

## Problem E (CLR Exercise 7.3-2) Model Solution

During the running time of the procedure RANDOMIZED-QUICKSORT, how many calls are made to the random number generator RANDOM in the worst case? How about in the best case? Give your answer in terms of  $\Theta$ -notation.

### Solution

$\Theta(n)$  calls are made to RANDOM in both cases.

### Explanation

RANDOM is called once for each time RANDOMIZED-PARTITION is called, so we can just consider the calls to RANDOMIZED-PARTITION by RANDOMIZED-QUICKSORT.

The worst case behavior occurs when the partitioning produces one subproblem of size  $n-1$  and one of size 0 each time it is called. The recurrence for the calls to RANDOMIZED-PARTITION is then

$$T(n) = T(n-1) + T(0) + \Theta(1)$$

$T(0) = 0$  because RANDOMIZED-PARTITION is not called on a subproblem of size 0, so

$$T(n) = T(n-1) + \Theta(1)$$

Therefore, in the worst case,  $T(n) = \Theta(n)$ .

The best case occurs when the partitioning produces two subproblems of size at most  $n/2$ . It is at most this because one will have  $\lfloor n/2 \rfloor$  elements and one will have  $\lceil n/2 \rceil - 1$ . The recurrence for the calls to RANDOMIZED-PARTITION is then

$$T(n) \leq 2T(n/2) + \Theta(1)$$

By case 1 of the Master Theorem,  $T(n)$  has the solution

$$T(n) = \Theta(n) \text{ in the best case.}$$

So,  $\Theta(n)$  calls are made to RANDOM in the best and worst cases.