CISC 320 Final Exam

Monday, May 23, 2011

Name: ________________________________

There are 24 questions. The first 19 questions count 3 points each.
The others each designate the points. The total is 100 points.
True or False: write T or F in the blank.
Multiple Choice: write the letter of the best choice in the blank.
Very Short Answer: write the very short answer in the blank.

1. ____

   (a) In the worst case, finding the maximum of an array of n numbers requires $n-1$ comparisons.
   (b) In the worst case, finding the maximum and minimum of an array of n numbers requires $2n-3$ comparisons.
   (c) both of the above
   (d) none of the above

2. Fill in the blank: There cannot be a comparison based algorithm for sorting an array of n numbers using fewer than $\mathrm{O}(n\log(n))$ comparisons. This is because any binary tree with k leaves has depth at least $\log(k)$, and the execution tree of any sorting algorithm must have at least

   ______________ leaves.

3. ____ To determine single source shortest paths in an *un*weighted graph, the way to go is

   (a) breadth first search
   (b) depth first search
   (c) Dijkstra's algorithm
   (d) none of the above

4. ____ A heap is stored in an array. If the array indexing begins at 1, for the node at location i (assuming i has two children), the left child is placed

   (a) at location ceil$(i/2+1)$
   (b) at location floor$(i/2+1)$
   (c) at location $2i$.
   (d) none of the above

5. ____ The edge weights of a weighted digraph may represent flow capacity. In seeking the maximum flow from node s to node t, we are also interested in a minimal cut that separates s from t. A minimal cut is

(a) a set of edges of minimal size such that there is no path from s to t when the set is removed.
(b) a set of edges of minimal total weight such that there is no path from s to t when the set is removed.
(c) a set of vertices of minimal size such that there is no path from s to t when the set is removed.
(d) a set of vertices constituting the interior vertices of a minimal weight path from s to t.

6. ____ Algorithm A manipulates an array of n items. It requires $\Theta(n^2)$ time. For arrays of 1000 items algorithm A takes about 1 second. On an array of 2,000 items how long do you expect algorithm A to take?

(a) about 2 seconds.
(b) about 4 seconds.
(c) about 1414 seconds.
(d) more time than the age of the universe.

7. ____ Algorithm B manipulates an array of n items. It requires $\Theta(2^n)$ time. For arrays of 20 items algorithm B takes about 1 second. On an array of 40 items how long do you expect algorithm B to take? Hint: $2^{20} \approx 10^6$ and $2^{40} \approx 10^{12}$.

(a) about 2 seconds.
(b) about 1414 seconds.
(c) about a million seconds.
(d) about a trillion seconds.

8. ____ Algorithm C manipulates an array of n items. It requires $\Theta(2^n)$ time. For arrays of 20 items algorithm C takes about 1 second. On an array of 352 items how long do you expect algorithm C to take? Hint: $2^{332} \approx 10^{100}$.

(a) about 100 seconds.
(b) about 1000 seconds.
(c) about 1000000 seconds.
(d) more time than the age of the universe.

9. ____ Which is true of the recurrence

$$T(n) = \begin{cases} 1, & \text{for } n = 1, \\ 3T(n/2) + O(n), & \text{for } n > 1 \end{cases} ?$$

(a) $T(n)$ describes the runtime of Karatsuba's integer multiplication algorithm.
(b) $T(n) = \mathrm{O}(n^{\log_2(3)})$.
(c) all of the above
(d) none of the above

10. ____ The min cut, max flow theorem states that.

(a) the total maximimum flow equals the total minimal cut
(b) A minimal cut is the residual graph after removal of a maximal flow.
(c) both of the above
(d) none of the above

11. ____ Prim's algorithm is for

(a) single source shortest paths on unweighted graphs
(b) minimal cuts in flow graphs.
(c) bipartite matching
(d) none of the above

12. ____ A directed graph G is a DAG (directed acyclic graph) if

(a) Each vertex has the same number of incident edges
(b) The vertices of G can be placed on a line so that all edges go from left to right.
(c) Each cycle of G has odd length.
(d) none of the above.

13. ____ Dijkstra's algorithm uses a priority queue. If it is run on a weighted graph with n vertices and m edges, which triple describes the number of calls made to insert(), extractmin(), and decreasekey(), respectively?

(a) n, n, and n.
(b) n, n, and m.
(c) n, m, and m.
(d) m, m, and m.

14. ____ Two of the following three statements are true. Which statement (of a,b,c) is false?

(a) Dijkstra's algorithm can be made to find single source longest paths just by reversing the signs of the edge weights.
(b) Prim's's algorithm can be made to find maximal weight spanning trees just by reversing the signs of the edge weights.
(c) Kruskal's's algorithm can be made to find maximal weight spanning trees just by reversing the signs of the edge weights.
(d) All three of a, b, c are true.

15. ____ This question concerns the containment relationshops among the problem complexity classes P (polytime), NP (nondeterministic polytime), and NPC (NP complete problems).

(a) $P \subseteq NP \subseteq NPC$.
(b) $NPC \subseteq NP \subseteq P$.
(c) $P \subseteq NP$ and $NPC \subseteq NP$.
(d) $P \subseteq NPC \subseteq NP$.

16. ____ Select the most accurate statement

(a) Cook's theorem shows that Circuit Satisfiability can be reducted to every NP complete problem.
(b) Cook's theorem shows that every problem in NP can be reduced to Circuit Satisfiability.
(c) Cook's theorem shows that P = NP.
(d) None of the above

17. ____ If a dynamic programming problem can be expressed in terms of building up an $n \times n$ table with each entry depending on *all* the entries before it in it's row and some entries above it in it's column (with a constant amount of arithmetic done with each entry accessed, for instance finding the minimum of those entries), then the run time of the algorithm is

(a) $O(n)$ (b) $O(n \log(n))$ (c) $O(n^2)$ (d) $O(n^3)$

18. ____ Consider edit distance when substitutions, insertions, and deletions are the three possible edit actions. What is the edit distance of these two strings: "summer" and "sampler"?

19. ____ Which of the following is *not* an invariant maintained in balanced binary search tree system called red-black trees?

(a) The root is black
(b) There are no two red nodes in a row on a path from a leaf to the root.
(c) There are no two black nodes in a row on a path from a leaf to the root.
(d) The key at each node is no less than any key in the tree rooted at it's left child and no greater than any key in the tree rooted at it's right child.

20. (9 points) Give definitions for each of these problem classes

(a) P (polynomial time)

(b) NP (nondeterministic polynomial time)

(c) NP-complete

21. (9 points) The Hamiltonian cycle problem (HAM) on an unweighted undirected graph is to determine if graph has a cycle that passes through each vertex exactly once. The travelling salesman problem (TSP) on a edge weighted undirected graph and number k, is to determine if the graph has a cycle with total weight less than k. HAM is NP Complete and TSP is in NP.

    (a) ____ To show that TSP is NP Complete, which reduction would you need to do?

        i. Reduce HAM to TSP.

        ii. Reduce TSP to HAM.

    (b) ____ To show that problem A reduces to problem B you construct an input y to problem B from an input x of problem A so that:

        i. If the answer to A on input x should be "true" then the answer to B on input y is "true".

        ii. If the answer to B on input y is "true" then the correct answer to A on input x is "true".

        iii. both of the above.

        iv. none of the above.

    (c) Explain the appropriate construction of input for one problem from input of the other to do the reduction that shows TSP is NP Complete, given that HAM is.

22. (9 points) Consider the polynomial $F(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, where the coefficients a, b, c, d, e, f are numbers.

    (a) What is the even part of $F(x)$?

    (b) What is the odd part of $F(x)$?

    (c) $G(x)$ is a polynomial with even part $G_e(x)$ and odd part $G_o(x)$. Thus $G(x) = G_e(x^2) + xG_o(x^2)$. If $G_e(4) = 7$ and $G_o(4) = 1$, what are the values of $G(2)$ and $G(-2)$?

23. (10 points) Kruskal's MST algorithm uses a dynamic disjoint set system called union-find. union(a,b) joins the sets containing a and b. Find(a) returns the root (CEO) of the set containing a. Assume the sets are maintained as trees with each node v having a parent pointer $p[v]$. The root node of a set is identified by $p[v] = v$. Weights are stored in $w[v]$, with initial value $w[v] = 1$ for each v.

   (a) implement union using the union by weight heuristic.

   (b) implement find using the path compression heuristic.

   (c) With those two heuristics, a sequence of n union and find operations runs in $O(n \log(n))$ time. Actually it is better than that. Give a better big-O bound for this.

24. (6 points) Assume you have access to an array Memo of size greater than the intended actual parameter n. Also the entries of Memo are initially 0. Memoize this function to achieve a dynamic programming implementation..

```
int f(int n) {
    if ( n < 4 ) ans = 1;



    else ans = min(2f(n-1)- f(n-3), f(n-2));



    return ans;
}
```