High-Resolution Measurements of the Specific Absorption Rate Produced by Small Antennas in Lossy Media

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Abstract—We describe a thermistor-based technique for measurement of the specific absorption rate (SAR) of radio frequency energy from a small antenna in lossy matter with high spatial resolution. The apparatus employs a small thermistor probe, that is moved about the antenna using a computer-controlled positioning system. The antenna is excited by pulses of radio frequency energy, and the SAR is obtained from the measured rate of temperature increase. We present a simplified thermal model to account for heating of the thermistor by the applied electromagnetic energy, which is a major potential artifact in the method. The apparatus is useful for studies of radio frequency ablation of tissue or near-field exposures in models of the body by wireless transmitters.

I. INTRODUCTION

The specific absorption rate (SAR), or distribution of heating produced in a medium by an antenna, is a fundamental property that characterizes the performance of antennas used to heat tissue. Several authors have described techniques for mapping the SAR of antennas in liquid media whose electrical properties approximate those of tissue [1]-[3]. One approach is to measure directly the electric field in the medium using a small probe. This offers high sensitivity, but the measurements are sensitive to the polarization of the field. An alternate approach is to use a small thermistor or fiber optic temperature sensor to measure the incremental temperature increases produced in the medium when the antenna is excited by a brief pulse of radio frequency (RF) energy. This latter approach provides a direct measurement of the SAR that is insensitive to polarization but is susceptible to thermal and electrical artifacts. Fiber optic temperature sensors (which are insensitive to RF interference) are commercially available, but are expensive and do not offer the short response time needed for the present purpose.

We describe a thermistor-based technique for SAR measurements with high spatial resolution, which has been in use in our laboratories at Catholic University and the University of Pennsylvania [4], [5]. We also present a simple model for thermal artifacts due to the heating of the thermistor by the RF field.

II. METHOD

The antenna under test is mounted in a tank filled with aqueous electrolyte whose conductivity is matched to that of tissue. The antenna is excited by brief (0.25 s) pulses from a suitable transmitter. Our studies have variously employed a 30 W, 915 MHz microwave or a 30 W, 500 kHz radio frequency generator.

The probe consists of a 0.25 mm diameter thermistor (thermal response time <0.1 s) fixed at the tip of a thin glass capillary with epoxy cement. The leads of the thermistor are a twisted pair of shielded wires aligned perpendicularly to the field of the antenna. This arrangement was found to be sufficiently immune from interference to permit accurate measurements of the SAR pattern even close to antennas.

The probe is mounted on a three-axis stage that is controlled by stepping motors. After each RF pulse, the thermistor records the transient temperature rise. After the thermistor has returned to local equilibrium, the probe is moved to a new location, and the process is continued until the SAR is mapped over the entire region of interest. The complete measurement process, including movement of the probe and the recording and analysis of the thermistor output, is controlled by a laboratory computer running ASYST®. This apparatus can map the three-dimensional SAR pattern of a small antenna with a 0.5 mm spatial resolution within a few hours.

III. THERMAL MODEL

The temperature rise of the thermistor is due both to direct heating by the field (i.e., an artifact) and by heat conduction from the external medium. Heat transfer models for similar problems have been developed by Bowman et al. [6] and Ryan et al. [7]. We develop a simple model, similar to the previous work but adapted to the present application, that can be used to model the thermal artifact.

The model (Fig. 1) consists of a spherical thermistor of radius $R$, electrical conductivity $\sigma_{th}$, thermal conductivity $k_{th}$, and thermal diffusivity $\alpha_{th}$, immersed in a medium with $\epsilon_0$ (initially unperturbed) electric field $E_0$. The external medium, assumed infinite in extent, has electrical conductivity $\sigma_m$, thermal conductivity $k_m$, specific heat $c_m$, and density $\rho_m$. Initially, the thermistor and external medium are at thermal equilibrium at temperature $T_i$. The field is turned on at time $t = 0$.

The internal field in the thermistor, $E_{th}$, is given in terms of the external field $E_0$ by

\[
E_{th} = \frac{3\epsilon_0 + j\sigma_{th}}{2\epsilon_0 + j\sigma_{th}} E_0
\]

where $\epsilon_0 = \epsilon_{th} - j\sigma_{th}/(\omega\sigma_0)$ and $\epsilon_0^* = \epsilon_m^* - j\sigma_m/(\omega\sigma_0)$ are the complex permittivity of the thermistor and external media respectively, $\epsilon_0$ is the permittivity of space, and $\omega$ is the frequency in rad/s.

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In addition, the thermistor will perturb the field in the surrounding medium, adding in effect a dipolar field contribution to the initial unperturbed field. As a consequence, the SAR near the probe will vary along its surface. Exact solution of the field equations (using Maple, from Waterloo Software) shows that the SAR is increased in the medium close to the bead, but this increase falls off very quickly with distance from the probe. In fact, the volume-averaged SAR in a spherical shell surrounding the probe whose thickness is one-half the radius of the thermistor is within 25% of the unperturbed SAR, regardless of the dielectric properties of the thermistor relative to those of the medium. This effect, and the heating of the medium by thermal conduction from the probe, will both lead to small systematic overestimates in the local SAR. We neglect these effects in the present analysis, in which the SAR data are normalized in any event. We note that the local enhancement of the SAR near the probe due to the dipolar field contribution is a potential source of error in fiber optic measurements as well, that may lead to errors in the measurement of the absolute SAR.

Under these assumptions, the temperature distribution inside the thermistor, \( T(r, \theta, \phi) \), is determined by solving the heat conduction equation with the appropriate boundary conditions. In spherical coordinates \((r, \theta, \phi)\)

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left( \sin(\theta) \frac{\partial T}{\partial \theta} \right) + \frac{1}{\rho c_m} \frac{\partial^2 T}{\partial \phi^2} + \frac{Q}{\alpha_{th}} = \frac{1}{\alpha_{th}} \frac{\partial T}{\partial t},
\]

which evaluates to

\[
T_{ave}(t) = T_i + \frac{6R^2}{\pi \alpha_{th}} \left[ \frac{Q}{\alpha_{th}} - \frac{S}{\alpha_{th}} \right] \sum_{n=1}^{\infty} \frac{1}{(n \pi)^2} \left[ 1 - e^{-\left(\frac{n \pi}{R}\right)^2 \alpha_{th} t} \right] + St.
\]

The boundary conditions are

\[
T(0, t) = \text{finite} \\
T(R_-, t) = T(R_+, t) = T_i + S(R_+)t
\]

where \(R_- \) and \(R_+\) correspond to points just inside and outside the thermistor. \(T_i\) is the initial temperature, and

\[
S(R_+) = \frac{|E_0|^2 \sigma_m}{2 \rho c_m}
\]

is the rate of temperature rise that would occur in the medium due to the field \(E_0\). This formulation assumes that the temperature just outside the thermistor is not significantly affected by the heating of the thermistor. This is clearly an oversimplification but appears to be a reasonable approximation, in part because of the small size of the thermistor and the relatively high thermal conductivity of the surrounding water.

The solution of (3) subject to boundary conditions (4) yields the temperature distribution within the thermistor

\[
(r, t) = -\frac{2R^3}{\pi} \left[ \frac{Q}{\alpha_{th}} - \frac{S}{\alpha_{th}} \right] \sum_{n=1}^{\infty} \frac{\cos(n \pi)}{(n \pi)^3} \left[ 1 - e^{-\left(\frac{n \pi}{R}\right)^2 \alpha_{th} t} \right] \times \sin \left( \frac{n \pi R}{R} \right) + St + T_i
\]

The average temperature of the thermistor is

\[
T_{ave}(t) = \frac{1}{\frac{4}{3} \pi R^3} \int_0^R 4\pi r^2 T(r, t) \, dr
\]
Fig. 3. The initial and second slopes identified in Fig. 2 versus input power.

Fig. 4. Measured normalized SAR distribution for a 5.0 mm (outer diameter) radio frequency ablation catheter electrode.

Fig. 5. Calculated normalized SAR distribution for 5.0 mm radio frequency catheter electrode using the finite-element method.

V. CONCLUSION

This work has shown that inexpensive thermistors can be used to make high-resolution measurements of the SAR pattern of antennas in liquid media. The simple thermal model developed here offers a useful way to account for and remove thermal artifacts from the measurements. One new use for this method is to study the SAR produced in models of the body by wireless transmitters.

REFERENCES