ELEG 648
Field Theory

Graduate Level Electromagnetic Theory

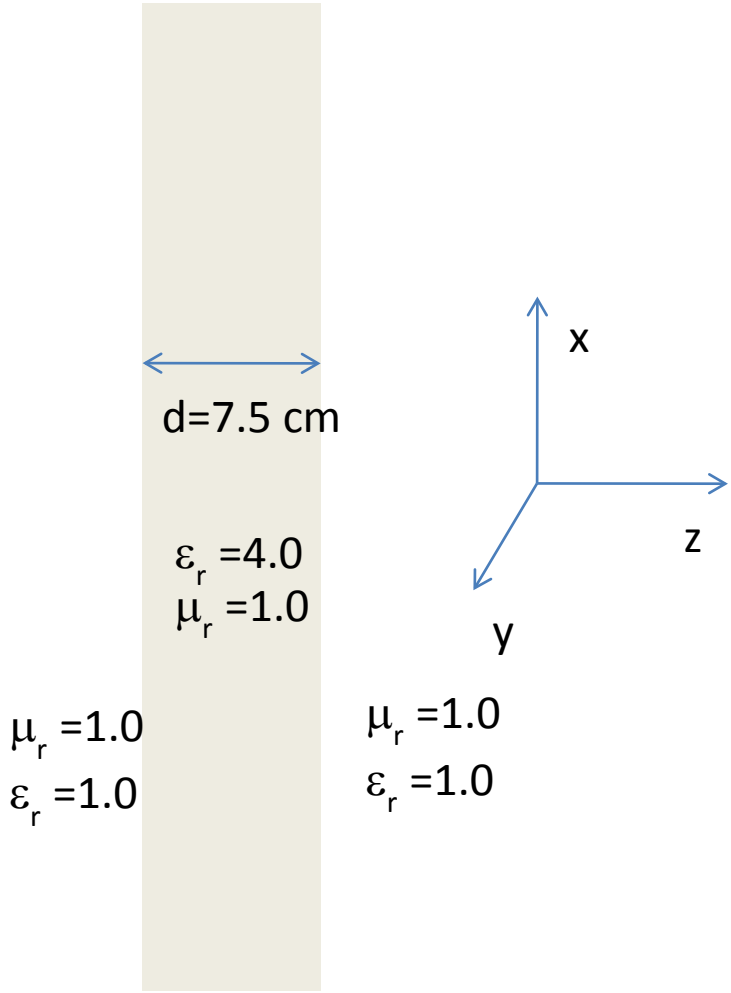
Midterm Take Home Exam
Spring 2015

Name _____

Directions: Do all five problems. Show your work clearly for partial credit.

1. The following electric field is incident from the left on the dielectric slab shown below :

$$\tilde{E}_{inc} = (0.866\hat{a}_x - j\hat{a}_y + 0.5\hat{a}_z)\, e^{-j54.4z} e^{-j31.4x}$$



Determine equations for both the reflected and transmitted electric fields.

2. Derive the wave equation in a source-free, lossless (i.e. σ=0) and non-magnetic (i.e. $\mu=\mu_o$) but inhomogenous medium whose relative dielectric constant is given by

$$\varepsilon_r(z) = 2 + \cos(z)$$

Hints: $\nabla \cdot (\varepsilon \tilde{E}) = \nabla \varepsilon \cdot \tilde{E} + \varepsilon \nabla \cdot \tilde{E}$

$\nabla(\tilde{f} \cdot \tilde{g}) = \tilde{f} \nabla \cdot \tilde{g} + \tilde{g} \cdot \nabla \tilde{f}$ where f and g are complex vectors

3. An electric field in a non-magnetic media is given as

$$\vec{E}(x,y,t) = 10\cos(3\cdot 10^7 t + 3x - 4y)\hat{a}_z$$

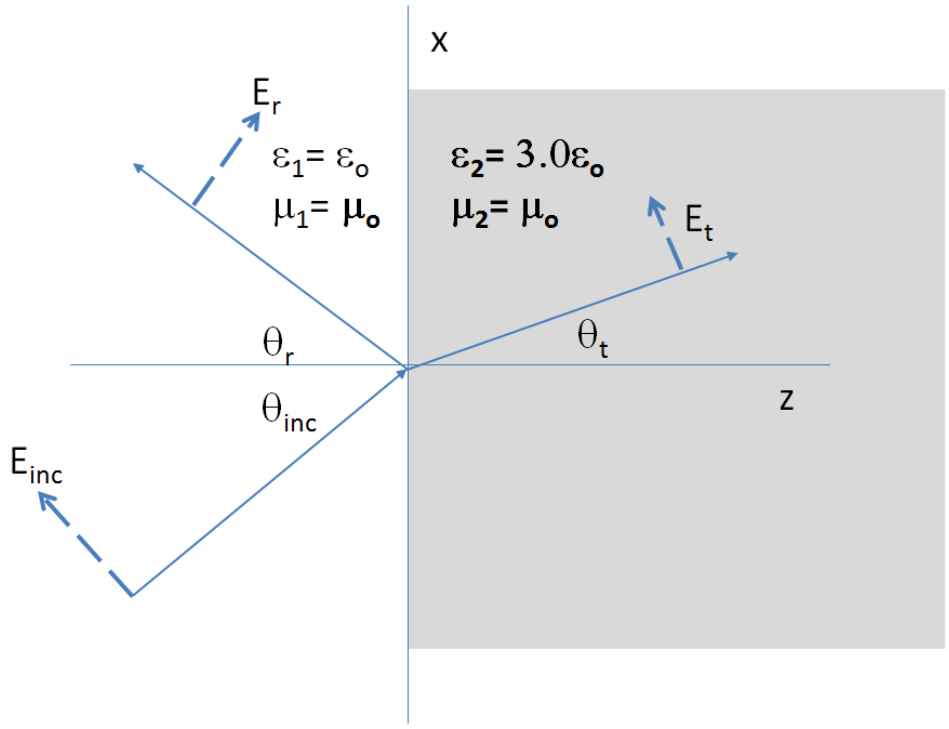
(a) Find the time varying magnetic field H(x,y,t)

(b) What direction is the wave traveling

(c) Find the relative dielectric constant ε_r of the media

(d) If you froze the wave in time and starting walking in the +y direction. How far would you travel (in meters) before the wave repeated itself?

4. A parallel polarized uniform plane wave is incident on a dielectric half space at an angle of θ_{inc}. Assume the material to the left of the interface is free-space and to the right is non-magnetic with a relative permittivity of $\varepsilon_r = 3.0$.

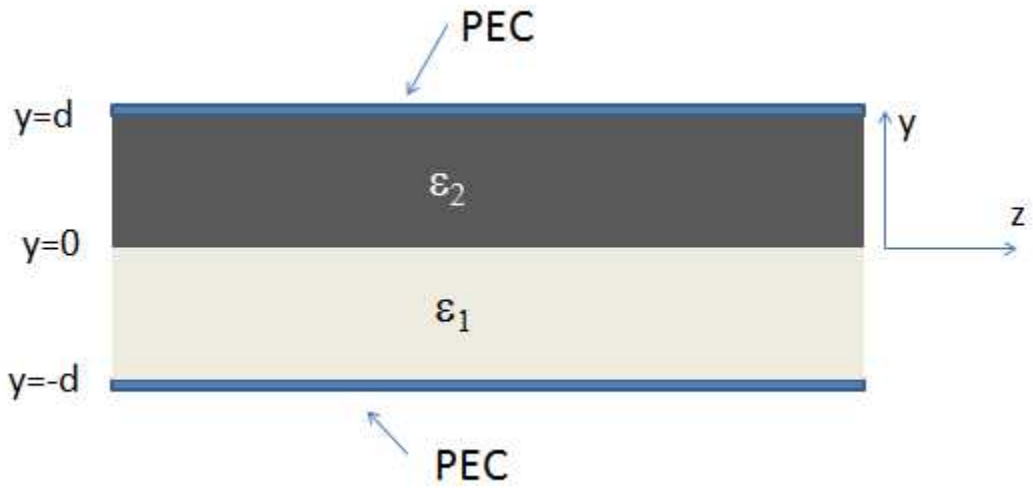


(a) Write down the equations that describe the incident, reflected and transmitted electric and magnetic fields.

(b) Using boundary conditions write down the set of equations that must be solved to find the unknowns (i.e. reflection and transmission coefficients)

(c) Derive the equations for reflection and transmission coefficients for the parallel polarization case.

5. Assume we have an inhomogeneous parallel plate waveguide with waves traveling in the +z direction and infinite in the x direction. Assume half of the waveguide is filled with material of dielectric constant ε_1 and the other half filled with material of dielectric constant ε_2.



The solution for the Ez component of the TM modes for each of the two regions shown above is given by:

$$E_z^{(1)}(y,z) = \left\{A_1 \cos(\beta_y^{(1)}(y+d)) + B_1 \sin(\beta_y^{(1)}(y+d))\right\} e^{-j\beta_z^{(1)} z} \qquad -d < y < 0$$

$$E_z^{(2)}(y,z) = \left\{A_2 \cos(\beta_y^{(2)}(y-d)) + B_2 \sin(\beta_y^{(2)}(y-d))\right\} e^{-j\beta_z^{(2)} z} \qquad 0 < y < d$$

Using boundary conditions, dispersion relationships and phase matching conditions determine all the unknowns

$$A_1, B_1, A_2, B_2, \beta_y^{(1)}, \beta_y^{(2)}, \beta_z^{(1)}, \beta_z^{(2)}$$