1. 
(a) What determines the highest energy of x-ray photons emitted from an x-ray tube?

Tube voltage

(b) In x-ray imaging for medical applications, why are low-energy photons undesired? What measures can be taken to reduce the number of low-energy photons entering the human body?

Low energy photons are absorbed by tissue and thus potentially harmful while not at all useful for imaging applications.

We can limit the number of low energy photons by using filters (e.g. such as thin aluminum plates) that screen the low energy photons while allowing higher energy photons to be transmitted.

(c) why are photons undergoing scattering undesired? What measures can be taken to prevent these photons form entering the detectors?

Scattered photons, created normally by Compton scattering, appear as noise in an x-ray image since their direction is random. We can prevent (or at least reduce them) by using grids in front of the detectors.
2. A slab of soft tissue with one blood vessel in the middle is imaged under an x-ray imaging system, as shown in the figure below. The blood vessel may be injected with a contrast agent. For ease of computation, assume the tissue and the vessel both have square shaped cross-sections, with dimensions shown in the figure. For the range of the photon energy of the x-ray used, the linear attenuation coefficients of the soft tissue, the blood vessel and the contrast agent are 0.4 cm⁻¹ and 0.2 cm⁻¹, and 20 cm⁻¹, respectively.

Determine the local contrast of the blood vessel (a) when the contrast agent is not injected and (b) when the contrast agent is injected.

(a) No contrast agent.

\[
I_t = e^{-0.4 \times 3.5 - 0.2 \times 0.5} = 0.2231 \\
I_b = e^{-0.4 \times 4.0} = 0.2019
\]

\[
\text{Contrast} = \frac{I_t - I_b}{I_b} = \frac{0.2231 - 0.2019}{0.2019} = 0.1052
\]

(a) With contrast agent.

\[
I_t = e^{-0.4 \times 3.5 - 20 \times 0.5} = 0.0000112 \\
I_b = e^{-0.4 \times 4.0} = 0.2019
\]

\[
\text{Contrast} = \frac{I_t - I_b}{I_b} = \frac{0.0000112 - 0.2019}{0.2019} = -0.9999
\]
3. Consider the x-ray imaging of a two-layer slab, illustrated below. Determine the intensity of detected photons along the y axis on the detector plane. Express your solution in terms of the y-coordinate.

Sketch this function. You should consider the inverse square law and the obliquity effect. Assume the x-ray source is an ideal monochromatic point source with intensity $I_0$. For simplicity, assume the slab is infinitely long in the y direction.

\[ I(y) = I_o \cos^3(\Theta) \cdot e^{-\mu_1 y \sin \Theta} \cdot e^{-\mu_2 y \sin \Theta} \]

\[ I(y) = I_o \left( \frac{D}{\sqrt{D^2 + y^2}} \right)^3 \cdot e^{-\frac{\rho \mu_1}{\sqrt{\rho^2 + y^2}}} \cdot e^{-\frac{\rho \mu_2}{\sqrt{\rho^2 + y^2}}} \]
4. The tissue slice being imaged by a parallel beam x-ray CT scanner is shown in the figure below.

(a) Assume the detector is a point detector. Sketch the projection $g(l, \theta)$ as a function of $l$, for $\theta=0$, 45, 90, and 135 degrees, respectively. You should indicate the magnitudes of the projected values where necessary on your sketch. Also apply any transition points in the horizontal axis.

(b) Sketch the image obtained by back projections from both 0 and 90 degree projections. You should normalize your back projection using the dimension of the imaged region as indicated on the figure.

(c) Determine the Fourier transform of the original image along a line with orientation $\theta=90$ degree.
(a) $g(l, 90)$

$g(l, 0)$

$g(l, 45)$
(c) for \( g(l, 90) \)

\[
\begin{align*}
g(l, 90) &= \text{rect}(l) + \text{rect}(l/3) \\
F(u) &= \int_{-\infty}^{\infty} g(l, 90) e^{-j2\pi lu} dl = \int_{-\infty}^{\infty} \left[ \text{rect}(l) + \text{rect}(l/3) \right] e^{-j2\pi lu} dl \\
F(u) &= \int_{-1.5}^{1.5} e^{-j2\pi lu} dl + \int_{0.5}^{0.5} e^{-j2\pi lu} dl \\
&= \frac{-1}{j2\pi u} \left[ e^{-j2\pi u \cdot 1.5} - e^{-j2\pi u \cdot 1.5} \right] - \frac{1}{j2\pi u} \left[ e^{-j2\pi u \cdot 0.5} - e^{-j2\pi u \cdot 0.5} \right] \\
&= 3 \frac{\sin(2\pi u \cdot 1.5)}{2\pi u \cdot 1.5} + \frac{\sin(2\pi u \cdot 0.5)}{2\pi u \cdot 0.5}
\end{align*}
\]