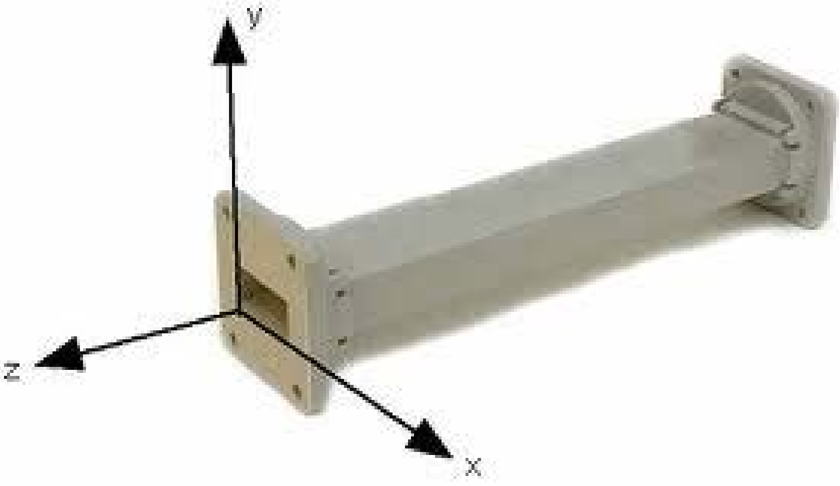
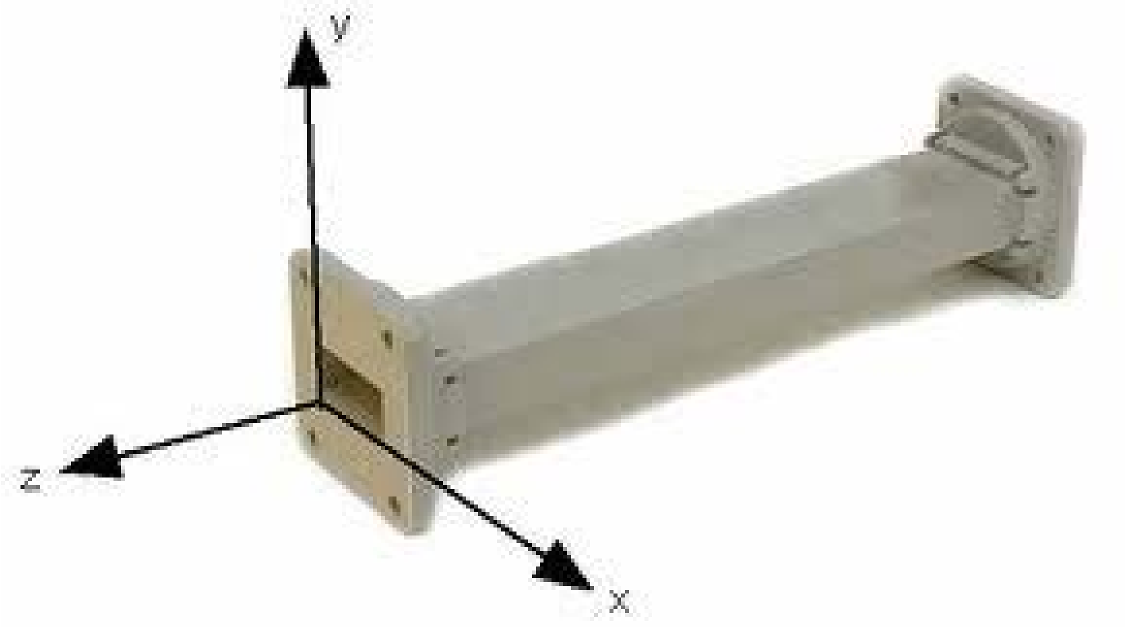
Waveguide Types



Uniform Waveguides



We are interested in finding what electromagnetic field solutions are possible in a uniform infinite waveguide with no sources.

We can always find those solutions by solving:

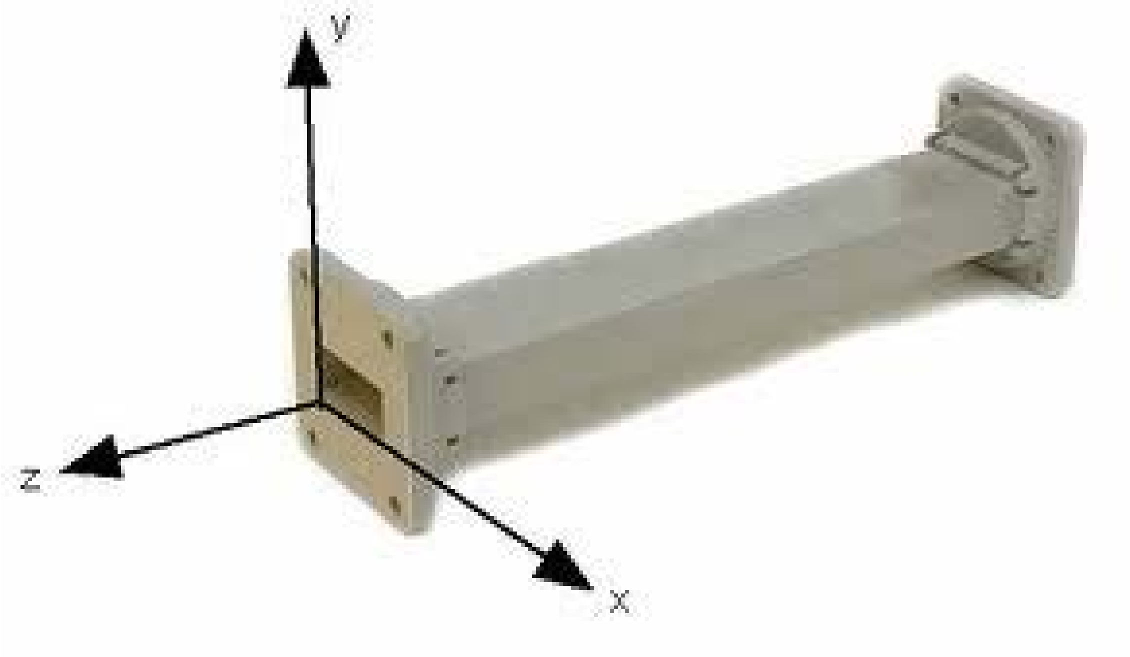
$$\nabla^2 \tilde{E}(x,y,z) + k^2 \tilde{E}(x,y,z) = 0$$

or

$$\nabla^2 \tilde{H}(x,y,z) + k^2 \tilde{H}(x,y,z) = 0$$

Subject to boundary conditions.

Uniform Waveguides



Because the cross section does not change in the z direction

$$\tilde{E}(x,y,z) = \tilde{E}_t(x,y)e^{-j\beta_z z}$$

$$\widetilde{H}(x,y,z) = \widetilde{H}_t(x,y)e^{-j\beta_z z}$$

Uniform Waveguides

$$E_x = \frac{-j}{(k_c{}^2)}\left[\omega\mu\frac{\partial H_z}{\partial y} + \beta_z\frac{\partial E_z}{\partial x}\right]$$

$$H_x = \frac{j}{(k_c{}^2)}\left[\omega\varepsilon\frac{\partial E_z}{\partial y} - \beta_z\frac{\partial H_z}{\partial x}\right]$$

$$E_y = \frac{j}{(k_c{}^2)}\left[\omega\mu\frac{\partial H_z}{\partial x} - \beta_z\frac{\partial E_z}{\partial y}\right]$$

$$H_y = \frac{-j}{(k_c{}^2)}\left[\omega\varepsilon\frac{\partial E_z}{\partial x} + \beta_z\frac{\partial H_z}{\partial y}\right]$$

Three Cases:

(1) E_z=0, $H_z \neq 0$ TE

$$E_x = \frac{-j}{(k_c{}^2)}\left[\omega\mu\frac{\partial H_z}{\partial y}\right]$$

$$E_y = \frac{j}{(k_c{}^2)}\left[\omega\mu\frac{\partial H_z}{\partial x}\right]$$

$$H_x = \frac{-j}{(k_c{}^2)}\left[\beta_z\frac{\partial H_z}{\partial x}\right]$$

$$H_y = \frac{-j}{(k_c{}^2)}\left[\beta_z\frac{\partial H_z}{\partial y}\right]$$

$$\nabla_t{}^2 H_z(x,y) + k_c{}^2 H_z(x,y) = 0$$

(2) $E_z \neq 0$, H_z = 0 TM

$$E_x = \frac{-j}{(k_c{}^2)}\left[\beta_z\frac{\partial E_z}{\partial x}\right]$$

$$E_y = \frac{-j}{(k_c{}^2)}\left[\beta_z\frac{\partial E_z}{\partial y}\right]$$

$$H_x = \frac{j}{(k_c{}^2)}\left[\omega\varepsilon\frac{\partial E_z}{\partial y}\right]$$

$$H_y = \frac{-j}{(k_c{}^2)}\left[\omega\varepsilon\frac{\partial E_z}{\partial x}\right]$$

$$\nabla_t{}^2 E_z(x,y) + k_c{}^2 E_z(x,y) = 0$$

Uniform Waveguides

$$E_x = \frac{-j}{(k_c{}^2)}\left[\omega\mu\frac{\partial H_z}{\partial y} + \beta_z\frac{\partial E_z}{\partial x}\right]$$

$$H_x = \frac{j}{(k_c{}^2)}\left[\omega\varepsilon\frac{\partial E_z}{\partial y} - \beta_z\frac{\partial H_z}{\partial x}\right]$$

$$E_y = \frac{j}{(k_c{}^2)}\left[\omega\mu\frac{\partial H_z}{\partial x} - \beta_z\frac{\partial E_z}{\partial y}\right]$$

$$H_y = \frac{-j}{(k_c{}^2)}\left[\omega\varepsilon\frac{\partial E_z}{\partial x} + \beta_z\frac{\partial H_z}{\partial y}\right]$$

Three Cases:

(3) $E_z=0$, $H_z = 0$ TEM

$$k_c = \sqrt{k^2 - \beta_z{}^2} = 0$$

$$\Rightarrow \beta_z = k = \omega\sqrt{\varepsilon\mu}$$

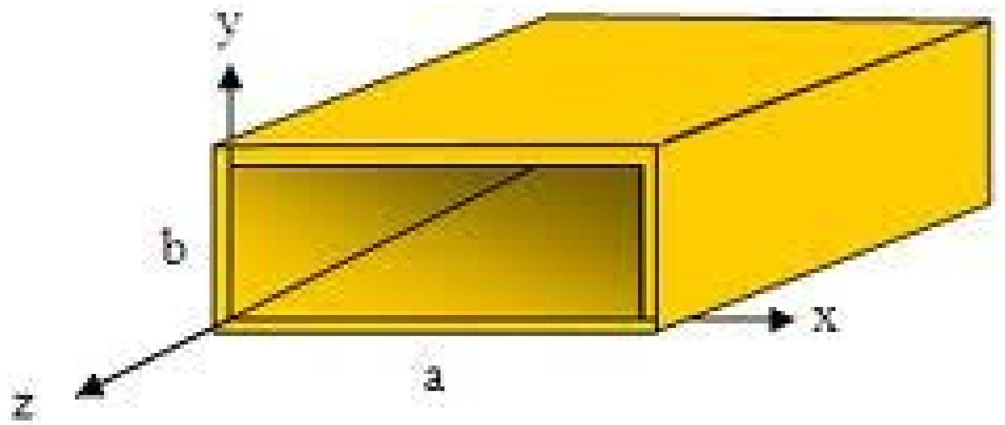
$$\nabla_t{}^2 E_x(x,y) + k_c{}^2 E_x(x,y) = 0$$

($k_c{}^2$ =0)

$$\Rightarrow \nabla_t{}^2 E_x(x,y) = 0$$

$$\nabla_t{}^2 E_y(x,y) = 0$$

Rectangular Waveguides



TM Modes

$$\nabla_t{}^2 E_z(x,y) + k_c{}^2 E_z(x,y) = 0$$

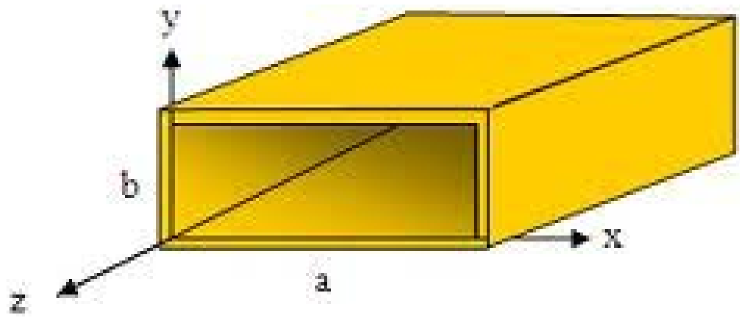
$$E_x = \frac{-j}{(k_c{}^2)}\left[\beta_z \frac{\partial E_z}{\partial x}\right]$$

$$E_y = \frac{-j}{(k_c{}^2)}\left[\beta_z \frac{\partial E_z}{\partial y}\right]$$

$$H_x = \frac{j}{(k_c{}^2)}\left[\omega\varepsilon \frac{\partial E_z}{\partial y}\right]$$

$$H_y = \frac{-j}{(k_c{}^2)}\left[\omega\varepsilon \frac{\partial E_z}{\partial x}\right]$$

Rectangular Waveguides



TM Modes

$$E_z{}^{nm}(x,y,z) = A_{mn}\left[sin\left(\frac{n\pi}{a}x\right)\right]\left[sin\left(\frac{m\pi}{b}y\right)\right]e^{-j\beta_z{}^{mn}z}$$

$$\beta_z{}^{mn} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}$$

$m \text{ and } n = \pm 1,2,3, \ldots$

$$E_x = \frac{-j}{(k_c{}^2)}\left[\beta_z\frac{\partial E_z}{\partial x}\right]$$

$$H_x = \frac{j}{(k_c{}^2)}\left[\omega\varepsilon\frac{\partial E_z}{\partial y}\right]$$

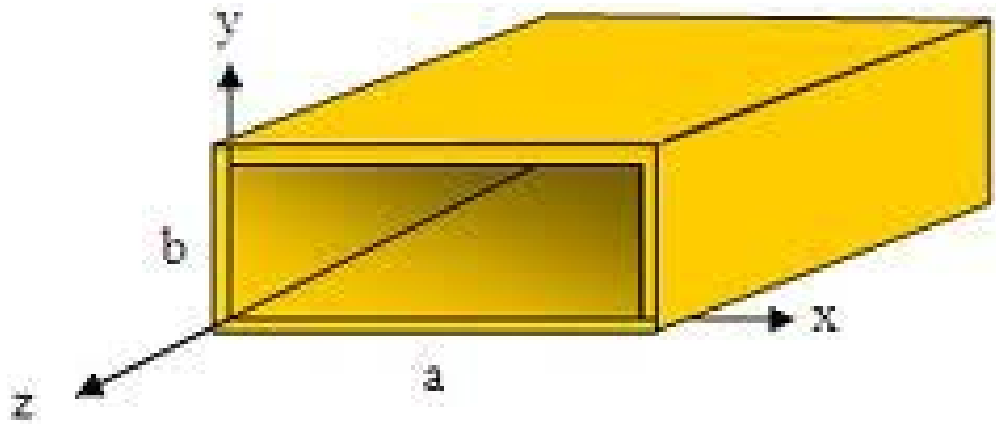
$$E_y = \frac{-j}{(k_c{}^2)}\left[\beta_z\frac{\partial E_z}{\partial y}\right]$$

$$H_y = \frac{-j}{(k_c{}^2)}\left[\omega\varepsilon\frac{\partial E_z}{\partial x}\right]$$

Summary of TM modes

Plane waves in the dielectric medium	Inside the waveguide
$\beta_{PW} = \omega\sqrt{\mu\varepsilon}$	$\beta = \beta_{PW}\sqrt{1-\left[\frac{f_c}{f}\right]^2}$
$\eta_{PW} = \sqrt{\mu / \varepsilon}$	$\eta_{TM} = \eta_{PW}\sqrt{1-\left[\frac{f_c}{f}\right]^2}$
$v_p = \omega / \beta_{PW} = f\lambda = 1/\sqrt{\mu\varepsilon} = c$	$v_p = \frac{\omega}{\beta_{PW}\sqrt{1-\left[\frac{f_c}{f}\right]^2}}$
$\lambda_{PW} = \frac{c}{f}$	$\lambda = \frac{\lambda_{PW}}{\sqrt{1-\left[\frac{f_c}{f}\right]^2}}$

Rectangular Waveguides



TE Modes

$$\nabla_t{}^2 H_z(x,y) + k_c{}^2 H_z(x,y) = 0$$

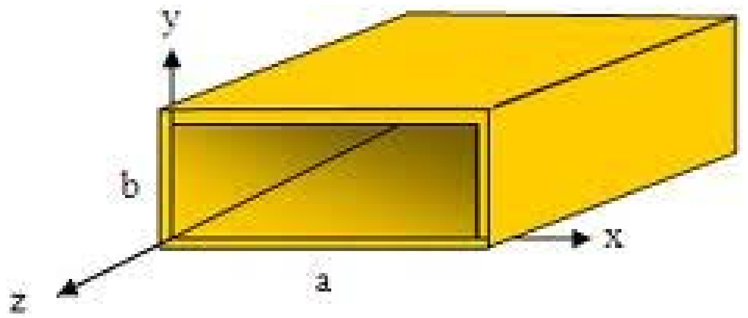
$$E_x = \frac{-j}{(k_c{}^2)}\left[\omega\mu \frac{\partial H_z}{\partial y}\right]$$

$$E_y = \frac{j}{(k_c{}^2)}\left[\omega\mu \frac{\partial H_z}{\partial x}\right]$$

$$H_x = \frac{-j}{(k_c{}^2)}\left[\beta_z \frac{\partial H_z}{\partial x}\right]$$

$$H_y = \frac{-j}{(k_c{}^2)}\left[\beta_z \frac{\partial H_z}{\partial y}\right]$$

Rectangular Waveguides



TE Modes

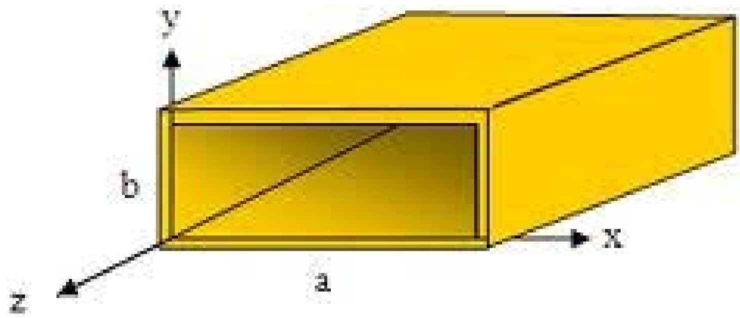
$$\nabla_t^2 H_z(x,y) + k_c^2 H_z(x,y) = 0$$

Solve using separation of variables

$$H_z(x,y,z) = \left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

How many unknowns do we have?

Rectangular Waveguides



TE Modes

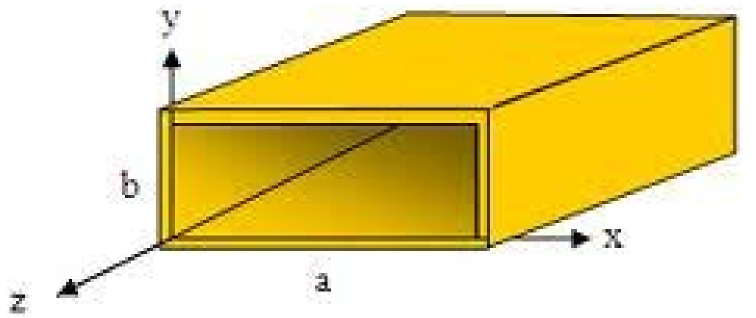
$$\nabla_t^{\,2} H_z(x,y) + k_c^{\,2} H_z(x,y) = 0$$

Solve using separation of variables

$$H_z(x,y,z) = \left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

What boundary conditions should we use?

Rectangular Waveguides



TE Modes

$$\nabla_t^{\,2} H_z(x,y) + k_c^{\,2} H_z(x,y) = 0$$

Solve using separation of variables

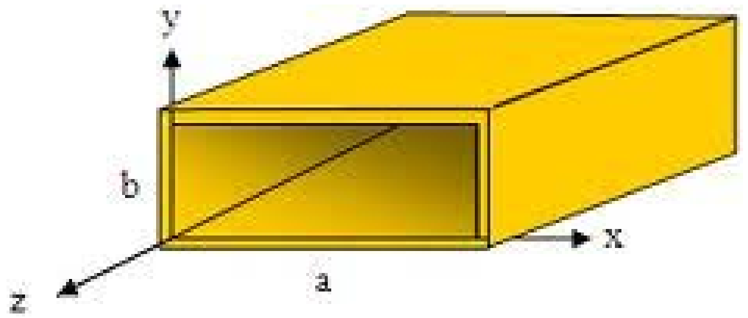
$$H_z(x,y,z) = \left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

Boundary Conditions

$$E_y(0,y,z) = E_y(a,y,z) = 0$$

$$E_x(x,0,z) = E_x(x,b,z) = 0$$

Rectangular Waveguides



TE Modes

$$H_z(x,y,z) = [A_1\cos(\beta_x x) + A_2\sin(\beta_x x)][B_1\cos(\beta_y y) + B_2\sin(\beta_y y)]e^{-j\beta_z z}$$

$$E_x = \frac{-j}{(k_c{}^2)}\left[\omega\mu\frac{\partial H_z}{\partial y}\right]$$

$$E_y = \frac{j}{(k_c{}^2)}\left[\omega\mu\frac{\partial H_z}{\partial x}\right]$$

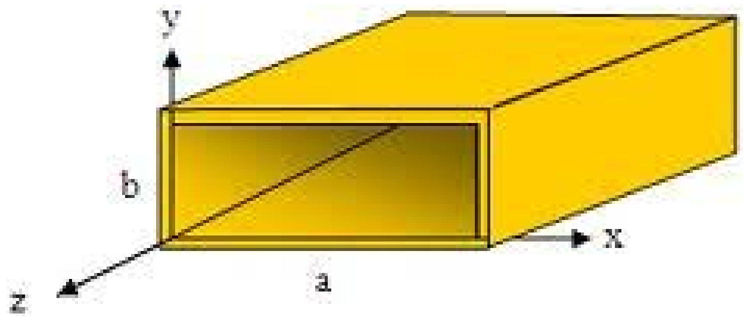
$$H_x = \frac{-j}{(k_c{}^2)}\left[\beta_z\frac{\partial H_z}{\partial x}\right]$$

$$H_y = \frac{-j}{(k_c{}^2)}\left[\beta_z\frac{\partial H_z}{\partial y}\right]$$

$$E_x(x,y,z) = -\frac{j\omega\mu}{k_c{}^2}\beta_y[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)][-B_1\sin(\beta_y y) + B_2\cos(\beta_y y)]e^{-j\beta_z z}$$

$$E_y(x,y,z) = \frac{j\omega\mu}{k_c{}^2}\beta_x[-A_1\sin(\beta_x x) + A_2\cos(\beta_x x)][B_1\cos(\beta_y y) + B_2\sin(\beta_y y)]e^{-j\beta_z z}$$

Rectangular Waveguides



TE Modes

$$H_z(x,y,z) = [A_1\cos(\beta_x x) + A_2\sin(\beta_x x)][B_1\cos(\beta_y y) + B_2\sin(\beta_y y)]e^{-j\beta_z z}$$

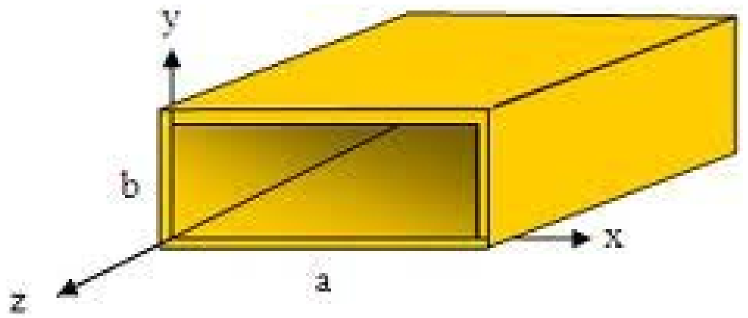
$$E_x(x,y,z) = -\frac{j\omega\mu}{k_c^{\,2}}\beta_y[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)][-B_1\sin(\beta_y y) + B_2\cos(\beta_y y)]e^{-j\beta_z z}$$

$$E_y(x,y,z) = \frac{j\omega\mu}{k_c^{\,2}}\beta_x[-A_1\sin(\beta_x x) + A_2\cos(\beta_x x)][B_1\cos(\beta_y y) + B_2\sin(\beta_y y)]e^{-j\beta_z z}$$

Boundary Conditions

$$E_y(0,y,z) = 0$$

Rectangular Waveguides



TE Modes

$$H_z(x,y,z) = \left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

$$E_x(x,y,z) = -\frac{j\omega\mu}{k_c^{\ 2}}\beta_y\left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[-B_1\sin(\beta_y y) + B_2\cos(\beta_y y)\right]e^{-j\beta_z z}$$

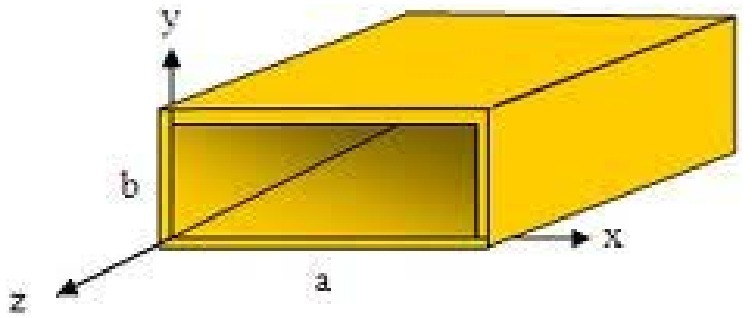
$$E_y(x,y,z) = \frac{j\omega\mu}{k_c^{\ 2}}\beta_x\left[-A_1\sin(\beta_x x) + A_2\cos(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

Boundary Conditions

$$E_y(0,y,z) = 0$$

$$E_y(0,y,z) = \frac{j\omega\mu}{k_c^{\ 2}}\beta_x\left[-A_1\sin(\beta_x 0) + A_2\cos(\beta_x 0)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z} = 0$$

Rectangular Waveguides



TE Modes

$$H_z(x,y,z) = \left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

$$E_x(x,y,z) = -\frac{j\omega\mu}{k_c^{\ 2}}\beta_y\left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[-B_1\sin(\beta_y y) + B_2\cos(\beta_y y)\right]e^{-j\beta_z z}$$

$$E_y(x,y,z) = \frac{j\omega\mu}{k_c^{\ 2}}\beta_x\left[-A_1\sin(\beta_x x) + A_2\cos(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

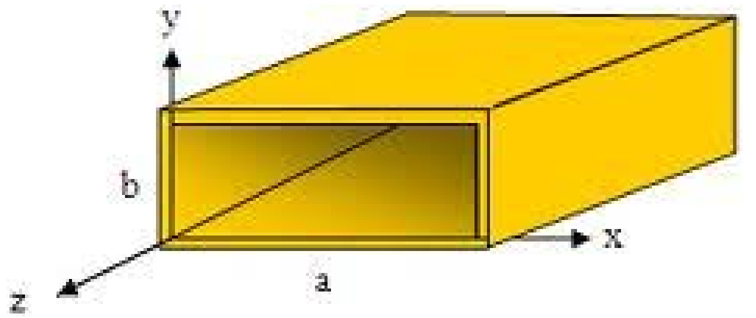
Boundary Conditions

$$E_y(0,y,z) = 0$$

$$E_y(0,y,z) = \frac{j\omega\mu}{k_c^{\ 2}}\beta_x\left[-A_1\sin(\beta_x 0) + A_2\cos(\beta_x 0)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z} = 0$$

$$\Rightarrow \quad A_2 = 0$$

Rectangular Waveguides



TE Modes

$$H_z(x,y,z) = \left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

$$E_x(x,y,z) = -\frac{j\omega\mu}{k_c^{\ 2}}\beta_y\left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[-B_1\sin(\beta_y y) + B_2\cos(\beta_y y)\right]e^{-j\beta_z z}$$

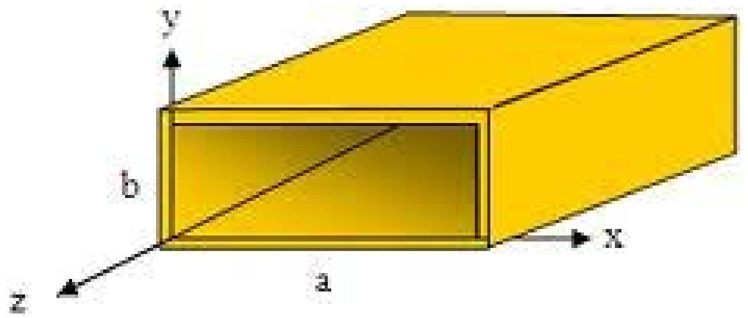
$$E_y(x,y,z) = \frac{j\omega\mu}{k_c^{\ 2}}\beta_x\left[-A_1\sin(\beta_x x) + A_2\cos(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

Boundary Conditions

$$E_y(a,y,z) = 0$$

$$E_y(a,y,z) = \frac{j\omega\mu}{k_c^{\ 2}}\beta_x\left[-A_1\sin(\beta_x a)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z} = 0$$

Rectangular Waveguides



TE Modes

$$H_z(x,y,z) = \left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

$$E_x(x,y,z) = -\frac{j\omega\mu}{k_c^{\;2}}\beta_y\left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[-B_1\sin(\beta_y y) + B_2\cos(\beta_y y)\right]e^{-j\beta_z z}$$

$$E_y(x,y,z) = \frac{j\omega\mu}{k_c^{\;2}}\beta_x\left[-A_1\sin(\beta_x x) + A_2\cos(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

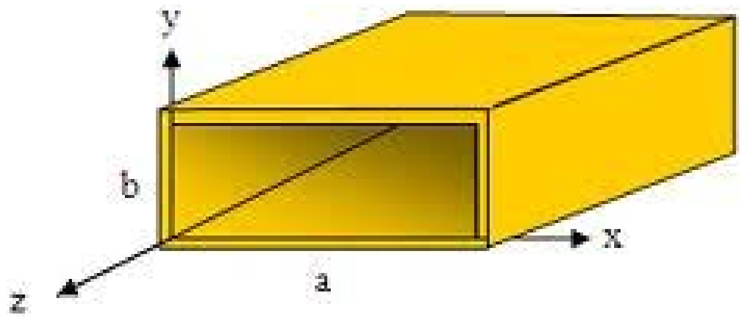
Boundary Conditions

$$E_y(a,y,z) = 0$$

$$E_y(a,y,z) = \frac{j\omega\mu}{k_c^{\;2}}\beta_x\left[-A_1\sin(\beta_x a)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z} = 0$$

$$\Rightarrow \quad \beta_x = \frac{n\pi}{a},\ n = 0,1,2,\ldots.$$

Rectangular Waveguides



TE Modes

$$H_z(x,y,z) = [A_1 \cos(\beta_x x) + A_2 \sin(\beta_x x)][B_1 \cos(\beta_y y) + B_2 \sin(\beta_y y)]e^{-j\beta_z z}$$

$$E_x(x,y,z) = -\frac{j\omega\mu}{k_c^{\ 2}}\beta_y [A_1 \cos(\beta_x x) + A_2 \sin(\beta_x x)][- B_1 \sin(\beta_y y) + B_2 \cos(\beta_y y)]e^{-j\beta_z z}$$

$$E_y(x,y,z) = \frac{j\omega\mu}{k_c^{\ 2}}\beta_x [- A_1 \sin(\beta_x x) + A_2 \cos(\beta_x x)][B_1 \cos(\beta_y y) + B_2 \sin(\beta_y y)]e^{-j\beta_z z}$$

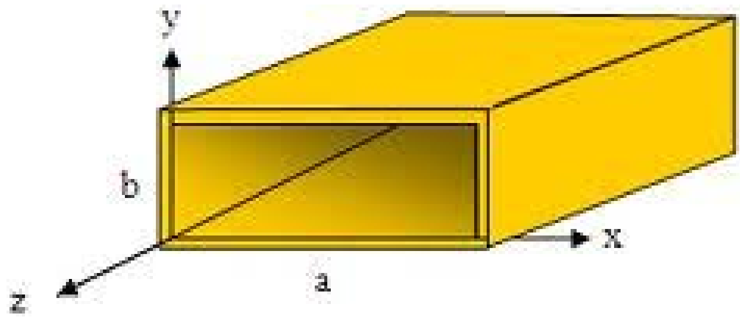
Boundary Conditions

$$E_x(x,0,z) = 0$$

$$E_x(x,0,z) = -\frac{j\omega\mu}{k_c^{\ 2}}\beta_y \left[A_1 \cos(\frac{n\pi}{a} x) \right][- B_1 \sin(\beta_y 0) + B_2 \cos(\beta_y 0)]e^{-j\beta_z z} = 0$$

$$\Rightarrow B_2 = 0$$

Rectangular Waveguides



TE Modes

$$H_z(x,y,z) = \left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

$$E_x(x,y,z) = -\frac{j\omega\mu}{k_c^{\ 2}}\beta_y\left[A_1\cos(\beta_x x) + A_2\sin(\beta_x x)\right]\left[-B_1\sin(\beta_y y) + B_2\cos(\beta_y y)\right]e^{-j\beta_z z}$$

$$E_y(x,y,z) = \frac{j\omega\mu}{k_c^{\ 2}}\beta_x\left[-A_1\sin(\beta_x x) + A_2\cos(\beta_x x)\right]\left[B_1\cos(\beta_y y) + B_2\sin(\beta_y y)\right]e^{-j\beta_z z}$$

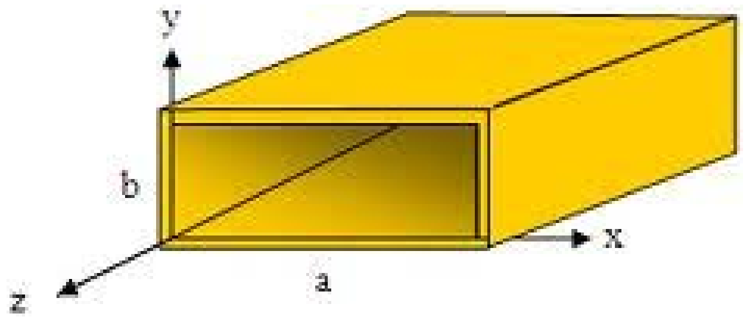
Boundary Conditions

$$E_x(x,b,z) = 0$$

$$E_x(x,b,z) = -\frac{j\omega\mu}{k_c^{\ 2}}\beta_y\left[A_1\cos(\frac{n\pi}{a}x)\right]\left[-B_1\sin(\beta_y b)\right]e^{-j\beta_z z} = 0$$

$$\Rightarrow \beta_y = \frac{m\pi}{b}, m = 0,1,2,....$$

Rectangular Waveguides

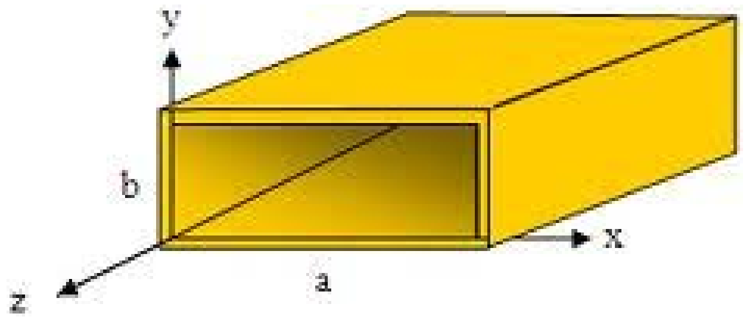


TE Modes

$$H_z(x,y,z) = \left[A_1 \cos(\frac{n\pi}{a}x) \right]\left[B_1 \cos(\frac{m\pi}{b}y) \right] e^{-j\beta_z z}$$

$$\Rightarrow \quad H_z^{nm}(x,y,z) = A_{nm} \cos(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y) e^{-j\beta_z z}$$

Rectangular Waveguides



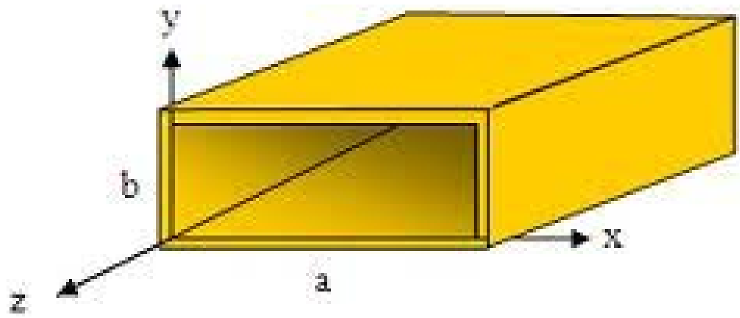
TE Modes

$$H_z(x,y,z) = \left[A_1 \cos\left(\frac{n\pi}{a}x\right) \right] \left[B_1 \cos\left(\frac{m\pi}{b}y\right) \right] e^{-j\beta_z z}$$

$$\Rightarrow \quad H_z^{nm}(x,y,z) = A_{nm} \cos\left(\frac{n\pi}{a}x\right) \cos\left(\frac{m\pi}{b}y\right) e^{-j\beta_z z}$$

How do we find βz ?

Rectangular Waveguides



TE Modes

$$H_z(x,y,z) = \left[A_1 \cos(\frac{n\pi}{a}x)\right]\left[B_1 \cos(\frac{m\pi}{b}y)\right]e^{-j\beta_z z}$$

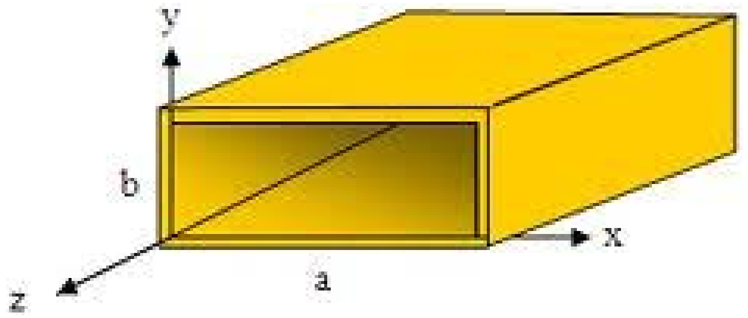
$$\Rightarrow \quad H_z^{nm}(x,y,z) = A_{nm} \cos(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y)e^{-j\beta_z z}$$

How do we find βz ?

$$\beta_z = \sqrt{\omega^2 \mu\varepsilon - \beta_x^2 - \beta_y^2}$$

$$\beta_z = \sqrt{\omega^2 \mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}$$

Rectangular Waveguides



TE Modes

$$H_z^{nm}(x,y,z) = A_{nm}\cos(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y)e^{-j\beta_z z}$$

$$\beta_z{}^{mn} = \sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}$$

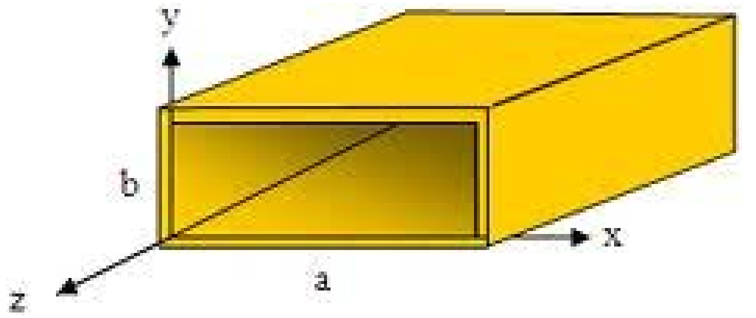
At the point $\omega_c{}^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2 = 0$

the mode changes from evanescent to propagating.

$$f_c{}^{nm} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}$$

Cutoff frequency

Rectangular Waveguides



TE Modes

$$H_z^{nm}(x,y,z) = A_{nm}\cos\left(\frac{n\pi}{a}x\right)\cos\left(\frac{m\pi}{b}y\right)e^{-j\beta_z z}$$

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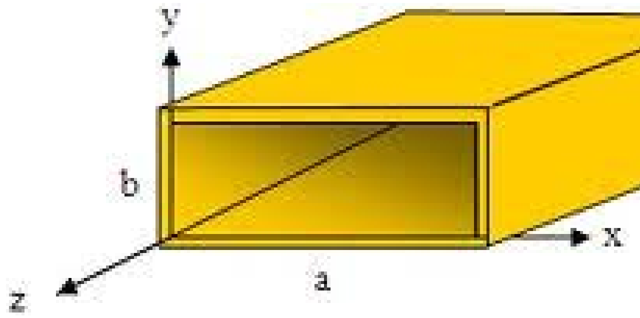
Cutoff frequency

What is the dominate mode for this waveguide?

Rectangular Waveguides



TE Modes

$$H_z^{nm}(x,y,z) = A_{nm}\cos\left(\frac{n\pi}{a}x\right)\cos\left(\frac{m\pi}{b}y\right)e^{-j\beta_z z}$$

$$f_c^{\,nm} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}$$

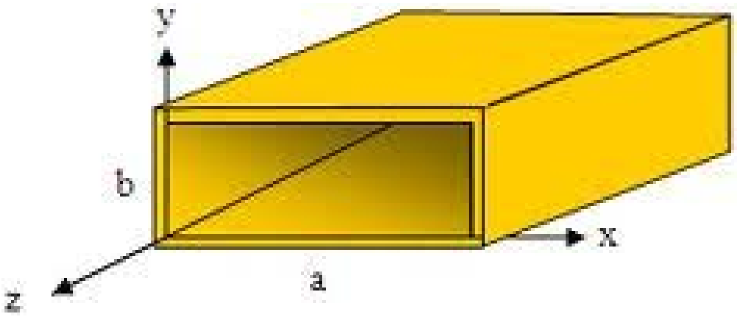
Cutoff frequency

What is the dominate mode for this waveguide?

$$f_c^{10} = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{1}{2a\sqrt{\mu\varepsilon}}$$

a>b

Rectangular Waveguides



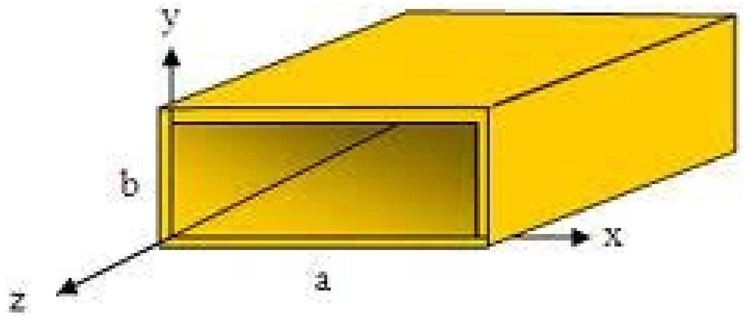
TE Modes

$$H_z^{nm}(x,y,z) = A_{nm}\cos\left(\frac{n\pi}{a}x\right)\cos\left(\frac{m\pi}{b}y\right)e^{-j\beta_z z}$$

$$\beta_z = \sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}$$

What is wavelength of the guided mode?

Rectangular Waveguides



TE Modes

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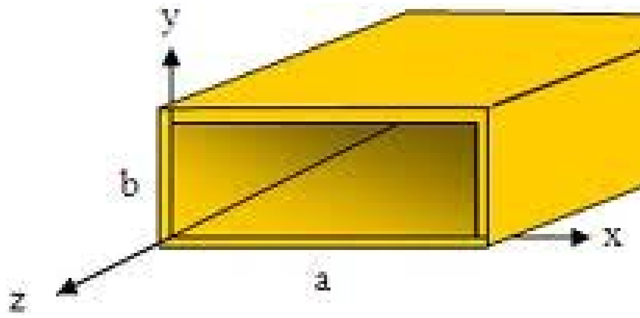
What is wavelength of the guided mode?

$$\lambda_b^{nm} = \frac{2\pi}{\beta_z^{nm}} = \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}}$$

Rectangular Waveguides



TE Modes

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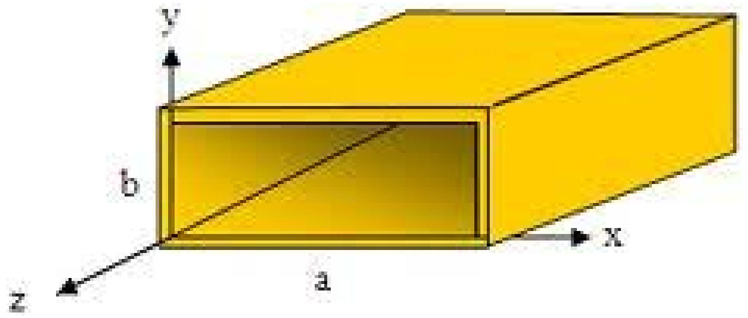
What is wavelength of the guided mode?

$$\lambda_b^{nm} = \frac{2\pi}{\beta_z^{nm}} = \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2}} = \frac{\lambda_o}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Rectangular Waveguides



TE Modes

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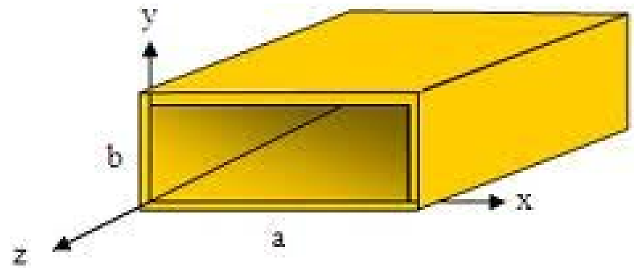
$$\beta_z = \sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}$$

What is the impedance of the guided mode?

$$Z_w^{nm} = \frac{E_x^{nm}}{H_y^{nm}} = \frac{-E_y^{nm}}{H_x^{nm}}$$

Rectangular Waveguides



TE Modes

$$E_x = \frac{-j}{(k_c{}^2)}\left[\omega\mu \frac{\partial H_z}{\partial y}\right]$$

$$E_y = \frac{j}{(k_c{}^2)}\left[\omega\mu \frac{\partial H_z}{\partial x}\right]$$

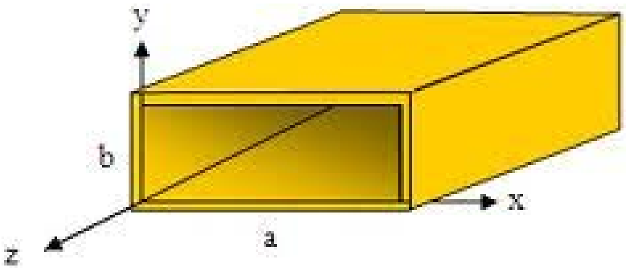
$$H_x = \frac{-j}{(k_c{}^2)}\left[\beta_z \frac{\partial H_z}{\partial x}\right]$$

$$H_y = \frac{-j}{(k_c{}^2)}\left[\beta_z \frac{\partial H_z}{\partial y}\right]$$

What is wavelength of the impedance of the guided mode?

$$Z_w^{nm} = \frac{E_x^{nm}}{H_y^{nm}} = \frac{\omega\varepsilon}{\beta_z^{nm}} = \frac{\omega\varepsilon}{\sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}}$$

Rectangular Waveguides



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$$E_y = \frac{j}{(k_c{}^2)}\left[\omega\mu \frac{\partial H_z}{\partial x}\right]$$

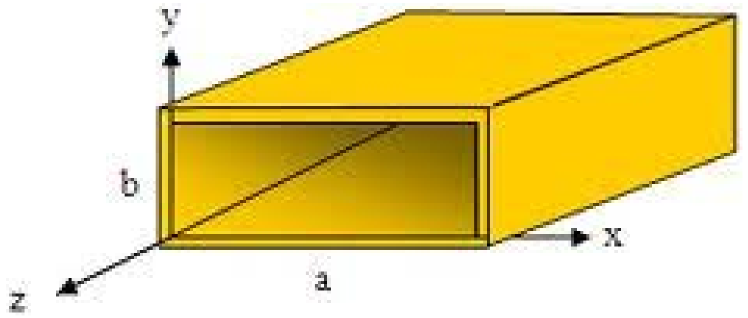
$$H_x = \frac{-j}{(k_c{}^2)}\left[\beta_z \frac{\partial H_z}{\partial x}\right]$$

$$H_y = \frac{-j}{(k_c{}^2)}\left[\beta_z \frac{\partial H_z}{\partial y}\right]$$

What is wavelength of the impedance of the guided mode?

$$Z_w^{nm} = \frac{E_x^{nm}}{H_y^{nm}} = \frac{\omega\varepsilon}{\beta_z^{nm}} = \frac{\omega\varepsilon}{\sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}} = \frac{\eta_{PW}}{\sqrt{1-\left(\frac{f_c^{nm}}{f}\right)^2}}$$

Rectangular Waveguides



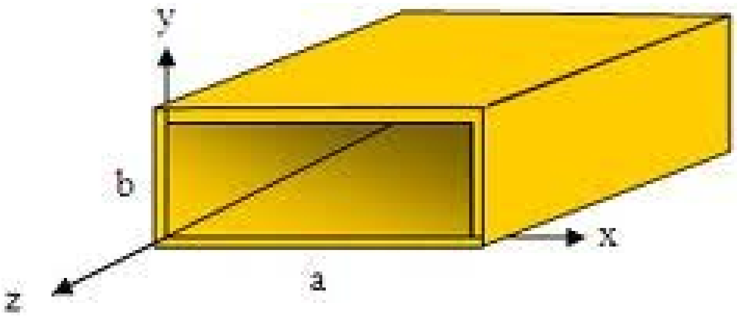
TE Modes

$$H_z^{nm}(x,y,z) = A_{nm}\cos\left(\frac{n\pi}{a}x\right)\cos\left(\frac{m\pi}{b}y\right)e^{-j\beta_z z}$$

$$\beta_z = \sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}$$

What is the phase velocity of the guided wave?

Rectangular Waveguides



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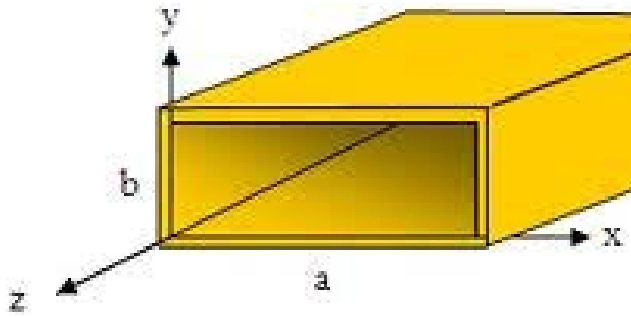
Rectangular Waveguides



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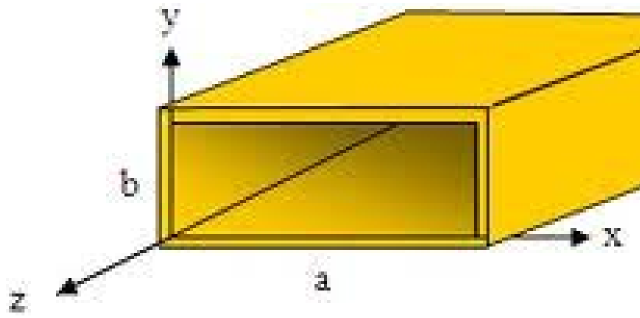
What is the phase velocity of the guided wave?

$$v_p = \frac{\omega}{\beta_z} = \frac{\omega}{\sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}}$$

Rectangular Waveguides



TE Modes

$$H_z^{nm}(x,y,z) = A_{nm}\cos(\frac{n\pi}{a}x)\cos(\frac{m\pi}{b}y)e^{-j\beta_z z}$$

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What is the phase velocity of the guided wave?

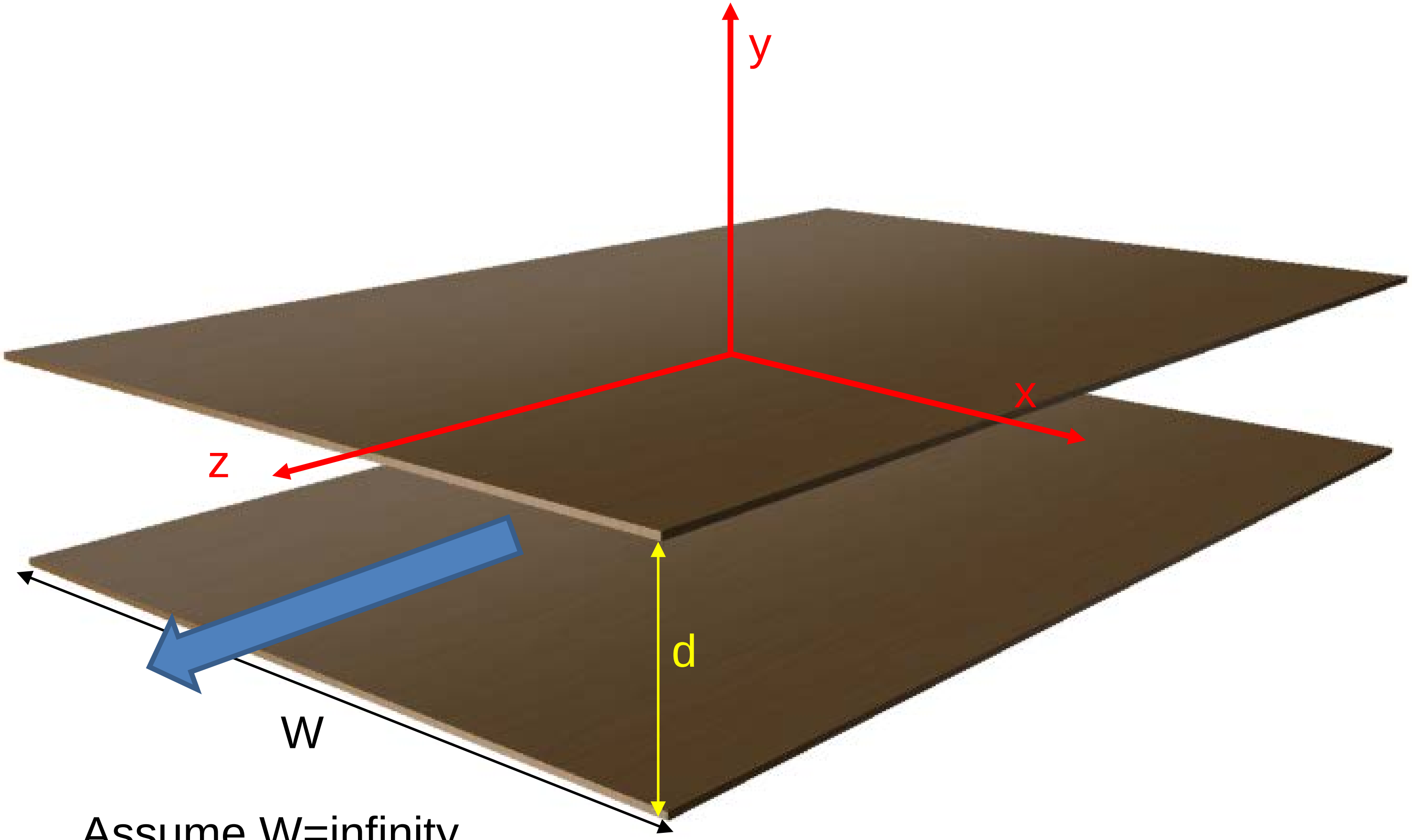
$$v_p^{nm} = \frac{\omega}{\beta_z^{nm}} = \frac{\omega}{\sqrt{\omega^2\mu\varepsilon - \left(\frac{n\pi}{a}\right)^2 - \left(\frac{m\pi}{b}\right)^2}} = \frac{c}{\sqrt{1-\left(\frac{f_c^{nm}}{f}\right)^2}}$$

Summary of TE modes

Plane waves in the dielectric medium	Inside the waveguide
$\beta_{PW} = \omega\sqrt{\mu\varepsilon}$	$\beta = \beta_{PW}\sqrt{1-\left[\frac{f_c}{f}\right]^2}$
$\eta_{PW} = \sqrt{\mu/\varepsilon}$	$\eta_{TE} = \frac{\eta_{PW}}{\sqrt{1-\left[\frac{f_c}{f}\right]^2}}$
$v_p = \omega/\beta_{PW} = f\lambda = 1/\sqrt{\mu\varepsilon} = c$	$v_p = \frac{\omega}{\beta_{PW}\sqrt{1-\left[\frac{f_c}{f}\right]^2}}$
$\lambda_{PW} = \frac{c}{f}$	$\lambda = \frac{\lambda_{PW}}{\sqrt{1-\left[\frac{f_c}{f}\right]^2}}$

Parallel Plate Waveguides

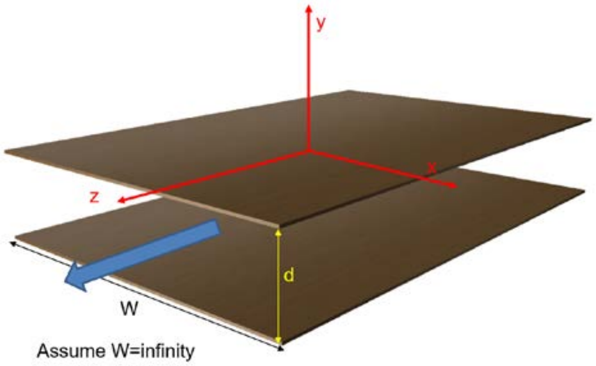
TM Modes (propagation in z direction)



Assume W=infinity

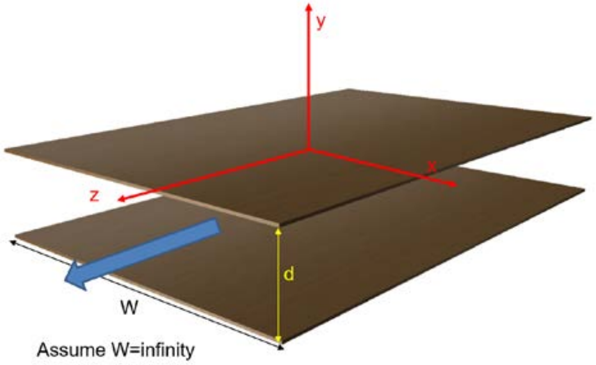
Parallel Plate Waveguides

TM Modes (propagation in z direction)



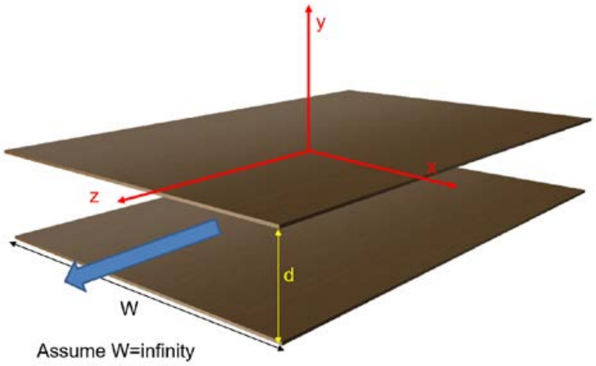
Parallel Plate Waveguides

TM Modes (propagation in z direction)



Parallel Plate Waveguides

TM Modes (propagation in z direction)



Parallel Plate Waveguides

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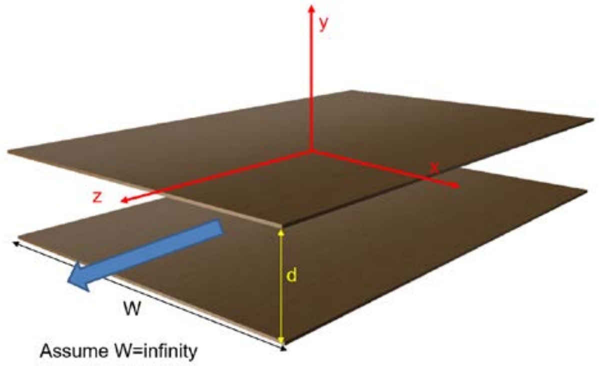

Parallel Plate Waveguides

TM Modes (propagation in z direction)