

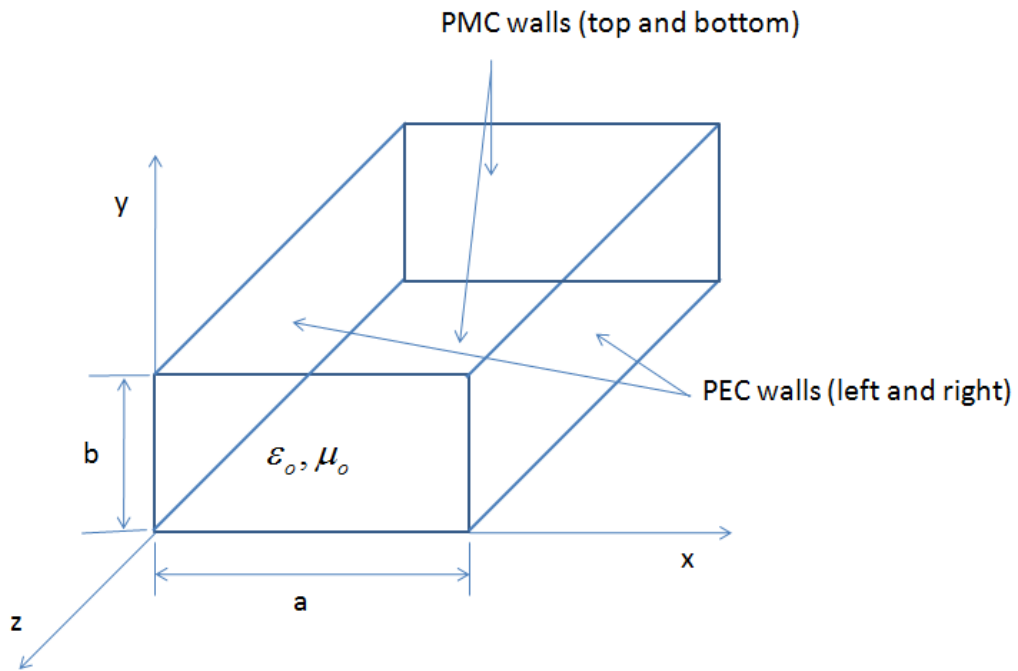
ELEG 648
Electromagnetic Theory

Final Exam
Spring, 2015

Name _____

Directions: Do all six problems. Show your work clearly for partial credit.

1. The rectangular waveguide shown below is constructed of two horizontal perfectly magnetic conducting (PMC) walls at $y=0$ and $y=b$ and two vertical perfectly electrical conducting walls (PEC) at $x=0$ and $x=a$.



(20 points) Determine all of the magnetic field components for the TE^Z waveguide modes:

2. (10 points) Assume you have an air filled square waveguide ($a=b$) with perfectly conducting side walls. Determine the surface current density on each of the four side walls (remember that surface current density J is a vector so give me all the components). Assume only the dominant mode propagates.

3. A number of important antennas use travelling waves of current instead of standing waves. Assume a long wire of length L (note that L is of the order of or more than the wavelength) is oriented along the z axis. A current on the wire is given by:

$$I(z) = I_0 e^{-j\beta z}$$

- (a) 10 points: Show that the net electric field at a far-field point is given by $E(r, \theta, \phi)$ is given by

$$E(r, \theta, \phi) = \hat{a}_\theta j \eta_0 \beta I_0 \cdot \frac{e^{-j\beta r}}{4\pi r} \sin \theta \frac{1 - e^{-j\beta L(1 - \cos(\theta))}}{j\beta(1 - \cos(\theta))}$$

- (b) 10 points: Derive an expression for the directive gain (i.e. $D(\phi, \theta)$) for the traveling wave antenna.

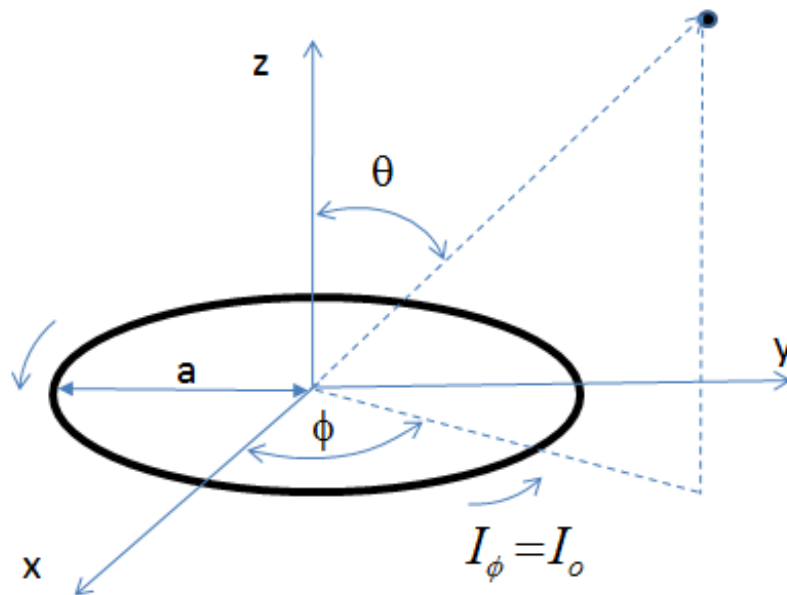
- (c) 10 points: Using Matlab (or any other program of your choice) plot the directive gain as a function of θ for antennas of lengths ; (1) $L/\lambda=2.0$, (2) $L/\lambda=10.0$ and (3) $L/\lambda=100.0$,

4. (10 points). You are designing a pair of walkie talkie radios that operate at 450 MHz. You are restricted to a maximum of 800 mW of transmit power and wish to communicate with an identical walkie talkie a distance of 1.0 mile away. The minimum detectable signal at the receiver is 1.0 nW. What is the minimum antenna gain needed for your design? Assume the antennas are well matched with negligible losses.

5. (30 points) A small loop antenna is shown below. The antenna is assumed to be made out of a very thin wire with a circular circumference much smaller than the wavelength (i.e. $2\pi a \ll \lambda$).

Under these conditions the current flowing in the wire can be approximated as a constant

$$I_\phi = I_o.$$



For this antenna show that the far-field electric field is given by

$$\vec{E} \approx \hat{a}_\phi \frac{\omega k \mu_o a^2 I_o}{4} \frac{e^{-jkr}}{r} \sin(\theta)$$

6. (30 points)

(2). An obliquely incident plane wave (polarized in the xz plane as shown below) impinges upon an opening in the very thin PEC plate shown below. Assume the plate to be infinitely thin and assume you can use the physical optics approximation. Determine the electric field in the far-field for $z > 0$.

