Summary of Last Lecture
X-ray Physics

- What are X-rays and when are they useful for medical imaging?
- How are X-rays generated?
- How do X-rays interact with matter (tissue)?
- How do we detect X-rays?
- How do we mathematically model X-ray projection radiography?
X-rays

- **Wavelength:** 0.01-10 nm
- **Frequency:** 30 petahertz to 30 exahertz (3x10^{16} to 3x10^{19})
- **Energies:** 120 ev to 120 keV

Soft X-rays: 0.12 keV to 30 keV
Hard X-rays: 30 keV to 120 keV

Soft X-rays do not penetrate matter where hard X-rays do.
X-ray spectra

- X-ray for general diagnostic radiology produced at 40 – 150 kVp

- Maximum photon energy:
  \[ E_p [\text{keV}] = h \nu_{\text{max}} = e \times \text{kVp} \]

- Characteristic radiation occurs only for anode voltages
  \[ e \times \text{kVp} > I_{K,L,M,\ldots} \]
Interaction of Photons with Matter

The dominant photon interaction mechanisms for $\gamma$ and x-rays:

- **Photoelectric absorption**
  - Interaction with (initially) bound atomic electron
  - Incident photon disappears
  - Photon energy absorbed by electron, momentum by atom
  - Probability increases at: low incident photon energy and high electron density in medium (mass density $\times Z$)

- **Compton scatter**
  - Interaction with “free” electron (photon energy $\gg$ Binding E.)
  - Scattered photon changes direction and loses energy
Attenuation

More Attenuation

Attenuation Coefficient

Less Attenuation

Photon Energy (keV)

Photoelectric effect dominates

Compton Scattering dominates

Bone
Muscle
Fat
Interaction of Photons with Matter: Attenuation

Homogeneous Slab

- Homogeneous slab: the attenuation rate is the same over the entire slab
  - Homogeneous slab thickness $\Delta x$
  - Fundamental photon attenuation law
    \[ N = N_0e^{-\mu\Delta x} \]
  - $\mu$ is linear attenuation coefficient
  - In terms of intensity:
    \[ I = I_0e^{-\mu\Delta x} \]
  - This is known as Beer’s Law
Interaction of Photons with Matter: Attenuation

X-rays image radiodensity = amount of absorption in material

Figure 1.12. Radiodensity as a function of composition, with thickness kept constant.

Attenuation is Energy dependent:

100 kVp

Bone | Muscle | Lung

74% 84% 98%

30 kVp

Bone | Muscle | Lung

21% 68% 96%

1cm layers

Lower energies can distinguish different material better than higher energies
Contrast Agents

* Goal: To create contrast where otherwise

When the x-ray energy exceeds the Kedge, the mu coefficient is much higher, providing high contrast
Contrast Agents

- Iodine:
  - Can be synthesized into soluble compounds that are safely introduced through intravascular injection or ingestion
  - Used for imaging of
    - Blood vessels, heart chambers, tumors, infections
    - Kidneys, bladder
  - Naturally exist in thyroid, and hence X-ray is very good for thyroid imaging
- Barium
  - Administered as a “chalky milkshake”
  - Used in the gastrointestinal tract,
    - Stomach, bowel
- Air
  - Does not absorb x-ray
  - “opposite” type of contrast
  - By inflating the lungs, air provides contrast for lung tissues
Interaction of Photons with Matter: Attenuation

Non-Homogeneous Slab

- The attenuation coefficient depends on $x$
  - Non-homogeneous slab:
    \[ \frac{dN}{N} = -\mu(x)\,dx \]
  - Integration yields:
    \[ N(x) = N_0 \exp\left\{-\int_0^x \mu(x')\,dx'\right\} \]
  - For intensity:
    \[ I(x) = I_0 \exp\left\{-\int_0^x \mu(x')\,dx'\right\} \]
Interaction of Photons with Matter: Attenuation Polyenergetic Photons

- The linear attenuation coefficient depends on the medium property as well the energy of incident photon (E).
- For a given material, $\mu$ can be denoted by $\mu(x; E)$.
- When the incident photons are polyenergetic, with spectrum $S(E)$, the outgoing photon spectrum is

$$S(x; E) = S_0(E) \exp \left\{ - \int_0^x \mu(x'; E) \, dx' \right\}$$

- In terms of intensity

$$I = \int_0^\infty E' S(E') \, dE'$$

$$I(x) = \int_0^\infty S_0(E') E' \exp \left\{ - \int_0^x \mu(x'; E') \, dx' \right\} \, dE'$$
Dose

As EM radiation passes through a material, it deposits energy into it by the photoelectric effect and Compton scattering.

- How much energy is deposited into material?
- **Dose** $D$, the energy deposited per unit volume
- SI unit: Gray (Gy)
  
  $1 \text{ Gy} = 1 \text{ J/kg}$ (energy per mass)

- Common unit: rad

  $1 \text{ Gy} = 100 \text{ rads}$

1 R of exposure yields 1 rad of absorbed dose in soft tissue.
Dose Equivalent

Different types of radiation, when delivering the same dose, can have different effects on the body.

• **Dose equivalent** $H$
  \[ H = D \times Q \]

• $Q = \text{quality factor}$,
  – $Q \approx 1$ for x-rays, gamma rays, electrons, beta,
  – $Q \approx 10$ for neutrons and protons,
  – $Q \approx 20$ for alpha particles.

• Since $Q \approx 1$, $H = D$

• SI unit, Sievert (Sv). More common, rems
<table>
<thead>
<tr>
<th>Diagnostic Procedure</th>
<th>Typical Effective Dose (mSv)</th>
<th>Number of Chest X rays (PA film) for Equivalent Effective Dose</th>
<th>Time Period for Equivalent Effective Dose from Natural Background Radiation</th>
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<tbody>
<tr>
<td>Chest x ray (PA film)</td>
<td>0.02</td>
<td>1</td>
<td>2.4 days</td>
</tr>
<tr>
<td>Skull x ray</td>
<td>0.1</td>
<td>5</td>
<td>12 days</td>
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<tr>
<td>Lumbar spine</td>
<td>1.5</td>
<td>75</td>
<td>182 days</td>
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<td>I.V. urogram</td>
<td>3</td>
<td>150</td>
<td>1.0 year</td>
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<tr>
<td>Upper G.I. exam</td>
<td>6</td>
<td>300</td>
<td>2.0 years</td>
</tr>
<tr>
<td>Barium enema</td>
<td>8</td>
<td>400</td>
<td>2.7 years</td>
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<tr>
<td>CT head</td>
<td>2</td>
<td>100</td>
<td>243 days</td>
</tr>
<tr>
<td>CT abdomen</td>
<td>8</td>
<td>400</td>
<td>2.7 years</td>
</tr>
</tbody>
</table>

* FDA Study
& Inevitable problems of not fully understanding the effects of ionizing radiation

Clarence Dally (1865-1904) Edison's assistant

Mihran Kassabian (1870-1910) x-ray pioneer

Some early “protection”
Today’s Lecture
Projection Radiography

- Overview of different systems for projection radiography
- Instrumentation
  - Overall system layout
  - X-ray sources
  - grids and filters
  - detectors
- Imaging Equations
  - Basic equations
  - Geometrical distortions
  - More complicated imaging equations
Projection Radiography

Static X-ray Systems (snap shots)

Traditional chest x-ray
- 2-3 mrem
- ~100 keV

Traditional dental x-ray
- 2-3 mrem
- 15-30 keV

Mammography
- 70 mrem
- ~30 keV
Projection Radiography

Dynamic X-ray Systems (Fluoroscopic)

- Digital Fluoroscopy
  - 10 mrem/min
  - ~50 keV

- Angiogram
  - 460-1,580 mrem
  - ~50 keV

- Barium Colon Image
  - 700 mrem
  - ~30 keV
Projection Radiography

Some new stuff

Digital Subtraction Angiography (DSA)
Projection Radiography
Generic System Description

- Remove photons that would be absorbed in the body anyway (beam hardening)
- Limit radiation to patient volume of interest
- Remove scattered photons (tungsten)

- X-ray source
- Collimator
- Detector

- Film-screen based
- Camera + image intensifier
- Storage phosphor plate
- Amorphous Si flat panel
- Amorphous Se direct detector
X-ray tube design

- Cathode w/ focusing cup, 2 filaments (different spot sizes)
- Anode
  - Tungsten, $Z_w = 74$, $T_{\text{melt}} = 2250 \, ^\circ\text{C}$
  - Embedded in copper for heat dissipation
  - Angled (see next slide)
  - Rotating to divert heat
Amount of radiation for a fixed voltage is proportional to the current in the tube. Radiation exposure is often given in units of mAs (milliampere-seconds). This gives an idea of how much current was used in the tube for how long. While mAs is usually proportional to dose (rems or rads) it is not easy to equate the two however.
Filtration

- Low energy x-ray will be absorbed by the body, without providing diagnostic information
- Filtration: Process of absorbing low-energy x-ray photons before they enter the patient

  - Inherent filtration
    - Within anode
    - Glass housing
  - Added filtration
    - Aluminum
    - Copper/Aluminum
    - Note: Cu has 8keV characteristic xrays
    - Measured in mm Al/Eq
Restriction

- Goal: To direct beam toward desired anatomy
Compensation Filters

- Goal: to even out film exposure
Scatter Control

- Ideal x-ray path: a line!
- Compton scattering causes blurring
- How to reduce scatter?
  - airgap
  - scanning slit
  - grid
Grids

- Effectiveness in scatter reduction?

\[ \text{grid ratio} = \frac{h}{b} \]

- 6:1 to 16:1 (radiography) or 2:1 (mammo)
Problem with Grids

- Radiation is absorbed by grid
  - grid conversion factor
    \[ GCF = \frac{\text{mAs w/ grid}}{\text{mAs w/o grid}} \]
  - Typical range \(3 < GCF < 8\)

- Grid visible on x-ray film
  - move grid during exposure
  - linear or circular motion
X-Ray Detectors

- Screen or Film
- Image Intensifier
- Storage Phosphors
- Direct Flat Panel Radiography
X-Ray Detectors

History

X-ray detectors started with film, screen, and film/screen systems (screen = scintillator)
X-Ray imaging with films and screens

- Film can be exposed directly (highest resolution)
- Intensifying screen can be placed in front of the film
  - Increased contrast
  - Reduced sharpness
X-Ray Detectors: Screen-Film

Quantum Efficiency (QE): \[ QE = \frac{\text{detected photons}}{\text{incoming photons}} \times 100 \text{ percent} \]

Photographic film: very inefficient (QE=2%)
- would require huge patient doses

Phosphor-based: Place film between two intensifying fluorescent screens
- made out of rare earth phosphors (gadolinium oxysulfide Gd$_2$O$_2$S)
- phosphor converts X-rays to scattered visible light
- light directed toward film is recorded (QE=25%)
Film with intensifying (fluorescent) screens

fluorescent screen #2

Film

fluorescent screen #1
Screen-film cassette

Assembled screen-film “sandwich” in light-tight cassette
Properties of film/screen assembly

- Intensifying screens absorb X-Rays and transfer into visible light.
- Film detects lights which is emitted by the screens
- Cassette is light-tight
- After exposure, film can be viewed as a semi-transparent on a light screen
X-Ray Detectors

What do we do in dynamic systems that can’t use film?
Image intensifiers produce images at high speeds (unlike film):

- photons → visible light → electrons → visible light

- limited spatial resolution due to limited camera resolution
- elevated noise due to additional conversions
- geometric distortions (pin-cushion distortion)
X-Ray Detectors: Direct Radiography

Shortcomings of image intensifier detectors

- camera was made out of Si-crystal technology, restricting its size to a small area (just like CCDs)
- this required the long chain from photons to camera (see before)

Newer (scintillator: high-energy x-rays $\rightarrow$ photons) technology: hydrogenated amorphous silicon detectors (a-Si:H)

- can be manufactured in flat, large sheets
- can be coupled directly with the phosphor plate
- but still need to convert photons to visible light, affecting resolution

Latest technology: amorphous selenium (a-Se)

- a photo-conducting layer (not a phosphor)
- a-Se electrical conductivity proportional to radiation energy
- before exposure: a homogenous charge is applied to Se-surface
- during exposure: photons are absorbed in the Se-layer, setting free electrons $\rightarrow$ electrons neutralize charge locally (pixels)
- resulting image can then be read by a photo-conductor matrix
- high QE and resolution (11-13 lp/mm, lp=line pairs=half-pixels)
10”x12”
85 micron pixel pitch
3600 x 3000 pixels
(10.8 Mpixels)
Quiz #1: March 18th, 2014

(1) Determine the transfer function of an imaging system defined by the following PSF

\[ h(x, y) = \text{rect}\left(\frac{x}{2}, \frac{y}{2}\right) \]

Recall: \[ \text{rect}(p, q) = \begin{cases} 
1 & |p| < 0.5, \ |q| < 0.5 \\
0 & \text{else} 
\end{cases} \]

(2) Calculate the value of the transfer function at \( u=0.25 \ \text{rad/cm} \) and \( v=0.25 \ \text{rad/cm} \). In words tell me what the value you just calculated means physically.

Equations:

\[ H(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y)e^{-j2\pi(ux+vy)} \, dx \, dy \]

\[ h(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(u, v)e^{j2\pi(ux+vy)} \, du \, dv \]
Quiz #1: March 18th, 2014

(1) Determine the transfer function of an imaging system defined by the following PSF

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\[ H(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y)e^{-j2\pi(ux+vy)} \, dx \, dy \]

\[ = \int_{-1}^{1} \int_{-1}^{1} e^{-j2\pi(ux+vy)} \, dx \, dy = \int_{-1}^{1} [e^{-j2\pi(ux)} - e^{j2\pi(ux)}] \cdot \int_{-1}^{1} [e^{-j2\pi(vy)} - e^{j2\pi(vy)}] \]

\[ = \frac{-1}{2\pi j u} [e^{-j2\pi(u)} - e^{j2\pi(u)}] \cdot \frac{-1}{2\pi j v} [e^{-j2\pi(v)} - e^{j2\pi(v)}] \]

\[ = \frac{e^{-j2\pi(u)} - e^{j2\pi(u)}}{2\pi j u} \cdot \frac{e^{-j2\pi(v)} - e^{j2\pi(v)}}{2\pi j v} \]

\[ = \frac{-2j \sin(2\pi u)}{2\pi j u} \cdot \frac{-2j \sin(2\pi v)}{2\pi j v} = 4 \cdot \sin c(2\pi u) \cdot \sin c(2\pi v) \]
(2) Calculate the value of the transfer function at $u=0.25$ rad/cm and $v=0.25$ rad/cm. In words tell me what the value you just calculated means physically.

$$H(u, v) = 4 \cdot \sin c(2\pi u) \cdot \sin c(2\pi v)$$

$$H(0.25, 0.25) = 4 \cdot \sin c(2\pi 0.25) \cdot \sin c(2\pi 0.25)$$

$$= 4 \cdot \frac{\sin(\pi / 2)}{\pi / 2} \cdot \frac{\sin(\pi / 2)}{\pi / 2} = \frac{16}{\pi^2}$$

If a sinusoidal input of spatial frequencies $u=0.25$ rad/cm and $v=0.25$ rad/cm is inputted into the system the output’s magnitude will be $16/\pi^2$ less and in phase.
Image Formation

- Basic imaging equation
- Geometric effects
- Extended source
- Film blurring
- Impact of noise and scattering
Basic Imaging Equations

Monochromatic

\[ I(x, y) = I_o e^{- \int_0^{r(x,y)} \mu(s;x,y) ds} \]

Polychromatic

\[ I(x, y) = \int_{E_{\text{min}}}^{E_{\text{max}}} S_o(E') e^{- \int_0^{r(x,y)} \mu(s;x,y,E') ds} dE' \]
1) What is the local contrast of the blood vessel?
2) What is the local contrast of the blood vessel when contrast agent is injected?
1) What is the local contrast of the blood vessel?

\[ I_{\text{min}} = I_o e^{-(0.4 \cdot 2.0)} \approx 0.449 I_o \]
\[ I_{\text{max}} = I_o e^{-(0.4 \cdot 1.5 + 0.2 \cdot 0.5)} \approx 0.4966 I_o \]

Local contrast: \[ C_l = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}}} = 0.0959 \]

Global contrast: \[ C = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = 0.0503 \]
Example

1) What is the local contrast of the blood vessel?

2) What is the local contrast of the blood vessel when contrast agent is injected?

\[
\begin{align*}
I_{\text{min}} &= I_o e^{-(0.4 \cdot 2.0)} \approx 0.449 I_o \\
I_{\text{max}} &= I_o e^{-(0.4 \cdot 1.5 + 20 \cdot 0.5)} \approx 0.0000249 I_o
\end{align*}
\]

Local contrast: \( C_l = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}}} = 0.999944 \)

Global contrast: \( C = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = 0.999889 \)
Geometric Effects

- X-rays are diverging from source
- Undesirable effects
  - $\cos^3(\theta)$ fall off in intensity across detector
  - magnification
Geometric Effects

- $I_0$ is intensity at $(0, 0)$
- $r$ is distance from $(x, y)$ to x-ray origin
- $\theta$ is angle between $(0, 0)$ and $(x, y)$
Inverse Square Law

- Net flux of photons decrease as $1/r^2$.
  Therefore
  \[
  I_0 = \frac{I_S}{4\pi d^2} \quad I_r = \frac{I_S}{4\pi r^2}
  \]

- Eliminate source intensity $I_S$
  \[
  I_r = I_0 \frac{d^2}{r^2}
  \]

- Since $\cos \theta = d/r$
  \[
  I_r = I_0 \cos^2 \theta
  \]

$I_s =$ photon flux at source
$I_o =$ photon flux at detector origin
$I_r =$ photon flux at $(x,y)$ point on detector
Obliquity

- Intensity is

\[ I_d = I_0 \cos \theta \]
Inverse Square Law + Obliquity

- Inverse square law and obliquity combine

\[ I_d(x_d, y_d) = I_0 \cos^3 \theta \]

How big of a deal is this?
Inverse Square Law + Obliquity

- Inverse square law and obliquity combine

\[ I_d(x_d, y_d) = I_0 \cos^3 \theta \]

How big of a deal is this?

Depends on (1) how far away is the detector from the source and (2) how large of a field of view are we looking at.
Inverse Square Law + Obliquity

- Inverse square law and obliquity combine

\[ I_d(x_d, y_d) = I_0 \cos^3 \theta \]

How big of a deal is this?

Depends on (1) how far away is the detector from the source and (2) how large of a field of view are we looking at.

Example

Chest x-ray system has a detector of 18" x 18"
The source is placed a distance of 6' (72") away
What is the maximum change in intensity across the detector?

\[ \theta = \tan^{-1}\left(\frac{9}{72}\right) = 7.12^\circ \]

\[ \cos(\theta)^3 = 0.977 \]

\[ I_{\text{min}} = 0.977I_o \]

Usually not that big of a problem
Path Length Variation

Since the rays are going at different angles the amount of material they go through varies depending on the location through the tissue. This difference in path length means more attenuation for points away from the origin.
Path Length Variation

This results in a prism effect if you are imaging a cube.

\[ I_d(x, y) = I_0 \exp\{-\mu L / \cos \theta\} \]

How big of a deal is this?
Inverse Square Law +Obliquity+Path Length

\[ I_d (x, y) = I_o \cos^3 (\theta) e^{-\mu L / \cos \theta} \]
Objects Magnification

- Size on detector depends on distance from source

\[ w_z = w \frac{d}{z} \]

Magnification factor:

\[ M(z) = \frac{d}{z} \]
Imaging of a Thin Non-Uniform Slab

- Assume a very thin slab at $z$
  - the linear absorption coefficient at $(x',y')$ is $\mu(x',y')$
  - Detector position $(x,y) \rightarrow$ slab position $(x',y')$

\[
\frac{x}{d} = \frac{x'}{z_o} \Rightarrow x' = x \cdot \frac{z_o}{d} = \frac{x}{M(z)}
\]
Let “transmittivity” given as \( t_z(x', y') \) be transmission through an infinitely thin object (e.g. thin piece of paper with a lead pattern printed on it)

Then at the detector we get
\[
I_d(x, y) = I_o \cos^3(\theta) \cdot t_z\left(\frac{x}{M(z)}, \frac{y}{M(z)}\right)
\]

After substitution we get:
\[
I_d(x, y) = I_o \left(\frac{d}{\sqrt{d^2 + x^2 + y^2}}\right)^3 \cdot t_z\left(\frac{xz}{d}, \frac{yz}{d}\right)
\]
Not done yet!

There are additional factors

(1) X-rays source are not really a point source but an extended source. This causes blurring.

(2) X-ray films are not perfect! A single x-ray photon causes a blurry spot on the film. This is equivalent to the “impulse response” or PSF of the film $h(x,y)$.
When we add all that together

\[ I_d(x, y) = I_o \cos^3(\theta) \cdot \left\{ \frac{1}{4\pi d^2 m^2} s\left(\frac{x}{m}, \frac{y}{m}\right) \right\} * t_z\left(\frac{x}{M}, \frac{y}{M}\right) * h(x, y) \]

Inverse square + obliquity

Extended source
m = source magnification
s = source intensity
distribution.

Transmittivity
of object

PSF of film

where

\[ M = \frac{d}{z_o} \]

\[ m = 1 - M \]