Auxiliary Potential Functions

Summary:

Given: $\vec{J}\,,\vec{M}$

Solve: $\nabla^2\vec{A}+k^2\vec{A}=-\mu\vec{J}$

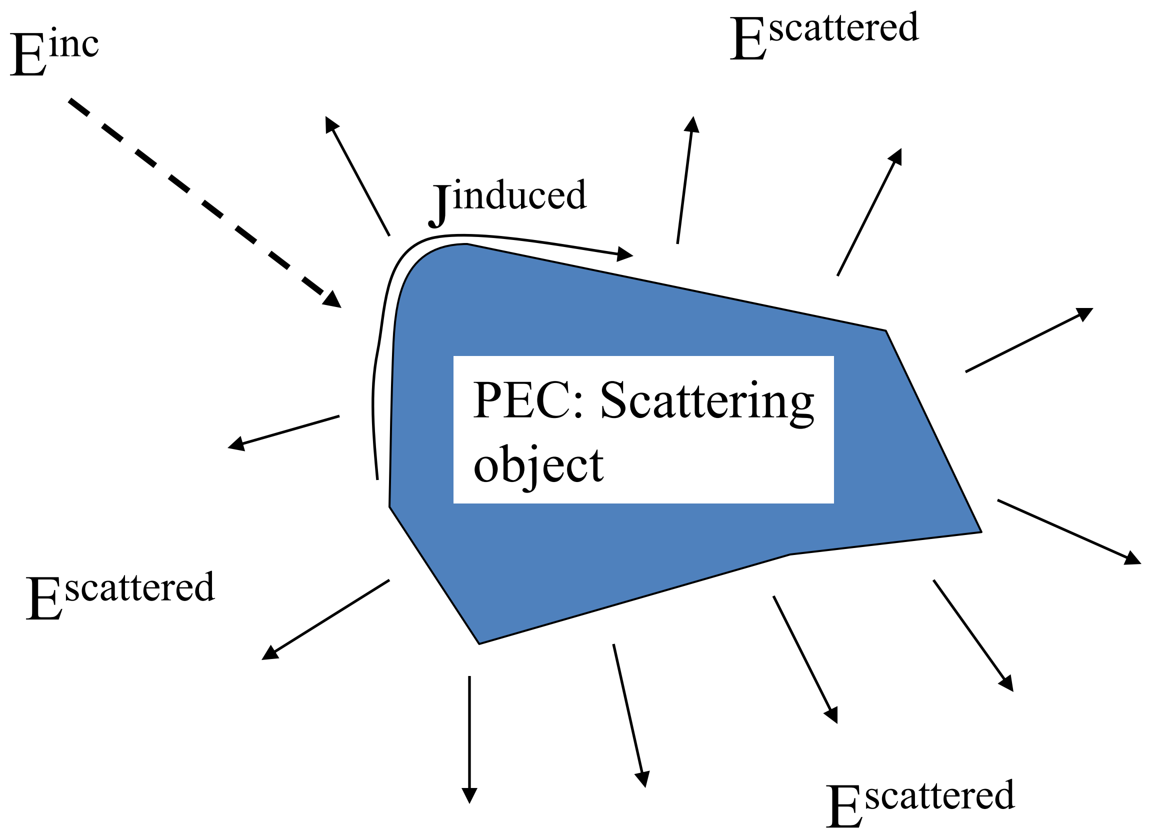
$$\nabla^2\vec{F}+k^2\vec{F}=-\varepsilon\vec{M}$$

Calculate:

$$\vec{E}=\vec{E}_e+\vec{E}_m=-j\omega\vec{A}-\frac{j}{\omega\mu\varepsilon}\nabla(\nabla\cdot\vec{A})-\frac{1}{\varepsilon}\nabla\times\vec{F}$$

$$\vec{H}=\vec{H}_e+\vec{H}_m=\frac{1}{\mu}\nabla\times\vec{A}-j\omega\vec{F}-\frac{j}{\omega\mu\varepsilon}\nabla(\nabla\cdot\vec{F})$$

Scattering from Conducting Objects



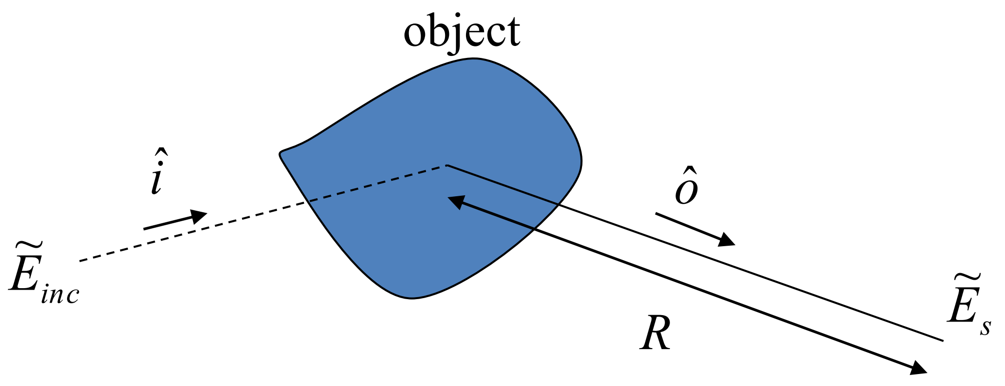
$$E^{total} = E^{scattered} + E^{inc}$$

Scattering from Conducting Objects

In the far field:

$$\tilde{E}_s = \tilde{f}(\hat{i},\hat{o})\frac{e^{-jkR}}{R}$$

where $\tilde{f}(\hat{i},\hat{o})$ represents the amplitude, phase and polarization of the scattered wave in the far field.



Radar Cross Sections

$$\sigma_d(\hat{i},\hat{o}) = \left|\tilde{f}(\hat{i},\hat{o})\right|^2 \frac{m^2}{solid\ angle}$$

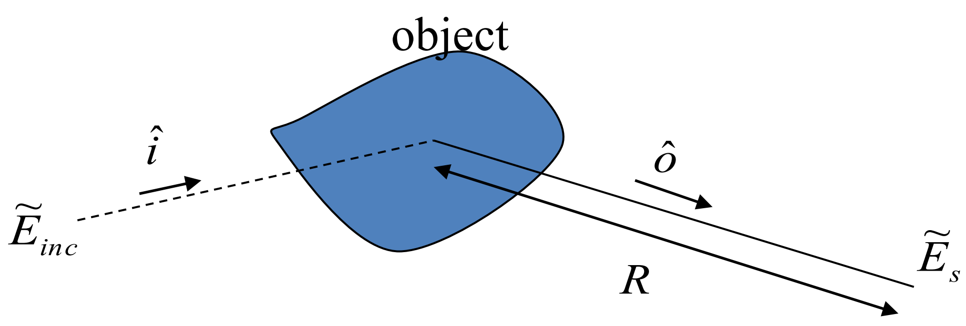
where $\sigma_d(\hat{i},\hat{o})$ is called the differential radar cross section.

$$\sigma_{bi}(\hat{i},\hat{o}) = 4\pi\,\sigma_d(\hat{i},\hat{o}) \quad m^2$$

where $\sigma_{bi}(\hat{i},\hat{o})$ is called the bistatic radar cross section.

$$\sigma_b = 4\pi\,\sigma_d(-\hat{i},\hat{i}) \quad m^2$$

where σ_b is called the monostatic radar cross section or RCS.



General Properties of Radar Cross Sections

$$\sigma_t = \int_{4\pi} \left| \tilde{f}(\hat{i}, \hat{o}) \right|^2 d\omega$$

where σ_t is called the total scattering cross section.

(a) Size of the object is large compared to the wavelength (λ)

$$\sigma_t \to 2\sigma_g$$

where σ_g is called the real geometrical cross section of the object.

(b) Size of the object is small compared to the wavelength (λ)

$$\sigma_t \to 2\sigma_g \frac{size}{\lambda^4}$$

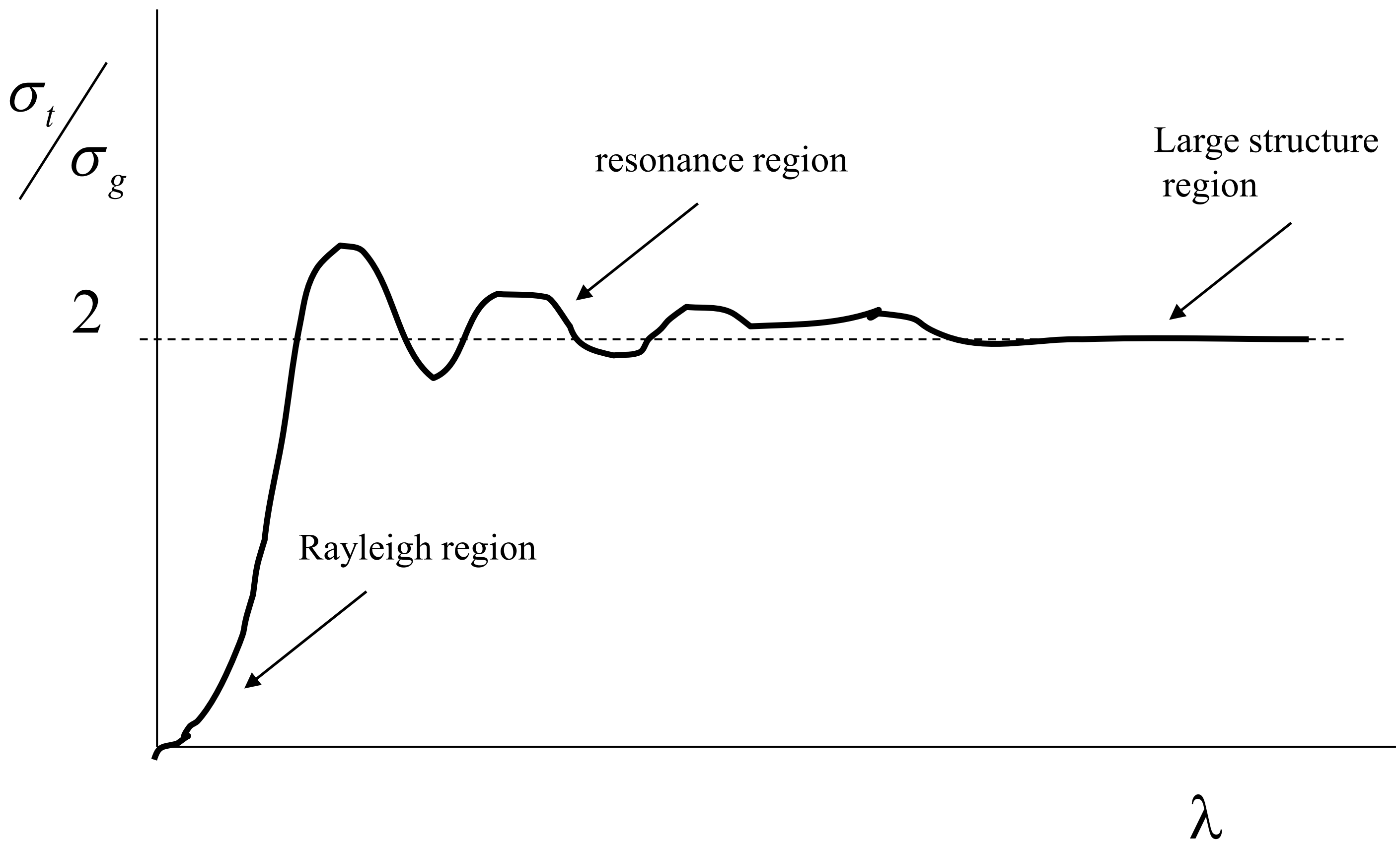
This is called Rayleigh scattering.

(c) Size of the object is comparable to the wavelength (λ)

$$\sigma_t \to complicated$$

This is called resonance or Mie scattering

Example: Scattering from Sphere



RCS Calculations
Governing Equations

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\iiint_V \vec{J}(\vec{r}\,')\frac{e^{-jkR}}{R}dv'$$

$$\vec{F}(\vec{r}) = \frac{\varepsilon}{4\pi}\iiint_V \vec{M}(\vec{r}\,')\frac{e^{-jkR}}{R}dv'$$

$$\vec{E} = -j\omega\vec{A} - \frac{j}{\omega\mu\varepsilon}\nabla(\nabla\cdot\vec{A}) - \frac{1}{\varepsilon}\nabla\times\vec{F}$$

$$\vec{H} = \frac{1}{\mu}\nabla\times\vec{A} - j\omega\vec{F} - \frac{j}{\omega\mu\varepsilon}\nabla(\nabla\cdot\vec{F})$$

Scattering from Conducting Objects

Step #1: Determine either rigorously or through approximations the current distribution induced on a conducting object due to an incident field.

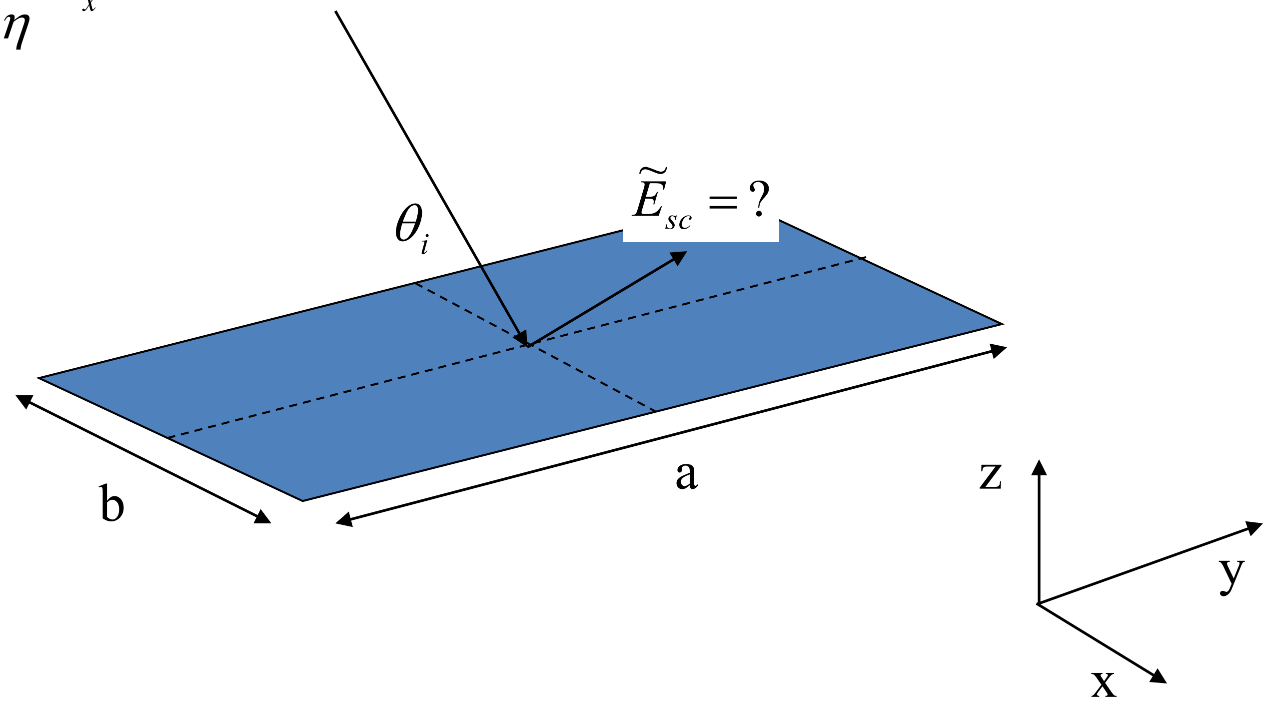
Step #2: Calculate the vector potential field A and then the far field electric field distribution scattered (or reradiated) from the induced current distribution.

Step #3: Calculate either the bistatic or monostatic RCS from the far field scattered electric field distribution.

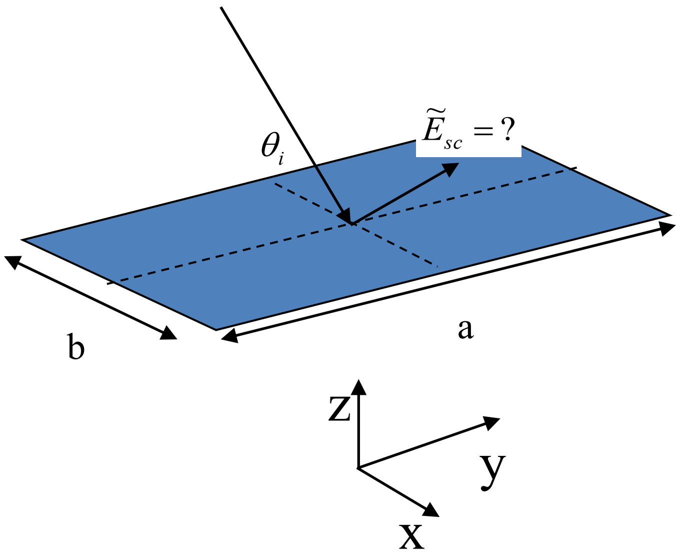
Example: Scattering from a flat PEC plate

$$\tilde{E}_{inc} = E_o(\hat{a}_y \cos(\theta_i) + \hat{a}_z \sin(\theta_i))e^{-j\beta(y\sin(\theta_i) - z\cos(\theta_i))}$$

$$\tilde{H}_{inc} = \frac{E_o}{\eta}\hat{a}_x e^{-j\beta(y\sin(\theta_i) - z\cos(\theta_i))}$$



Example: Scattering from a flat PEC plate



Step #1: What is the induced current on the plate?

$$\widetilde{J}_s = \hat{n} \times \widetilde{H}_{total}\Big|_{z=0} = \hat{n} \times (\widetilde{H}_{inc} + \widetilde{H}_{sc})\Big|_{z=0}$$

This is true but we still do not know $\widetilde{H}_{sc}$

Physical Optics (PO) approximation:

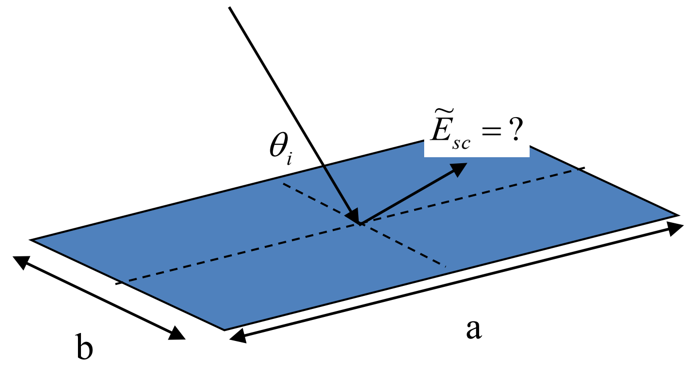
$$\widetilde{H}_{sc}\Big|_{z=0} \approx \widetilde{H}_{inc}\Big|_{z=0}$$

This is not true in general since it essentially assumes the plate is infinitely large and only produces a specular reflection. All edge effects are ignored!!

$$\widetilde{J}_s = \hat{n} \times (\widetilde{H}_{inc} + \widetilde{H}_{sc})\Big|_{z=0} \approx \hat{n} \times (\widetilde{H}_{inc} + \widetilde{H}_{inc})\Big|_{z=0} = 2\hat{n} \times \widetilde{H}_{inc}\Big|_{z=0}$$

$$\widetilde{J}_{sc} = \frac{2E_o}{\eta} \hat{a}_y e^{-j\beta(y\sin(\theta_i))}$$ on the surface of the plate

Example: Scattering from a flat PEC plate



Step #2: Calculate A from J

$$\tilde{J}_{sc} = \frac{2E_o}{\eta}\hat{a}_y e^{-j\beta(y\sin(\theta_i))}$$

$$\tilde{A} = \frac{\mu}{4\pi}\iint_S \tilde{J}_s \frac{e^{-j\beta R}}{R} ds'$$

$$\tilde{A}(x,y,z) = \hat{a}_y \frac{\mu}{4\pi}\int_{-b/2}^{b/2}\int_{-a/2}^{a/2} 2\frac{E_o}{\eta} e^{-j\beta y'\sin(\theta_i)} \frac{e^{-j\beta\sqrt{(x-x')^2+(y-y')^2+z^2}}}{\sqrt{(x-x')^2+(y-y')^2+z^2}} dx'dy'$$

In the far-field

$$\tilde{A} = \hat{a}_y \frac{\mu}{2\pi}\frac{E_o}{\eta}\frac{e^{-j\beta r}}{r}\int_{-b/2}^{b/2}\int_{-a/2}^{a/2} e^{-j\beta y'\sin(\theta_i)} e^{-j\beta\sqrt{x'^2+y'^2}\cos(\psi)} dx'dy'$$

$$r = \sqrt{x^2+y^2+z^2}$$

Example: Scattering from a flat PEC plate

Step #2: Calculate A from J

$$\widetilde{A} = \hat{a}_y \frac{\mu}{2\pi} \frac{E_o}{\eta} \frac{e^{-j\beta r}}{r} \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} e^{-j\beta y' \sin(\theta_i)} e^{-j\beta \sqrt{x'^2 + y'^2} \cos(\psi)} dx' dy'$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

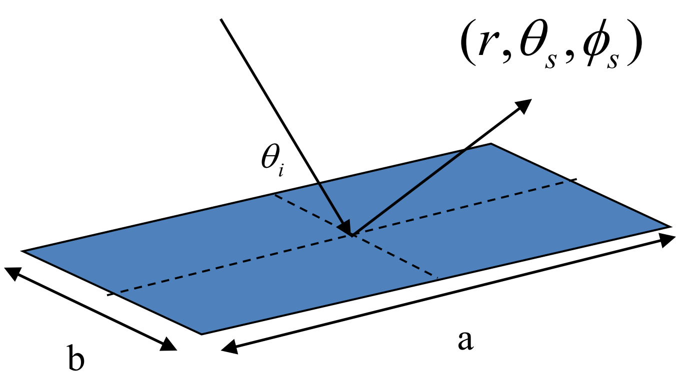
after much calculus and algebra and a conversion to spherical coordinates

$$A_\theta = \frac{\mu e^{-j\beta r}}{2\pi r} ab \frac{E_o}{\eta} \cos(\theta_s) \sin(\phi_s) \frac{\sin(X)}{X} \frac{\sin(Y)}{Y}$$

$$A_\phi = \frac{\mu e^{-j\beta r}}{2\pi r} ab \frac{E_o}{\eta} \cos(\phi_s) \frac{\sin(X)}{X} \frac{\sin(Y)}{Y}$$

$$X = \frac{\beta a}{2} \sin(\theta_s) \cos(\phi_s)$$

$$Y = \frac{\beta b}{2} (\sin(\theta_s) \sin(\phi_s) - \sin(\phi_{inc}))$$



Example: Scattering from a flat PEC plate

Step #3: Calculate E from A

In the far-field:

$$E_\theta = -j\omega A_\theta$$

$$E_\phi = -j\omega A_\phi$$

$$E_r = 0$$

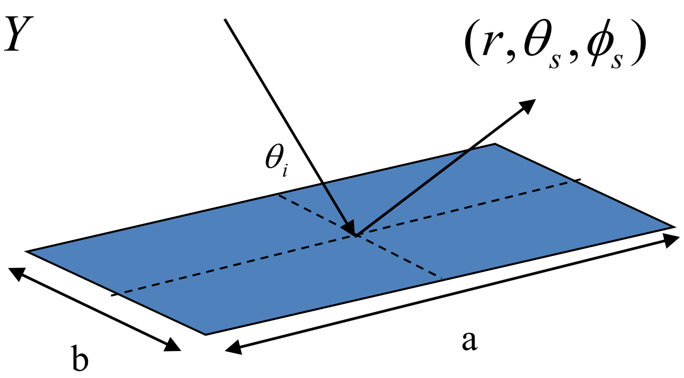
$$E_\theta = \frac{-j\omega\mu e^{-j\beta r}}{2\pi r} ab \frac{E_o}{\eta} \cos(\theta_s)\sin(\phi_s) \frac{\sin(X)}{X} \frac{\sin(Y)}{Y}$$

$$E_\phi = \frac{-j\omega\mu e^{-j\beta r}}{2\pi r} ab \frac{E_o}{\eta} \cos(\phi_s) \frac{\sin(X)}{X} \frac{\sin(Y)}{Y}$$

$$E_r = 0$$

$$X = \frac{\beta a}{2}\sin(\theta_s)\cos(\phi_s)$$

$$Y = \frac{\beta b}{2}(\sin(\theta_s)\sin(\phi_s) - \sin(\phi_{inc}))$$



Example: Scattering from a flat PEC plate

Step #3: Calculate RCS

$$\tilde{E}_s = \tilde{f}(\hat{i},\hat{o})\frac{e^{-jkR}}{R}$$

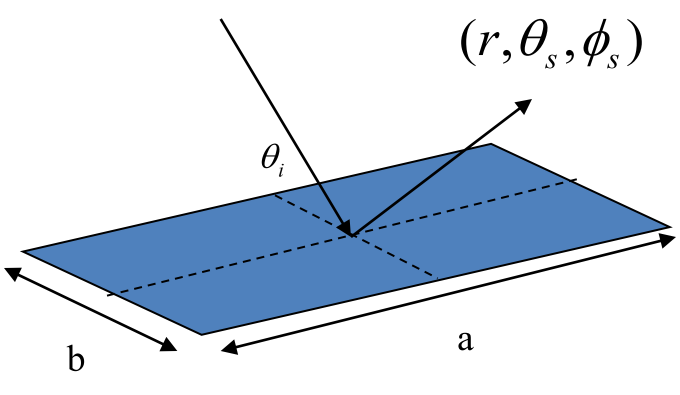
$$f_\theta = \frac{-j\omega\mu}{2\pi} ab \frac{E_o}{\eta} \cos(\theta_s)\sin(\phi_s)\frac{\sin(X)}{X}\frac{\sin(Y)}{Y}$$

$$f_\phi = \frac{-j\omega\mu}{2\pi} ab \frac{E_o}{\eta} \cos(\phi_s)\frac{\sin(X)}{X}\frac{\sin(Y)}{Y}$$

$$f_r = 0$$

$$X = \frac{\beta a}{2}\sin(\theta_s)\cos(\phi_s)$$

$$Y = \frac{\beta b}{2}(\sin(\theta_s)\sin(\phi_s) - \sin(\phi_{inc}))$$



Example: Scattering from a flat PEC plate

Step #3: Calculate RCS

The bistatic RCS is given by

$$\sigma_{bi}(\theta_{inc},\theta_s,\phi_s) = 4\pi\,\sigma_d(\theta_{inc},\theta_s,\phi_s) = 4\pi\left|\tilde{f}\right|^2 = 4\pi(\left|f_\theta\right|^2 + \left|f_\phi\right|^2) \quad m^2$$

The monostatic RCS is given by

$$\sigma_b(\theta_{inc}) = 4\pi\,\sigma_d(\theta_{inc},\theta_s = \theta_{inc},\phi_s = 0)$$

$$f_\theta = \frac{-j\omega\mu}{2\pi}ab\frac{E_o}{\eta}\cos(\theta_s)\sin(\phi_s)\frac{\sin(X)}{X}\frac{\sin(Y)}{Y}$$

$$f_\phi = \frac{-j\omega\mu}{2\pi}ab\frac{E_o}{\eta}\cos(\phi_s)\frac{\sin(X)}{X}\frac{\sin(Y)}{Y}$$

$$f_r = 0$$

$$X = \frac{\beta a}{2}\sin(\theta_s)\cos(\phi_s)$$

$$Y = \frac{\beta b}{2}(\sin(\theta_s)\sin(\phi_s) - \sin(\phi_{inc}))$$