = 0$

ii. $\mathcal{A}(A) = 0$ and $\mathcal{A}((A \land B) \iff (A \lor B)) = 0$

iii. $\mathcal{A}((A \to \neg B) \to (C \to B)) = 0$

3. In the following, F, G, and H stand for formulae, and S, S_1, and S_2 stand for sets of formulae. Prove or disprove (by giving concrete counter-examples).

a. $(F \to (G \to F)$ is valid.

b. $(F \to (G \to H)) \to ((F \to G) \to F \to H))$ is valid.

c. $(F \to (G \to \neg F)) \to (G \to \neg F)$ is valid.

d. $(F \land (G \lor H)) \to ((F \land G) \lor (F \land H))$ is valid.

e. $(F \lor (G \land H)) \to ((F \lor G) \land (F \lor H))$ is valid.

f. $((F \land (G \lor H)) \to ((F \land G) \lor (F \land H)))$ is valid.

g. $(F \to (G \lor H)) \to (F \to G)$ is valid.

h. $F \to \neg F$ is equivalent to $\neg F$.

i. If F and G are equivalent then $F \to G$ is valid.

j. If $(F \land G) \to H$ is valid, F is satisfiable and G is satisfiable then H is satisfiable.

k. If $(F \land G) \to H$ is valid, F is satisfiable and G is valid then H is satisfiable.

l. If $F \to G$ is satisfiable, and G is unsatisfiable then F is satisfiable.

m. If $(F \to (G \lor H))$ and $(F \to \neg G)$ are both valid then $(F \to H)$ is also valid.

n. If $(F \land G) \to H$ is valid, F is satisfiable and G is valid then H is satisfiable.

o. If $(F \to (G \lor H))$ is valid and G is unsatisfiable then $F \to H$ is valid.

p. If $F \to (G \lor H)$ is unsatisfiable then $(G \to F)$ is valid.

q. If $(F \land G) \to H$ is valid, F is satsfiable and G is valid then H is satisfiable.

r. If $\mathcal{A}$ is a model of $S_1 \bigcup S_2$ then $\mathcal{A}$ is a model of S_1.

s. If S_1 is satisfiable and $S_1 \subseteq S_2$ then S_2 is satisfiable.

4. Again, F, G and H stand for formulae and S stands for set of formulae. Prove or disprove (giving suitable counter-examples).

a. F is a consequence of $\{\neg F \rightarrow G, \neg G\}$.

b. H is a consequence of $\{F \rightarrow G, G \rightarrow H, F\}$.

c. For every satisfiable set, S, there is a formula, F, such that F is a consequence of S.

d. For every satisfiable set, S, there is a formula, F, such that F is not a consequence of S.

e. If G is a consequence of $S \bigcup \{F\}$ then $\neg F$ is a consequence of $S \bigcup \{\neg G\}$.

f. If G is a consequence of $S \bigcup \{F\}$ then F is a consequence of $S \bigcup \{G\}$.

g. If H is a consequence of $S \bigcup \{F\}$ and H is also a consequence of $S \bigcup \{G\}$ then H is a consequence of $S \bigcup \{(F \vee G)\}$.

h. If F as well as $\neg F$ are consequences of S then S is unsatisfiable.

i. $(F \rightarrow H)$ is a consequence of $\{(F \rightarrow (G \rightarrow H)), (F \rightarrow G)\}$.