
In this report, we compare the prediction algorithms we use with the scheme (theoretical and simulation results) proposed in [1], which we call the MD scheme.

I. Capacity of the MD Scheme

In the MD scheme, three time slots are used to transmit four symbols (i.e., two desired symbols for each user). In order to detect the symbol vector $x_1$ with two symbols for the first user, we need to solve the following linear equations:

$$y_1[1] = h_{1,1}^H x_1 + n_1[1]$$
$$y_1[3] = h_{1,3}^H \begin{bmatrix} h_{1,2}^H x_2 + h_{2,1}^H x_1 \\ 0 \end{bmatrix} + n_1[3],$$

where $h_{i,k}$ represents the channel vector (length 2) for the $i$th user in the $k$th time slot. Since the interference term $h_{1,2}^H x_2$ can be eliminated completely, we can rewrite the received signals as

$$y_1 = \begin{bmatrix} h_{1,1}^H \\ h_{1,3}^H \end{bmatrix} x_1 + n = H_{e1}^H x_1 + n$$

where $H_{e1}$ is the effective channel matrix. The achievable rate for the first user can be calculated based on (1). Similarly, for the second user, we have

$$y_2 = H_{e2}^H x_2 + n.$$  \hspace{1cm} (2)

Note that the average total transmit power $E(x_1^H x_1) = E(x_2^H x_2) = P$. Applying the same procedure of calculating the capacity of a MIMO channel, (given in Appendix A), the achievable rate of the first user or the second user is

$$2 \log_2 \left(1 + \frac{P}{2P_n} \right).$$  \hspace{1cm} (3)

As three time slots are used, the capacity achieved by the MD scheme is

$$\frac{4}{3} \log_2 \left(1 + \frac{P}{2P_n} \right).$$  \hspace{1cm} (4)

II. Simulation Results

In Figs. 1 and 2, the prediction algorithms we use in our paper and the MD scheme are compared. We see in Fig. 2 that the analytical bound for MD at 20 dB is 10% higher than the corresponding simulation result. Another important observation is that the performance of SU is always better than the simulation result of the MD scheme, indicating that SU is favored over MU transmission with the MD scheme.

In addition, Fig. 1 shows that, when $K = 3$, i.e., using three previous samples for prediction, both the MMSE and polynomial fitting algorithms outperform the MD scheme. Fig. 2 reveals that the MD scheme is independent of the channel correlation, as expected. If the channel changes slowly and the correlation between adjacent channel samples is close to 1, MU transmission with a prediction-based estimation schemes is preferable; otherwise, SU transmission is favorable.
Figure 1. Comparison between prediction algorithms we use and the MD scheme for different values of normalized total transmit power. $N_t = 2, N_r = 1, L = 2, f_{\text{fdtd}} = 0.12$.

Figure 2. Comparison between prediction algorithms we use and the MD scheme for different values of $f_{\text{fdtd}}$. $N_t = 2, N_r = 1, L = 2$, normalized total transmit power $= 20$ dB.
Appendix

Here, we prove why we use $P_2$ instead of $P$ in (3) and (4) by considering a point-to-point MIMO link. We also show that, when the transmit power (or the normalized transmit power $\gamma = \frac{P}{P_n}$) goes to infinity, the following equality holds.

$$E[\log(1 + \gamma|h|^2)] = \log(1 + \gamma E[|h|^2]).$$

(5)

A. Multiplexing Gain for a MIMO Channel

Consider a $2 \times 2$ MIMO channel, shown in Fig. 3. The received signal is

$$y = \mathbf{H}^\mathbf{H}\mathbf{x} + n,$$

where $E(x^Hx) = P$ is the average total transmit power. The received signals at each antenna are

$$y_1 = h_{11}x_1 + h_{21}x_2 + n_1$$

$$y_2 = h_{12}x_1 + h_{22}x_2 + n_2$$

respectively, and we can get the receive power at each antenna $E[jy_1j] = E[jy_2j] = P$. However, the desired signal power at each antenna is not $P$. This is because $h_{21}x_2$ and $h_{12}x_1$ need to be canceled if we want to detect two symbols $x_1$ and $x_2$, which implies that the desired signal power on average is $P/2$ (assuming uniform power allocation over these two streams/symbols).

From the information theoretic point of view, the MIMO channel capacity can be achieved by using SVD ($\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}$) and water-filling power allocation. The transmitter sends out $\mathbf{U}\mathbf{x}$ instead of $\mathbf{x}$. As $\mathbf{U}$ is an orthonormal matrix ($\mathbf{U}^\mathbf{H}\mathbf{U} = \mathbf{I}$), the total transmit power is still $P$. The received signal

$$y = \mathbf{H}^\mathbf{H}\mathbf{x} + n = \mathbf{V}^\mathbf{H}\mathbf{U}^\mathbf{H}\mathbf{U}\mathbf{x} + n = \mathbf{V}^\mathbf{H}\mathbf{Ax} + n,$$

which is multiplied by $\mathbf{V}$ at the receiver

$$y' = \mathbf{V}y = \Lambda\mathbf{x} + n'. $$

As $\mathbf{V}$ is orthonormal, $n' = \mathbf{V}n$ and $n$ have the same distribution and noise power $P_n$.

$$y_1' = \lambda_1 x_1 + n_1'$$

$$y_2' = \lambda_2 x_2 + n_2',$$

where $\lambda_i$ is the singular value of $\mathbf{H}$. If the channel and the noise are complex Gaussian with zero mean and unit variance, the mean of $\lambda_i$ is 1. Assuming uniform power allocation over all streams/symbols, the average transmit power is $|x_1|^2 = |x_2|^2 = P/2$. Thus, we can get the average sum rate of the two streams as

$$E[2\log_2(1 + \frac{P|h|^2}{2P_n})] \approx 2\log_2 \left(1 + \frac{P}{2P_n}\right).$$

(7)
Note that this approximation holds when the transmit power goes to infinity, which is proved in Part B below.

For a MU-MIMO channel with a two-antenna AP and two single-antenna users, the sum rate is also $2 \log_2(1 + \frac{P}{2P_n})$ if we assume the total transmit power is $P$. Instead of SVD, zero-forcing beamforming is used.

**B. Mean of $\log$ Versus $\log$ Mean**

For simplicity, we use the natural logarithm. We will show that

$$\lim_{\gamma \to \infty} \frac{\mathbb{E}[\log(1 + \gamma|h|^2)]}{\log(1 + \gamma\mathbb{E}[|h|^2])} = 1. \quad (8)$$

Assume $g = |h|^2$ and the distribution of $g$ is $f(g)$, where $g \in [0, \infty)$. When $\gamma$ is sufficiently large,

$$\mathbb{E}[\log(1 + \gamma|h|^2)] = \int_0^{\infty} \log(1 + \gamma g) f(g) dg$$
$$\approx \int_0^{\infty} \log(\gamma g) f(g) dg$$
$$= \log \gamma \int_0^{\infty} f(g) dg + \int_0^{\infty} f(g) \log g dg$$
$$= \log \gamma + \mathbb{E}[\log g],$$

and

$$\log(1 + \gamma\mathbb{E}[|h|^2]) \approx \log \gamma + \mathbb{E}[\log g].$$

Note that $\mathbb{E}[\log g]$ and $\log \mathbb{E}[g]$ are constant for any given distributions of $g$. If we consider the channel as complex Gaussian with zero mean and unit variance, $g$ is an exponential distributed random variable and $f(g) = e^{-g}$. In this case, $\int_0^{\infty} \log g e^{-g} dg \approx -0.577216$ is a constant, same as $\log \mathbb{E}[g] = \log 1 = 0$.

Therefore, we can get

$$\lim_{\gamma \to \infty} \frac{\mathbb{E}[\log(1 + \gamma|h|^2)]}{\log(1 + \gamma\mathbb{E}[|h|^2])} = 1.$$

**References**