Grammar Class Inclusion Tree

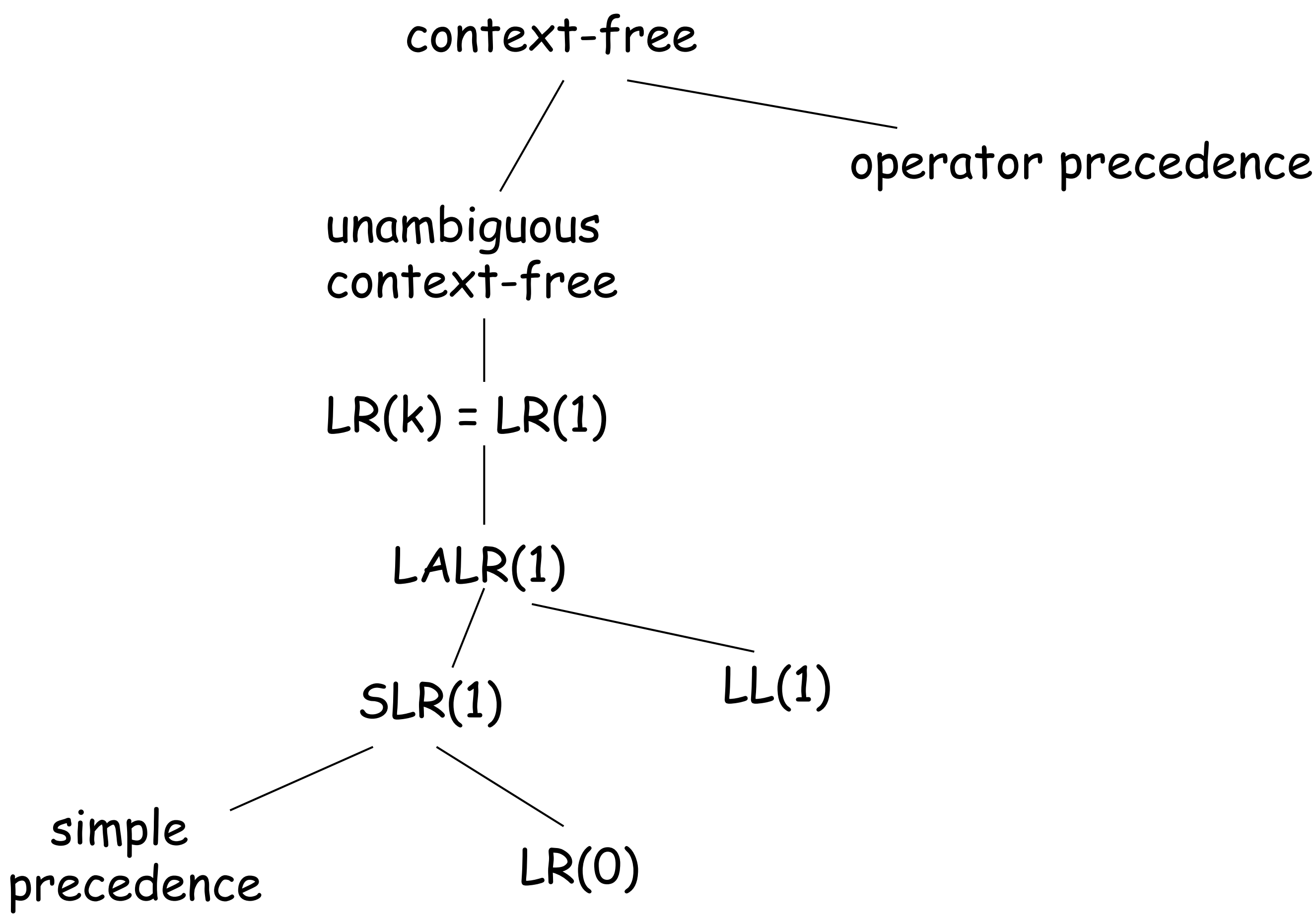
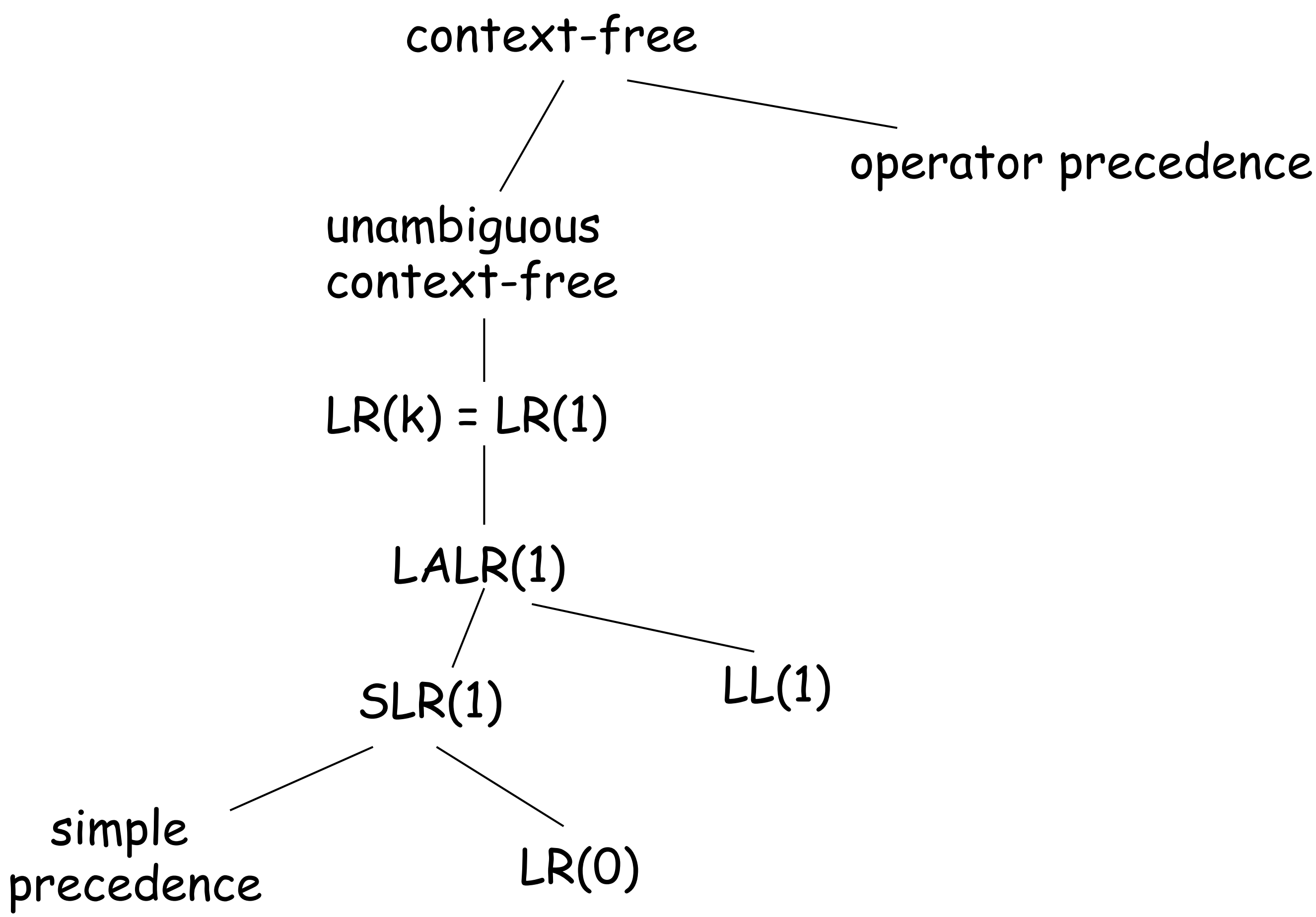


Table-driven Bottom-up Parsing

- Start at the leaves and grow toward root
- Bottom-up parsers handle a large class of grammars
- Most prevalent is based on LR(k)
- Why LR Parsing ?
  - Recognize many programming languages
  - Detect Syntax Errors
  - No backtracking

Grammar Class Inclusion Tree



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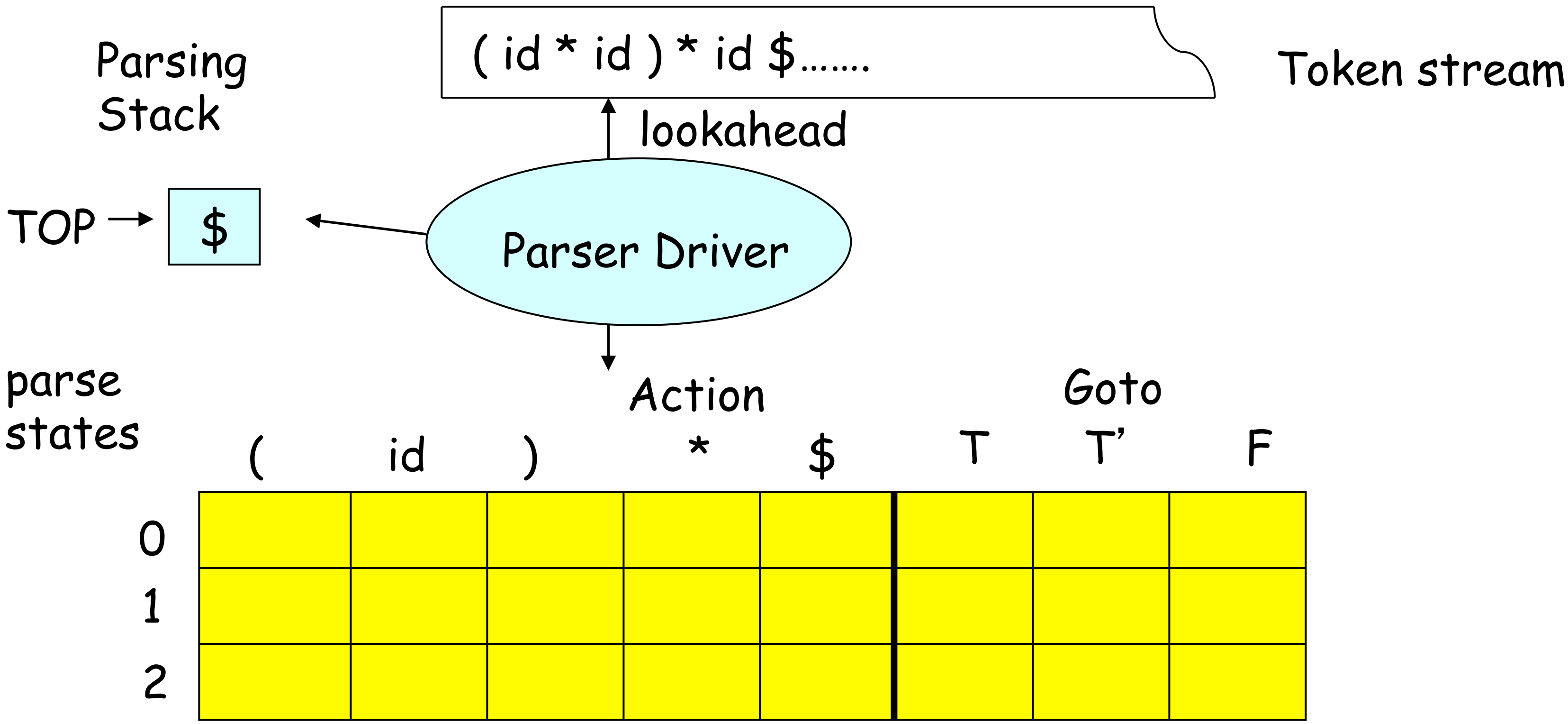


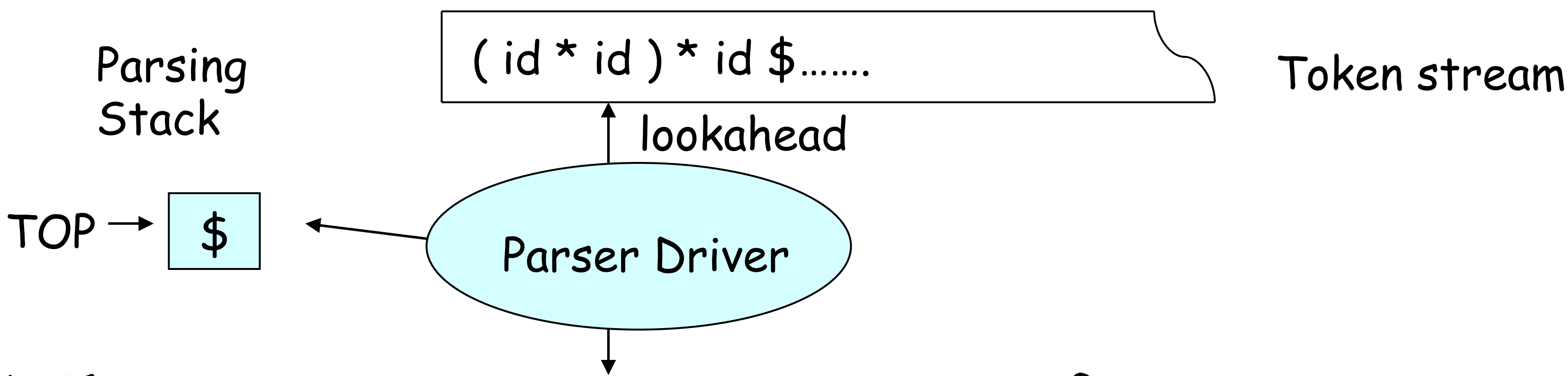
Table[state,terminal] =
- shift token and state onto stack.
- reduce by production A -> β
  pop rhs from stack; push A; push next state given by Goto[exposed state,A]
- accept
- error

Handle

- The parser must find a substring β of the tree's frontier that
  - matches some production $A \rightarrow \beta$ that occurs as one step in the rightmost derivation

- We call this substring β a handle

Table-driven Bottom-up Parsing



parse states	Action: (	id	)	*	$	Goto: T	T'	F
0								
1								
2								

Table[state,terminal] =

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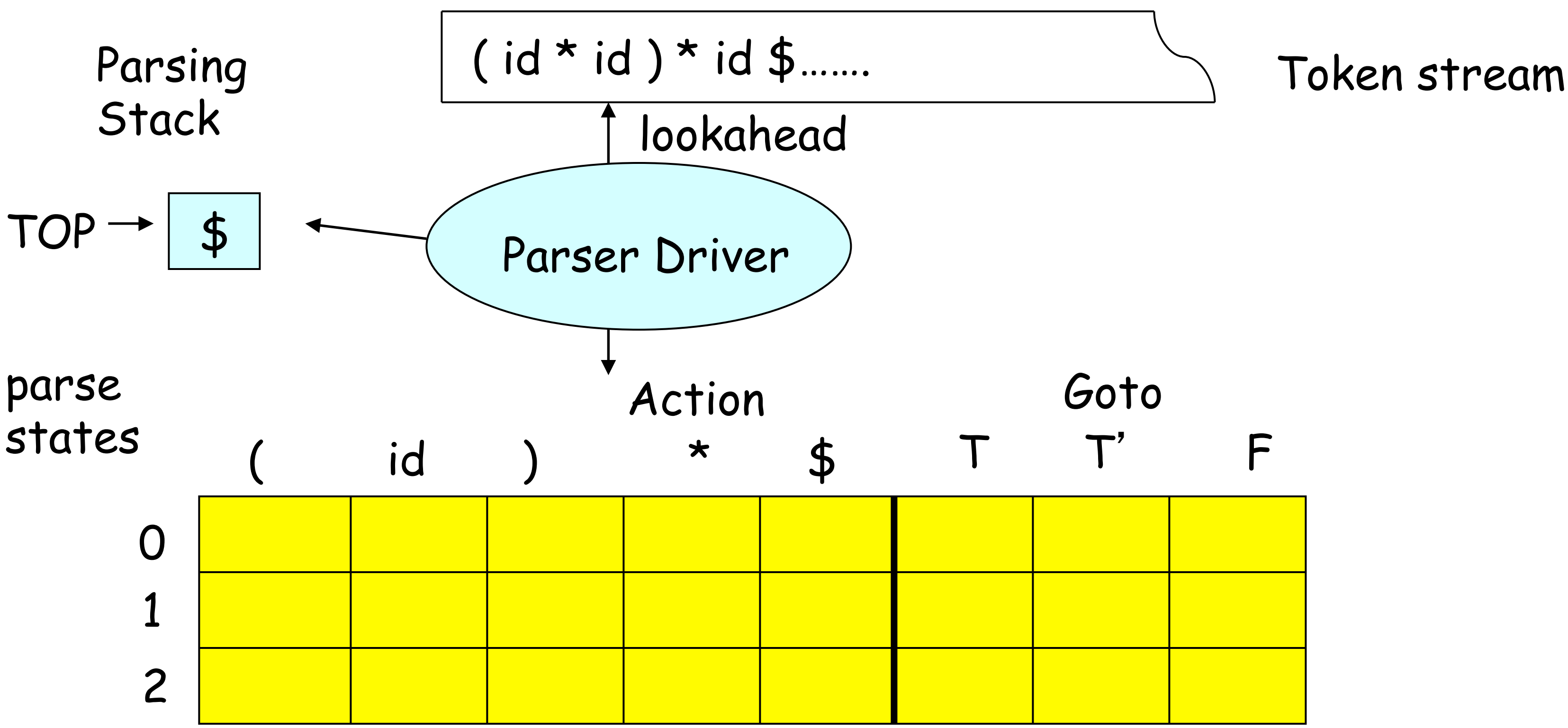

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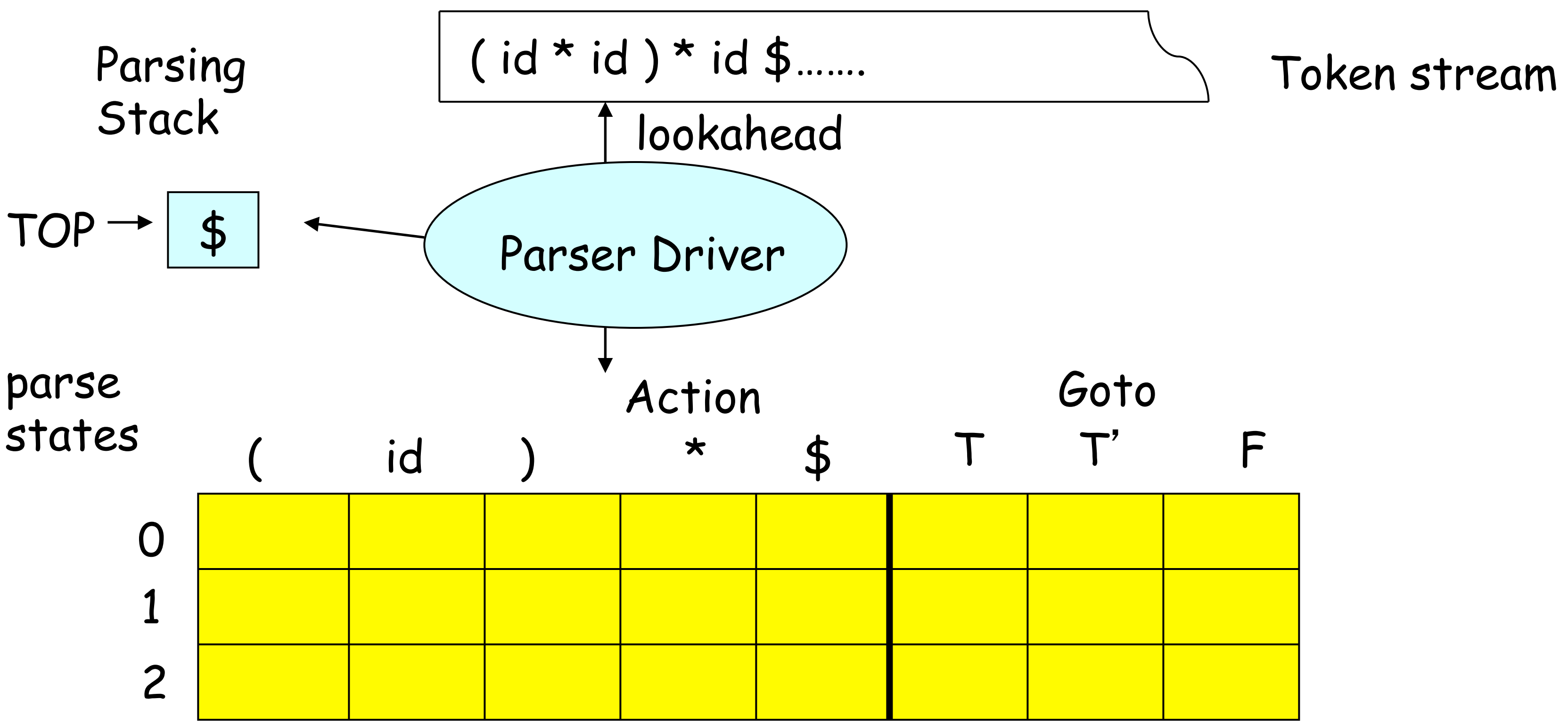

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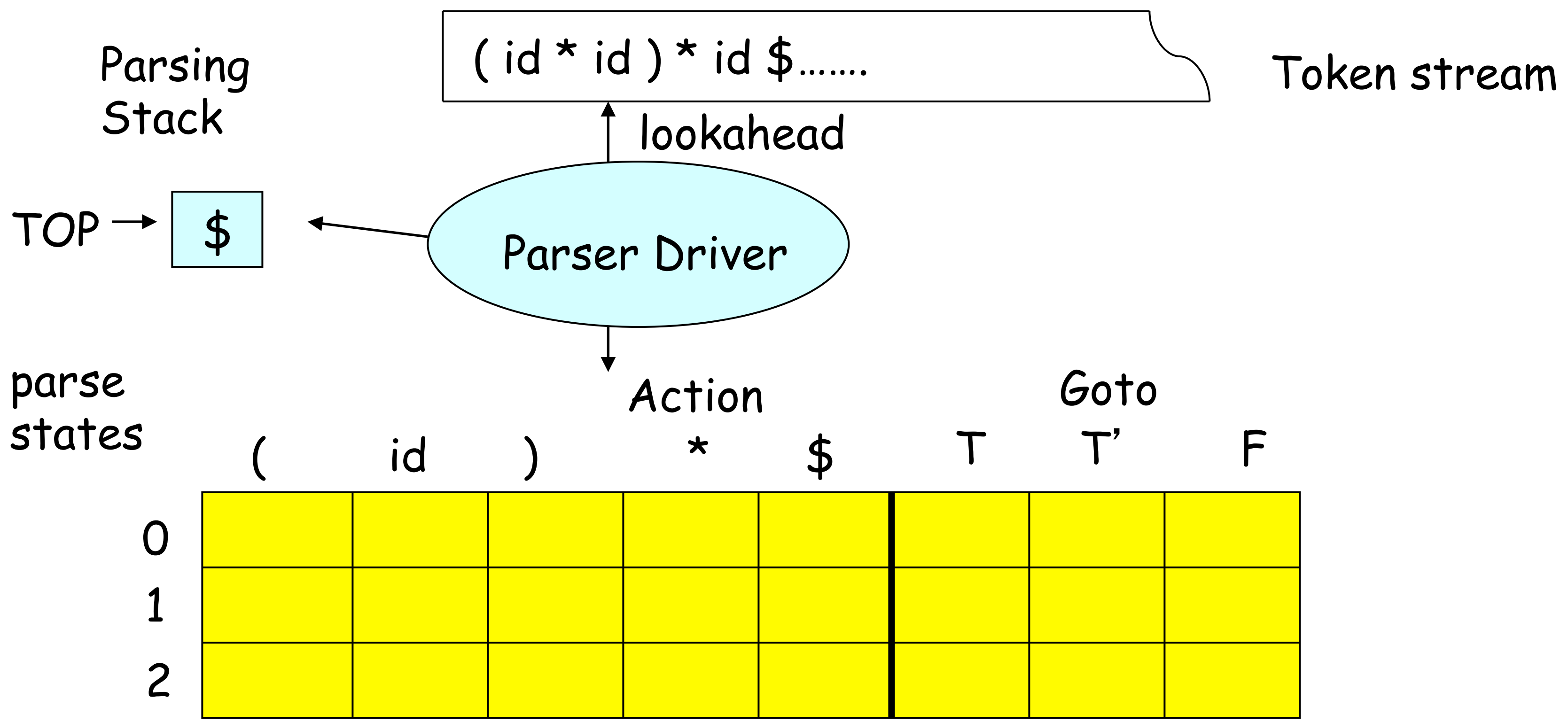

Table[state,terminal] =
- shift token and state onto stack.
- reduce by production A -> β
  pop rhs from stack; push A; push next state given by Goto[exposed state,A]
- accept
- error

LR Parsing Example

1: P -> b S e
2: S -> a ; S
3: S -> b S e ; S
4: S -> ε

Parse Table

state	b	e	a	;	$	P	S
0	s1						
1	s4	r4	s5				2
2		s3					
3					accept		
4	s4	r4	s5				7
5				s6			
6	s4	r4	s5				10
7		s8					
8				s9			
9	s4	r4	s5				11
10		r2					
11		r3					

Stack	Input
0	ba;a;e$
0b1	a;a;e$
0b1a5	;a;e$
0b1a5;6	a;e$
0b1a5;6a5	;e$
0b1a5;6a5;6	e$
0b1a5;6a5;6S10	e$
0b1a5;6S10	e$
0b1S2	e$
0b1S2e3	$
accept!	

LR Parsing Example

1: P -> b S e
2: S -> a ; S
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Stack	Input
0	ba;a;e$
0b1	a;a;e$
0b1a5	;a;e$
0b1a5;6	a;e$
0b1a5;6a5	;e$
0b1a5;6a5;6	e$
0b1a5;6a5;6S10	e$
0b1a5;6S10	e$
0b1S2	e$
0b1S2e3	$
accept!	

Parse Table

state	b	e	a	;	$	P	S
0	s1						
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2		s3					
3					accept		
4	s4	r4	s5				7
5				s6			
6	s4	r4	s5				10
7		s8					
8				s9			
9	s4	r4	s5				11
10		r2					
11		r3					

LR Parsing Example

1: P -> b S e
2: S -> a ; S
3: S -> b S e ; S
4: S -> ε

Parse Table

state	b	e	a	;	$	P	S
0	**s1**						
1	s4	r4	s5				2
2		s3					
3					accept		
4	s4	r4	s5				7
5				s6			
6	s4	r4	s5				10
7		s8					
8				s9			
9	s4	r4	s5				11
10		r2					
11		r3					

Stack	Input
0	**b**a;a;e$
0**b**1	a;a;e$
0b1a5	;a;e$
0b1a5;6	a;e$
0b1a5;6a5	;e$
0b1a5;6a5;6	e$
0b1a5;6a5;6S10	e$
0b1a5;6S10	e$
0b1S2	e$
0b1S2e3	$
accept!	

LR Parsing Example

1: P -> b S e
2: S -> a ; S
3: S -> b S e ; S
4: S -> ε

Parse Table

state	b	e	a	;	$	P	S
0	s1						
1	s4	r4	**s5**				2
2		s3					
3					accept		
4	s4	r4	s5				7
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6	s4	r4	s5				10
7		s8					
8				s9			
9	s4	r4	s5				11
10		r2					
11		r3					

Stack	Input
0	ba;a;e$
0**b**1	**a**;a;e$
0b1**a**5	;a;e$
0b1a5;6	a;e$
0b1a5;6a5	;e$
0b1a5;6a5;6	e$
0b1a5;6a5;6S10	e$
0b1a5;6S10	e$
0b1S2	e$
0b1S2e3	$
accept!	

LR Parsing Example

1: P -> b S e
2: S -> a ; S
3: S -> b S e ; S
4: S -> ε

Stack	Input
0	ba;a;e$
0**b**1	a;a;e$
0b1a5	;a;e$
0b1a5;6	a;e$
0b1a5;6a5	;e$
0b1a5;6a5;**6**	**e**$
0b1a5;6a5;6S10	e$
0b1a5;6S10	e$
0b1S2	e$
0b1S2e3	$
accept!	

Parse Table

state	b	e	a	;	$	P	S
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1	s4	r4	**s5**				2
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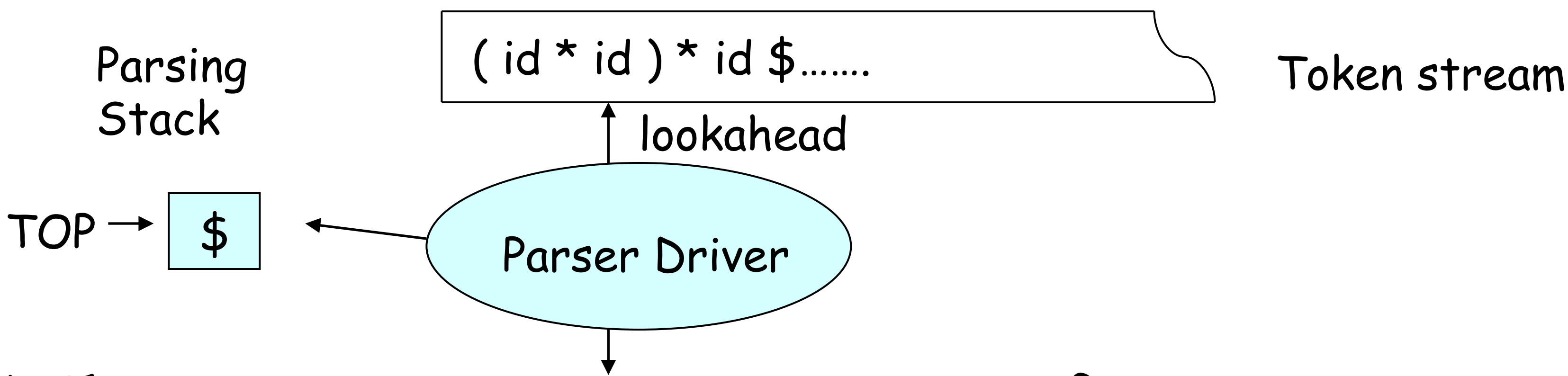
Table-driven Bottom-up Parsing



parse states	(	id	)	*	$	T	T'	F
	Action					Goto		
0								
1								
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Table[state,terminal] =
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LR Parsing Example

1: P -> b S e
2: S -> a ; S
3: S -> b S e ; S
4: S -> ε

Parse Table

state	b	e	a	;	$	P	S
0	s1						
1	s4	r4	**s5**				2
2		s3					
3					accept		
4	s4	r4	s5				7
5				s6			
6	s4	**r4**	s5				10
7		s8					
8				s9			
9	s4	r4	s5				11
10		r2					
11		r3					

Stack	Input
0	ba;a;e$
0**b**1	a;a;e$
0b1a5	;a;e$
0b1a5;6	a;e$
0b1a5;6a5	;e$
0b1a5;6a5;**6**	**e**$
0b1a5;6a5;6S10	e$
0b1a5;6S10	e$
0b1S2	e$
0b1S2e3	$
accept!	

LR Parsing Example

1: P -> b S e
2: S -> a ; S
3: S -> b S e ; S
4: S -> ε

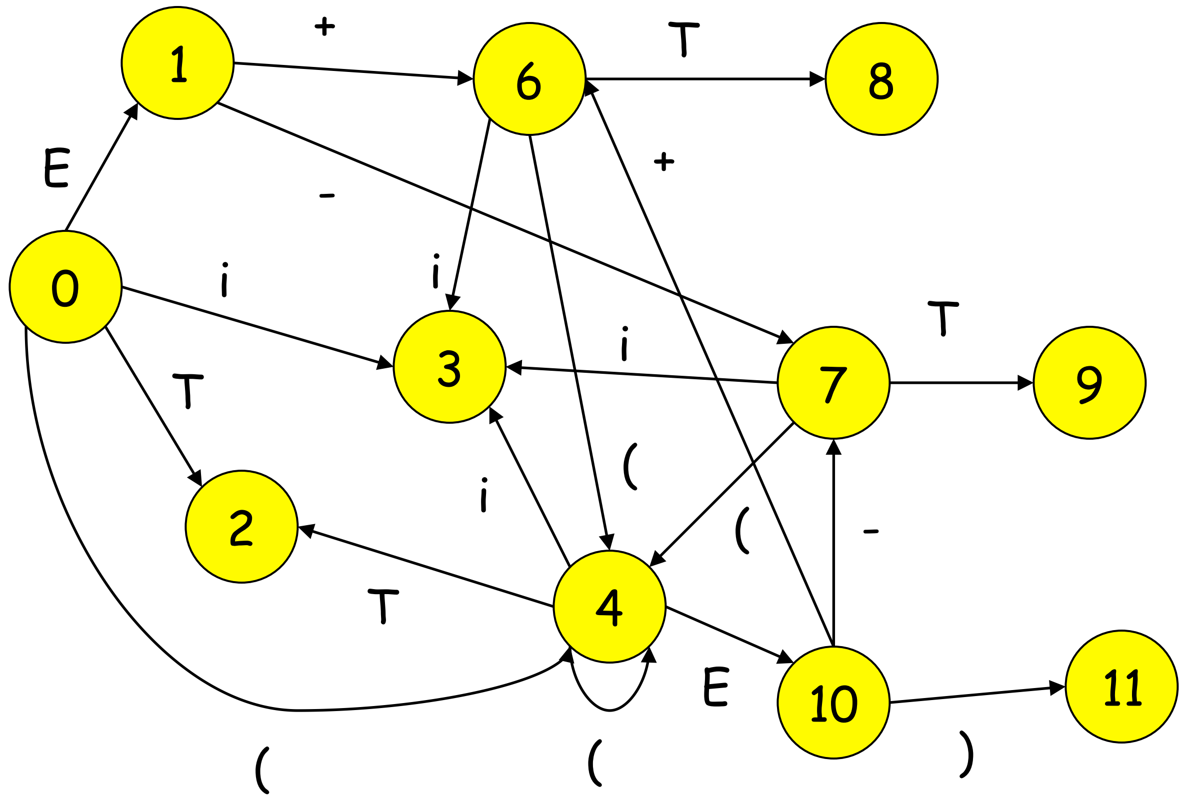
Parse Table

state	b	e	a	;	$	P	S
0	s1						
1	s4	r4	**s5**				2
2		s3					
3					accept		
4	s4	r4	s5				7
5				s6			
6	s4	r4	s5				10
7		s8					
8				s9			
9	s4	r4	s5				11
10		**r2**					
11		r3					

Stack	Input
0	ba;a;e$
0**b**1	a;a;e$
0b1a5	;a;e$
0b1a5;6	a;e$
0b1a5;6a5	;e$
0b1a5;6a5;6	e$
0b1a5;6**a5;6S10**	**e**$
0b1a5;6S10	e$
0b1S2	e$
0b1S2e3	$
accept!	

DFA for parser

S -> E
E -> T | E + T | E - T
T -> I | (E)



Reduce States:

3: T -> i
2: E -> T
8: E -> E + T
9: E -> E - T
11: T -> (E)
1: (on $) S -> E

stack	input
0	i-(i+i)$
0i3	-(i+i)$
0T2	-(i+i)$
…	

LR Parsing Another Example

1: E -> E + T
2: E -> T
3: T -> T * F
4: T -> F
5: F -> (E)
6: F -> id

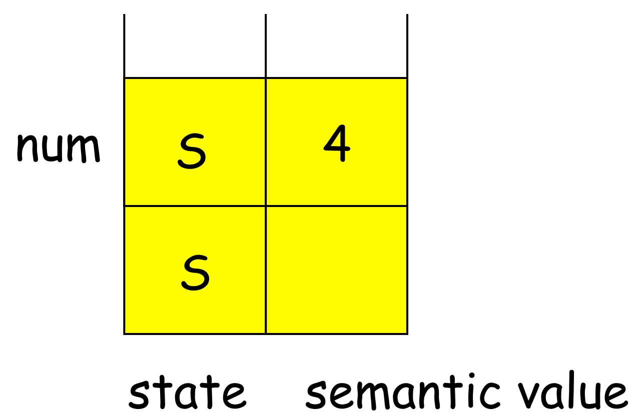
Parse Table

State	action						goto		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Semantic Actions during Parsing

```
S -> E          { $$ = $1;  root = $$; }
E -> E + T      { $$ = makenode('+', $1, $3);}  // E is $1, - is $2, T is $3
E -> E - T      { $$ = makenode('-', $1, $3);}
E -> T          { $$ = $1;}                       // $$ is top of stack
T -> ( E )      { $$ = $2;}
T -> id         { $$ = makeleaf('idnode', $1);}
T -> num        { $$ = makeleaf('numnode', $1);}
```

Consider parsing 4 + (x - y)



Parsing Stack

Items and States

LR(0) item - of a grammar G is a production of G with a dot at some position of the body

For example: A-> XYZ

A -> . XYZ
A -> X.YZ
A -> XY.Z
A -> XYZ.

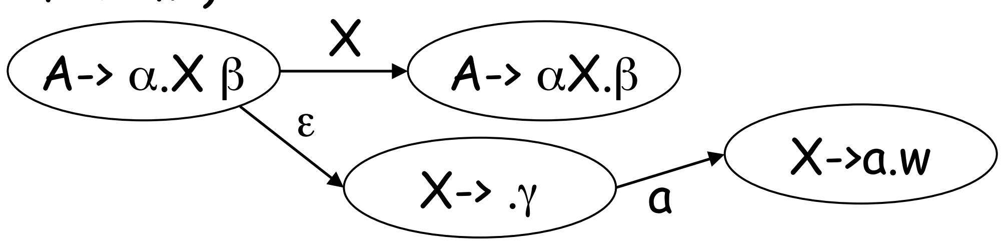
Building LR(0) and SLR(1) Parse Tables

1. Augment grammar
   - Add a production S' -> S, where S is original start state
   - Causes one ACCEPT table entry when reduce S' ->S on $.

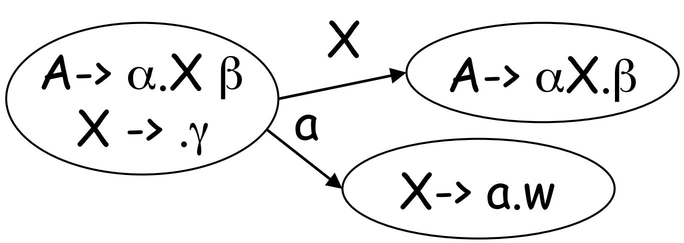
2. Create DFA from grammar

   item A -> α . β
   - just seen a string derivable from α
   - expect to see a string derivable from β

NFA : Each state represents a set of recognized viable prefixes (kernel set of items)



DFA: Subset construction to go from NFA to DFA = closure(kernel)



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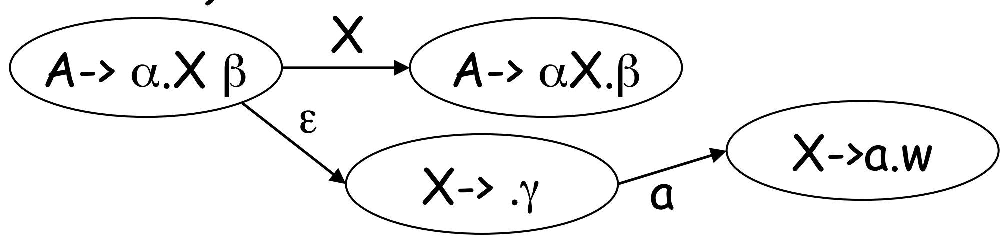

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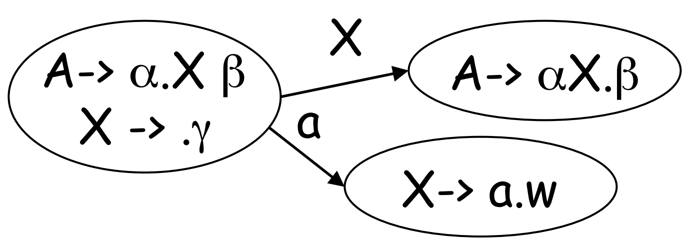

Closure(item set I)

Given a set of kernel items I for a DFA state,

Closure(I) = { kernel items I

if A-> α.Bβ in I and B-> γ
then add B-> . γ to I }

Intuitively, we expect to see strings derivable from all nonterminals immediately to the right of the dot in any item in I.

Example: S-> E
E -> T | E + T | E - T
T -> i | (E)

Let I = {S-> .E}
Closure(I)=

Let I = {E-> E+.T}
Closure(I) =

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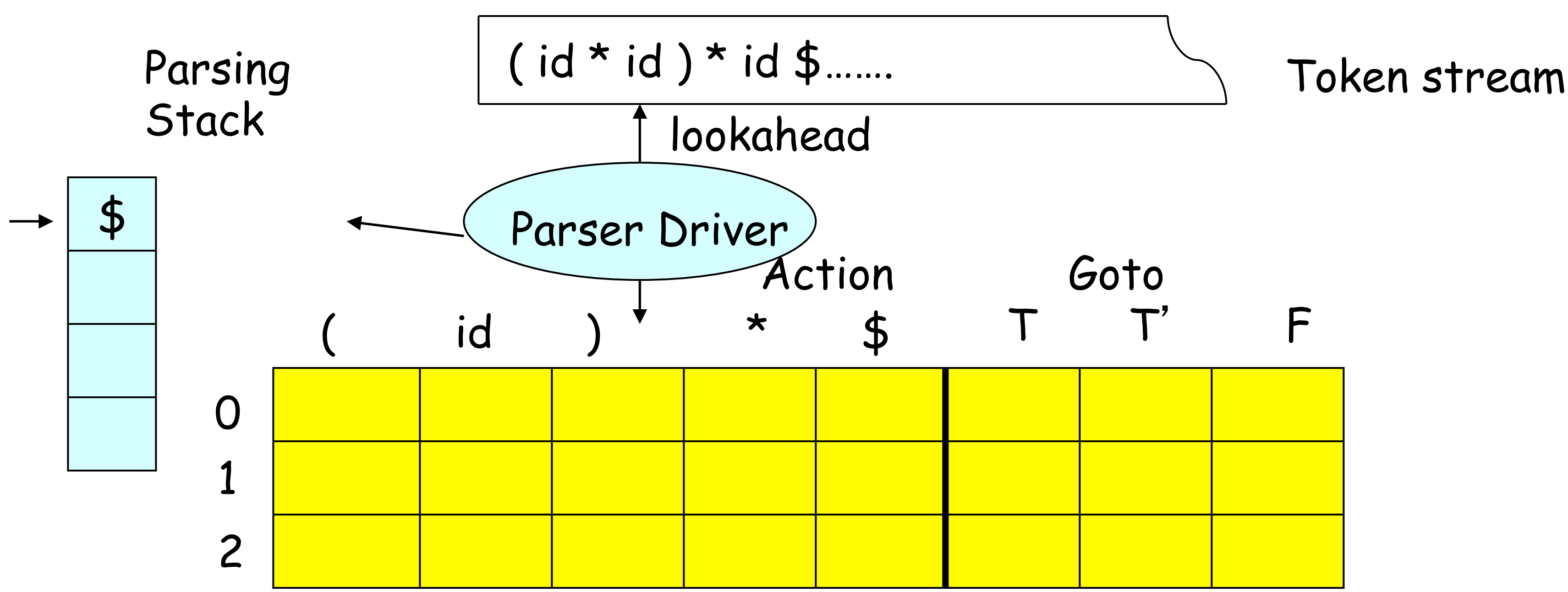
S-> .E
E -> .T
E-> .E + T
E-> .E - T
T -> .i
T->. (E)

Let I = {E-> E+.T}
Closure(I) =

Recap

- LR(k) Parsing
  - L : left to right scanning
  - R : rightmost derivation in reverse
  - k : number of input symbols of lookahead



Recap

- **Shift**
  - pushes a terminal onto the stack
- **Reduce**
  1. pops 0 or more symbols off of the stack
     - ✓ production rhs
  2. pushes a non-terminal on the stack
     - ✓ production lhs
- **Accept**
- **Error**

Recap

- LR Parsing
  - LR(0)
  - SimpleLR(1)
  - LR(1)
  - LALR
- Parser Driver is the same for all LR parsers only parsing table changes

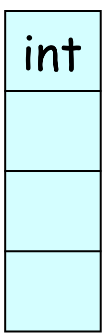
Handles

- How do we decide when to shift or reduce?
- Example grammar:

  E → T + E | T

  T → int * T | int | (E)

- Consider input: int * int + int
  - We could reduce by T → int : T * int + int



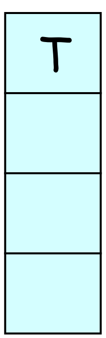
Handles

- How do we decide when to shift or reduce?
- Example grammar:

  $E \rightarrow T + E \mid T$

  $T \rightarrow \text{int} * T \mid \text{int} \mid (E)$

- Consider input: int * int + int
  - We could reduce by $T \rightarrow$ int : T * int + int
  - Mistake!
    - No way to reduce to the start symbol E



Recap

- Handle
  - A handle is a string that can be reduced and also allows further reductions back to the start symbol
- Item
  - An item is a production with a "." somewhere on the rhs

    A -> . XYZ
- State
  - Set of items

Item

- The item(s) for $X \rightarrow \varepsilon$??

Item

- The only item for $X \rightarrow \varepsilon$ is $X \rightarrow .$

Closure(item set I)

Given a set of kernel items I for a DFA state,

Closure(I) = { kernel items I

if $A \rightarrow \alpha.B\beta$ in I and $B \rightarrow \gamma$
then add $B \rightarrow .\gamma$ to I }

Intuitively, we expect to see strings derivable from all nonterminals immediately to the right of the dot in any item in I.

Example: S-> E
E -> T | E + T | E - T
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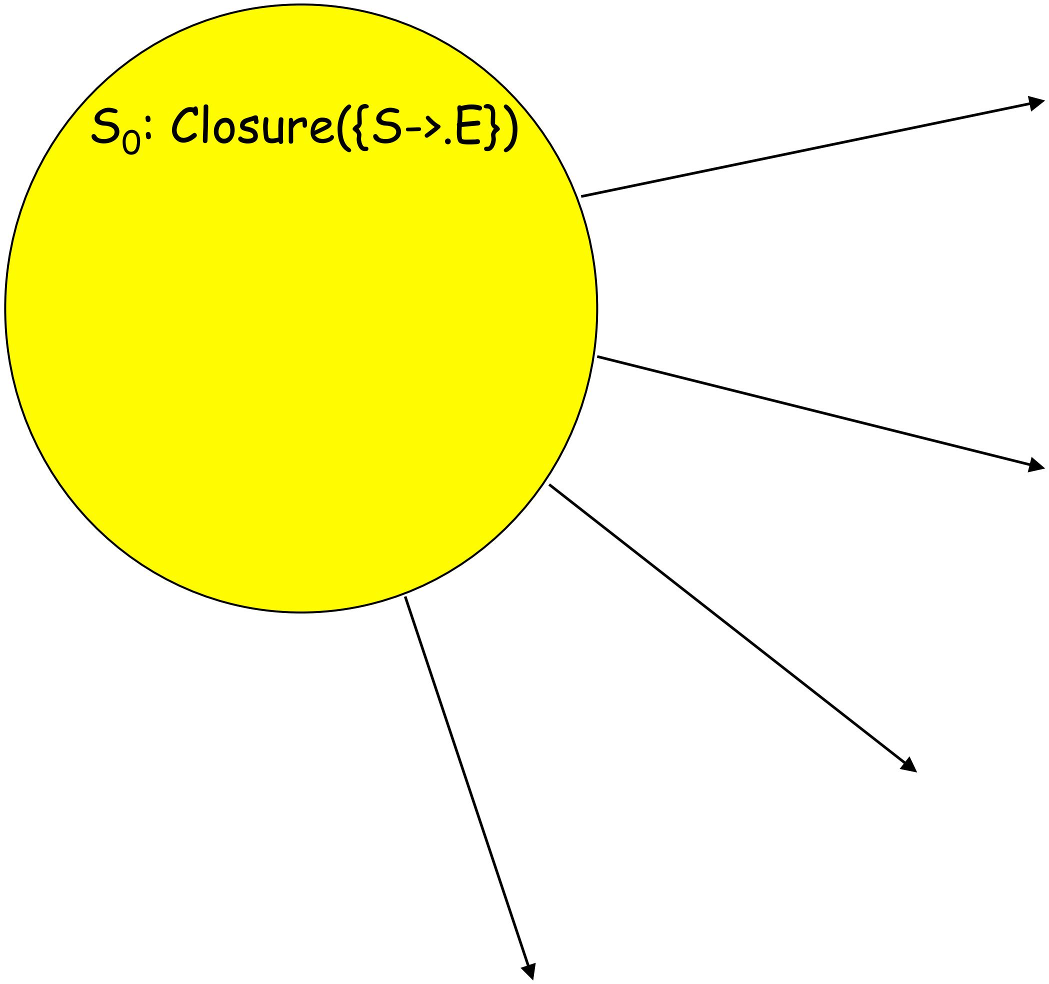
Let I = {S-> .E}
Closure(I)=

Let I = {E-> E+.T}
Closure(I) =

Example of DFA Construction



DFA Construction Algorithm

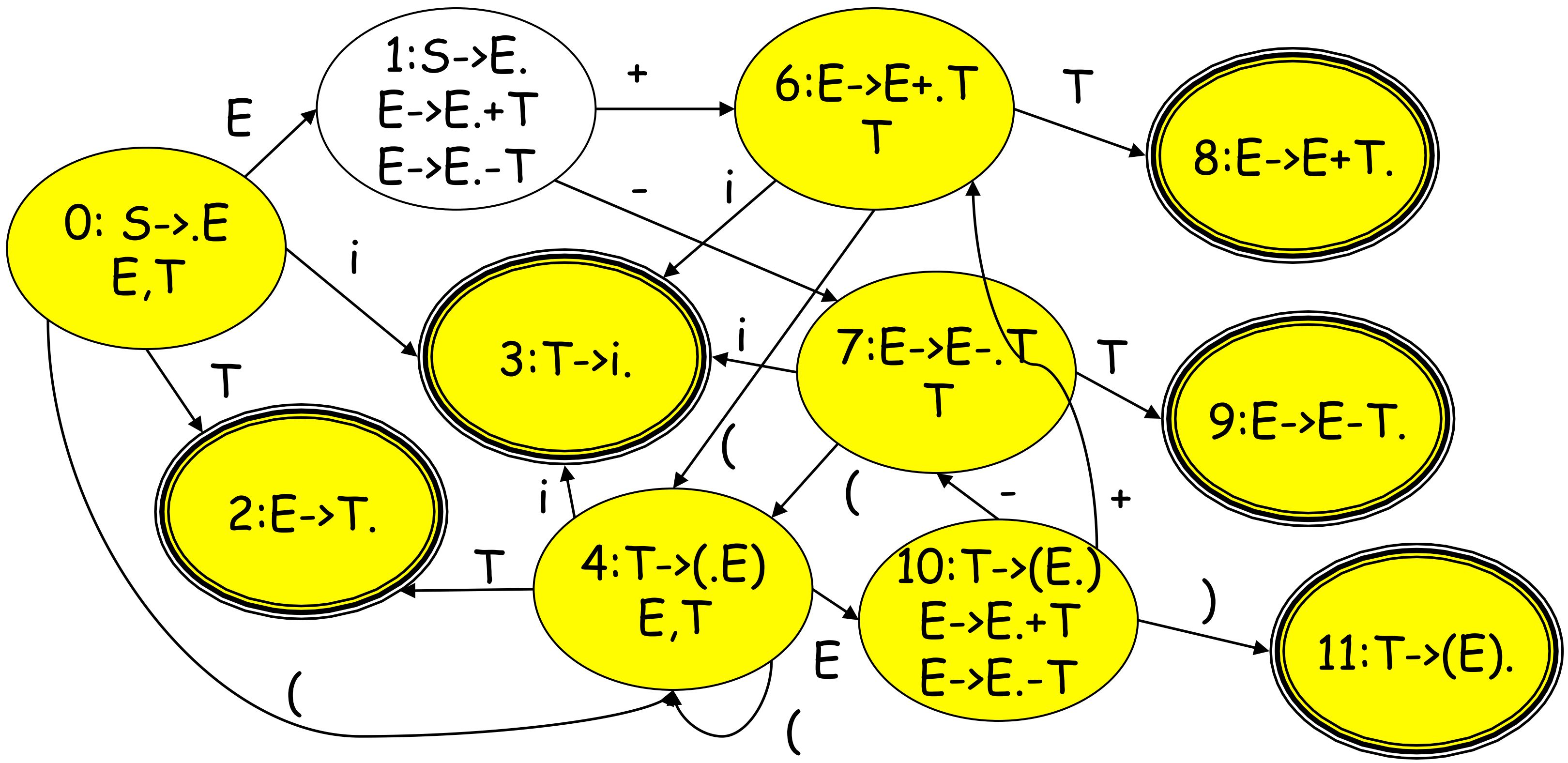
S_0 = Closure({S' -> .S});

Todo = {S_0};

```
WHILE Todo not empty DO
        Remove an item set (ie, state) Si from Todo;
        FOR each grammar symbol X DO
                FOR each A -> α.Xβ in Si DO
                        S_new = Closure(A -> αX .β);
                        If S_new is unique thus far,
                                then Add S_new to DFA
                                     Add S_new to Todo;
                        Add edge Si -> S_new labeled by X
```

Final DFA for Example

S-> E
E-> T|E+T|E-T
T-> i|(E)



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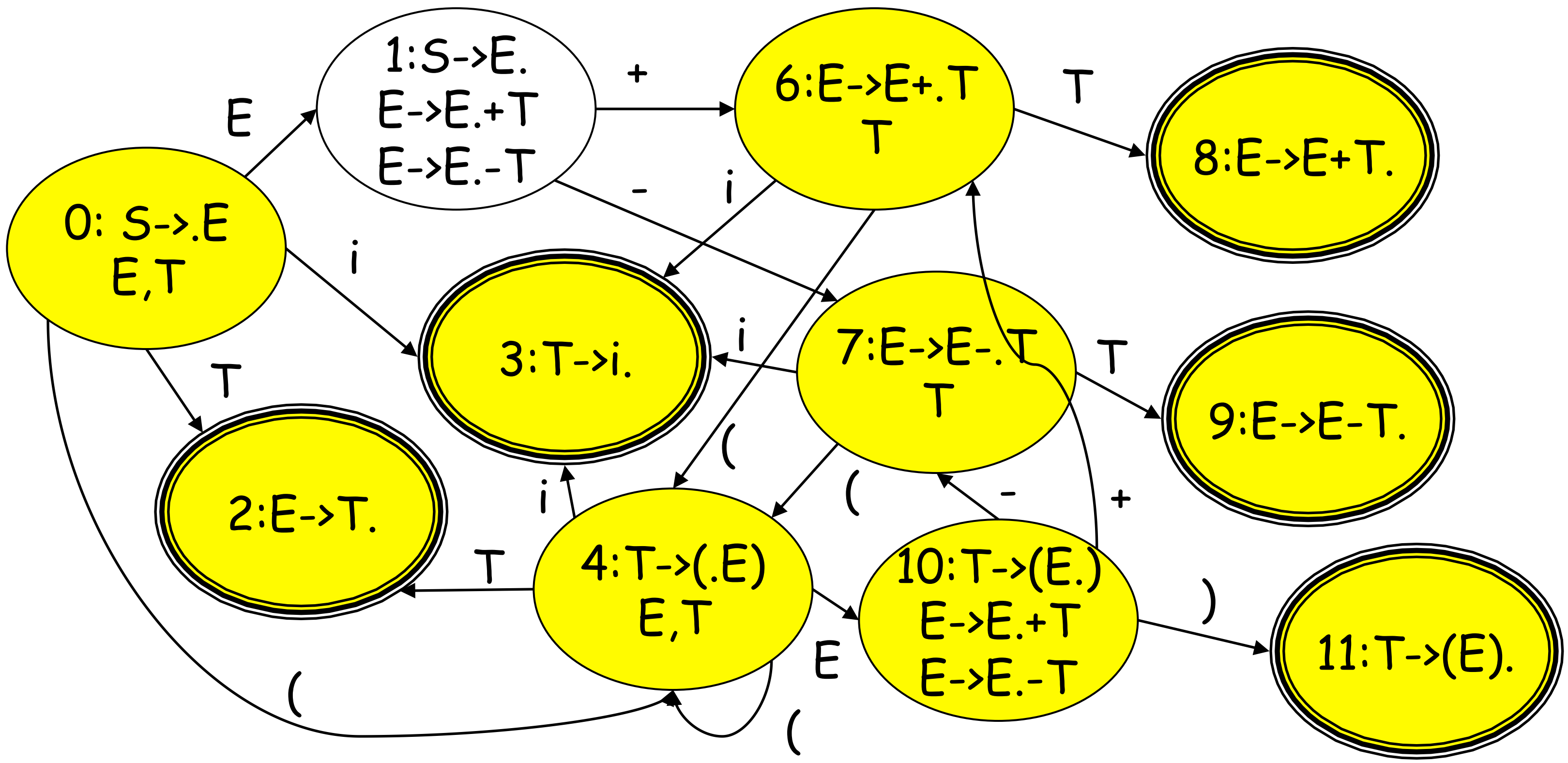
Final DFA for Example

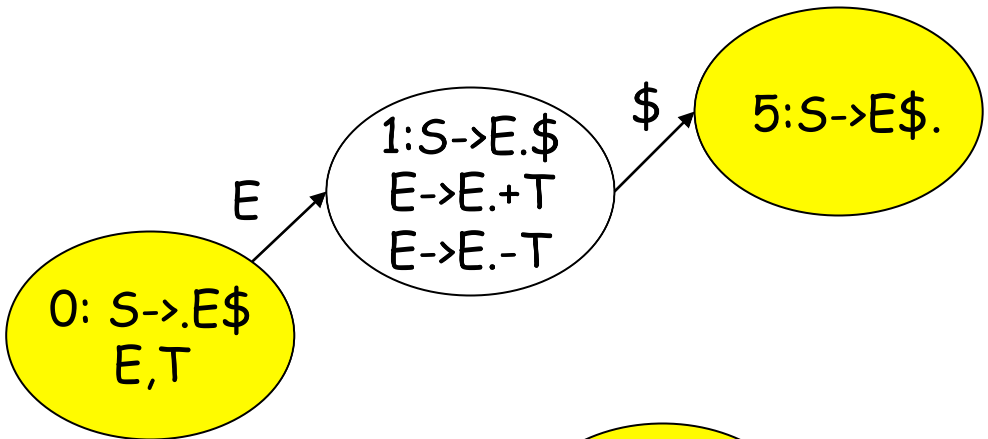


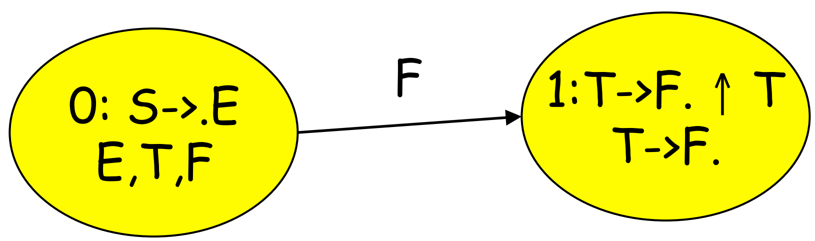
LR(0) grammar = DFA with no inadequate states,
where inadequate state has shift/reduce or reduce/reduce conflict
(e.g., state 1 is inadequate above)

SLR(1) grammar = Can resolve any inadequate states by FOLLOW info:
A -> α. and B-> β.Xδ in same state, but FOLLOW(A) $\cap$ {X} is empty.
A -> α. and B-> β. in same state, but FOLLOW(A) $\cap$ FOLLOW(B) =0

LR(0) versus SLR(1)

To convert previous grammar to LR(0): Replace S-> E by S-> E$



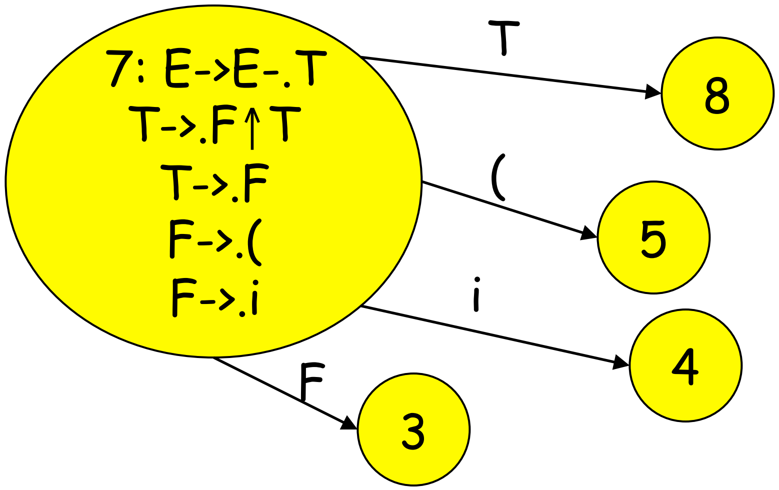


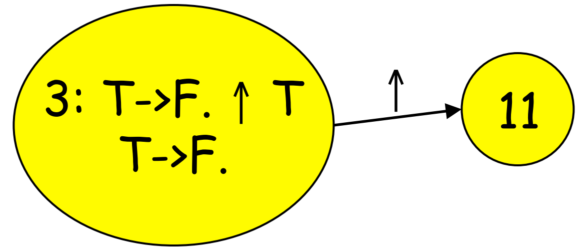
Now consider:

S-> E
E-> E-T | T
T-> F↑T | F
F-> (E) | i

- State 1 is inadequate, so not LR(0)
- FOLLOW(T) = {-,),$}
- FOLLOW(T) $\cap$ {↑} is empty, so it is SLR(1)

From DFA to SLR(1) Parse Table

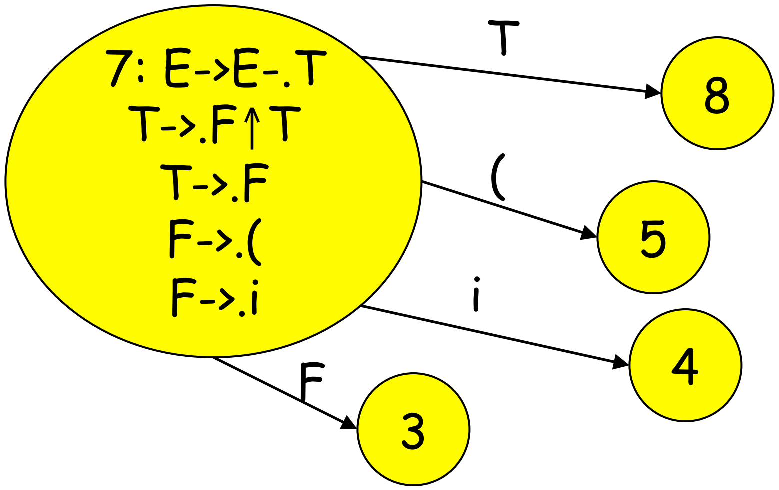


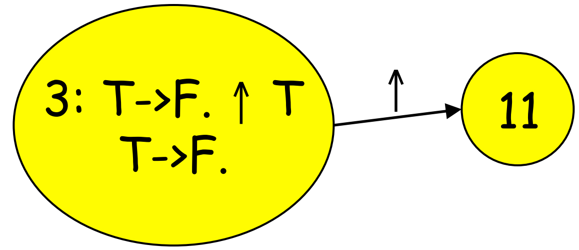


	FOLLOW
S	$
E	i,),$
T	-,),$
F	↑, -,),$

state	i	-	↑	(	)	$	S	E	T	F
3										
7										

From DFA to SLR(1) Parse Table





	FOLLOW
S	$
E	i,),$
T	-,),$
F	↑, -,),$

state	i	-	↑	(	)	$	S	E	T	F
3		r [T->F]	s11		r [T->F]	r [T->F]				
7	s4			s5					8	3

Is the grammar LR(0),SLR(1)?

LR(0):

- construct parse table with no lookahead/FOLLOW info
  If there are no multidefined entries, then LR(0)

- construct DFA. If there are no inadequate states, then LR(0).

SLR(1):

- construct parse table with FOLLOW info
  If there are no multidefined entries, then SLR(1)

- construct DFA. If there are no inadequate states, or
  for each inadequate state of the form:

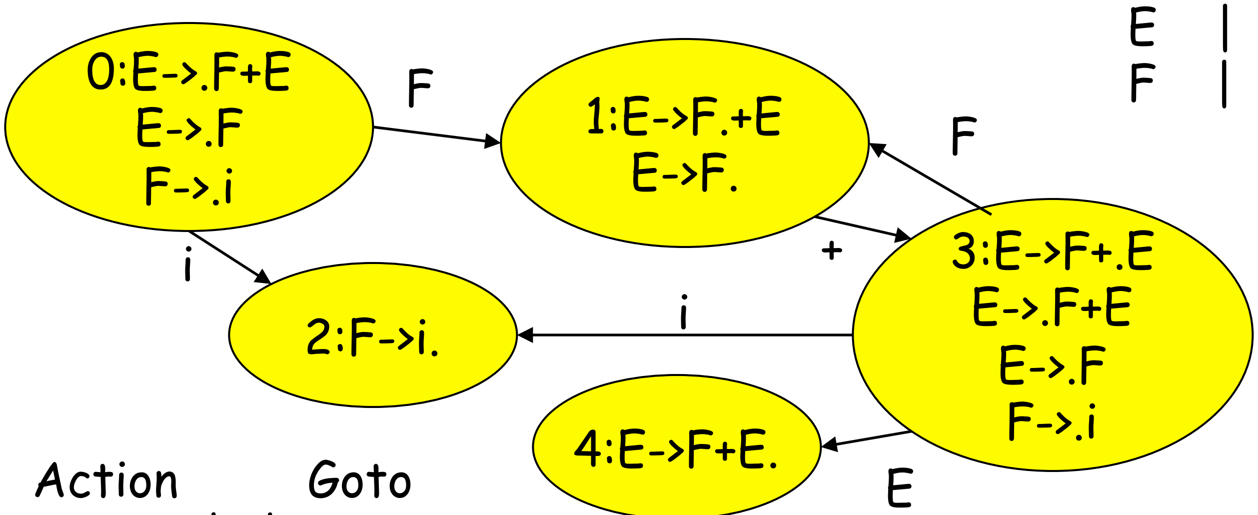
A-> α. B-> β.	FOLLOW(A) $\cap$ FOLLOW(B) is empty, AND
A-> α. B-> β.a γ	FOLLOW(A) $\cap$ {a} is empty

THEN SLR(1)

Why augment the grammar?

Consider E-> F + E | F
F -> i

FOLLOW	
E	$
F	$, +



	Action			Goto	
	+	i	$	E	F
0		s2			1
1	s3		?		
2	r3		r3		
3		s2		4	1
4			?		

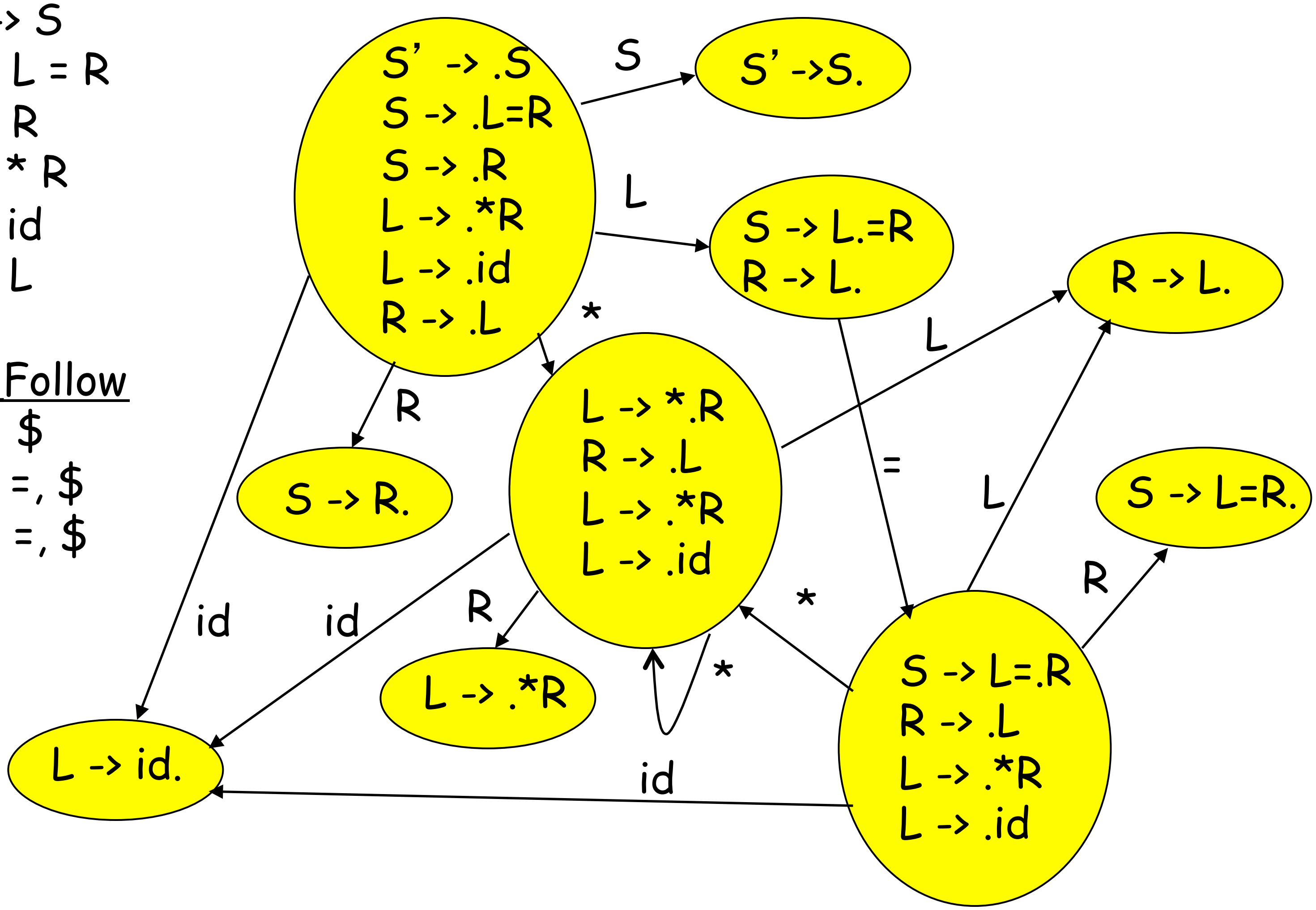
? Reduce E->F or Accept

? Reduce E-> F + E or Accept

Example – not SLR(1)

S' -> S
S -> L = R
S -> R
L -> * R
L -> id
R -> L

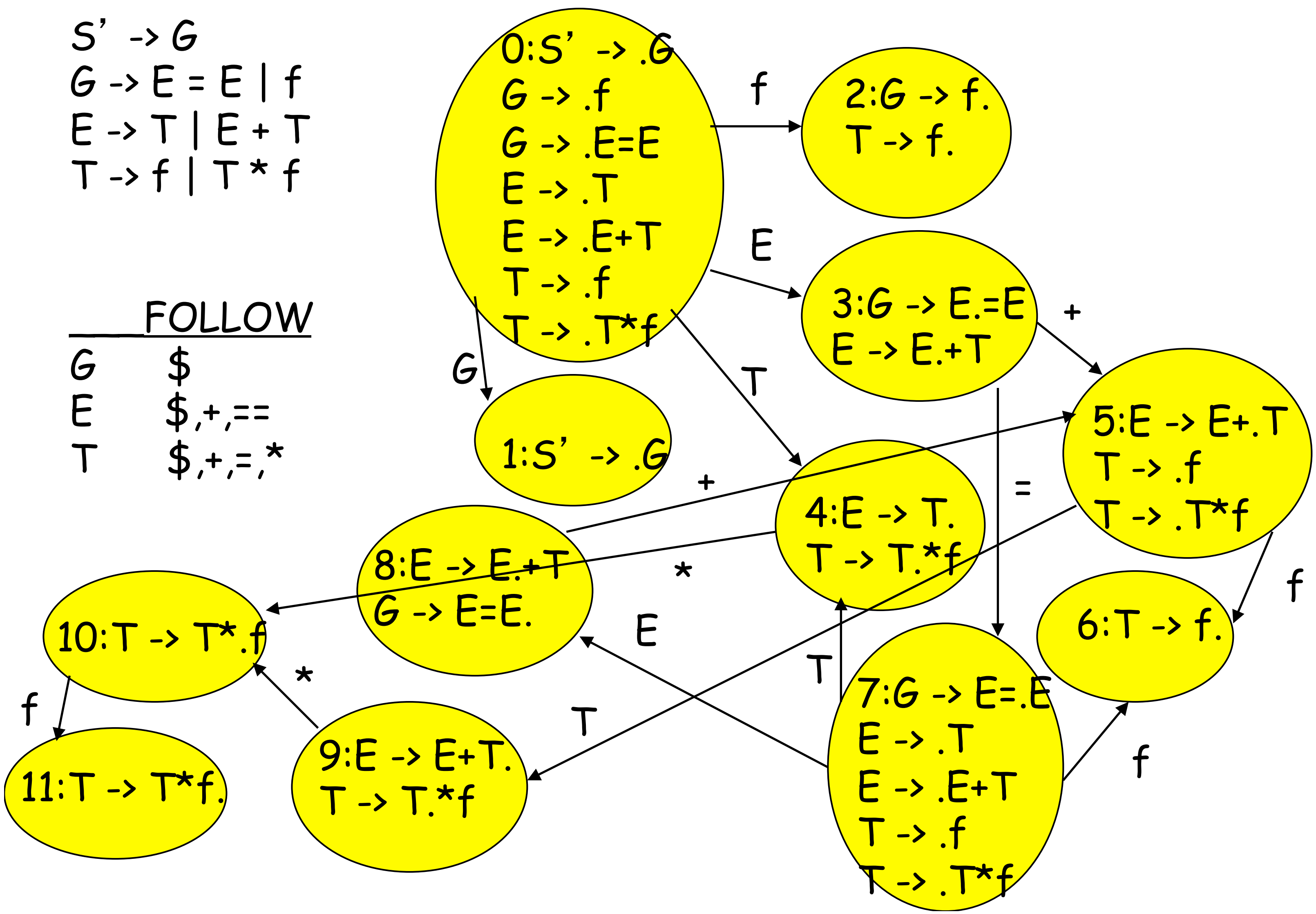
	Follow
S	$
L	=, $
R	=, $



Another Example – not SLR(1)

S' -> G
G -> E = E | f
E -> T | E + T
T -> f | T * f

	FOLLOW
G	$
E	$,+,==
T	$,+,=,*



LR(1) Parser (for same grammar)