

**ELEG 413**  
**Spring 2017**  
**Lecture #1**

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# ELEG 413 Engineering Electromagnetics

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Text: Engineering Electromagnetics, Constantine Balanis

[http://www.eecis.udel.edu/~mirotzni/ELEG413/ELEG413\\_2017.htm](http://www.eecis.udel.edu/~mirotzni/ELEG413/ELEG413_2017.htm)

Office Hours: Tuesday/Thursday 1:00-2:00 in Evans 106

**Grading** Your grade will be based on, quizzes and exams broken down as follows:

Homework	0%
In class quizzes	20%
1st Exam	25%
2nd Exam	25%
Final Exam	30%
	100%

# Maxwell's Equations



(1831 – 1879)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

## What was physics like at the time of Maxwell?

- ❑ Newton's laws had been around for almost 200 years and seemed to explain almost everything.
- ❑ Most felt that physics was largely a solved problem with just some odds and ends to figure out.
- ❑ It was felt that everything when broken down to its most fundamental level was just an application of mechanics (Newton's laws).

What was known about electromagnetics to Maxwell?

- (1) That electrical charges come in two types (negative and positive) and that they produce forces between them that proportional to the amount of charge and inversely proportional to the square of the distance between the charges. Like charges repel and unlike charges attract.

• Coulomb (1725)

$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$F_g = G \frac{m_1 m_2}{r^2} \quad (\text{Newton, 1680})$$

For two electrons,

$$e = 1.6 \times 10^{-19} \text{ Coulomb}$$

$$m = 9.1 \times 10^{-31} \text{ Kg}$$

# Introduction (continued)

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$$\frac{F_e}{F_g} = \left( \frac{e}{m} \right)^2 \frac{1}{4\pi\epsilon_0 G} \approx 4 \times 10^{42}!$$

**How big is the number  $10^{42}$  ?**

a. Dimension of universe  $\approx 10^{27}$  m ( $10^{10}$  light years)

Radius of a nucleus  $\approx 10^{-15}$  m

Ratio  $\Rightarrow 10^{42}$

b. Age of universe  $\approx 10^{18}$  s (30 M years )

Life of strange particles  $\approx 10^{-24}$  s

Ratio  $\Rightarrow 10^{42}$

What was known about electromagnetics to Maxwell?

- (2) That magnets have two poles (N and S) and also produce a force between magnets (like poles repel and unlike poles attract) but unlike electrical charges we cannot separate the two poles. If a break a magnet into two parts I get two magnets each having a N/S poles.

What was known about electromagnetics to Maxwell?

- (3) That electrical currents on two wires would create a force between them (like magnets) - Ampere 1820

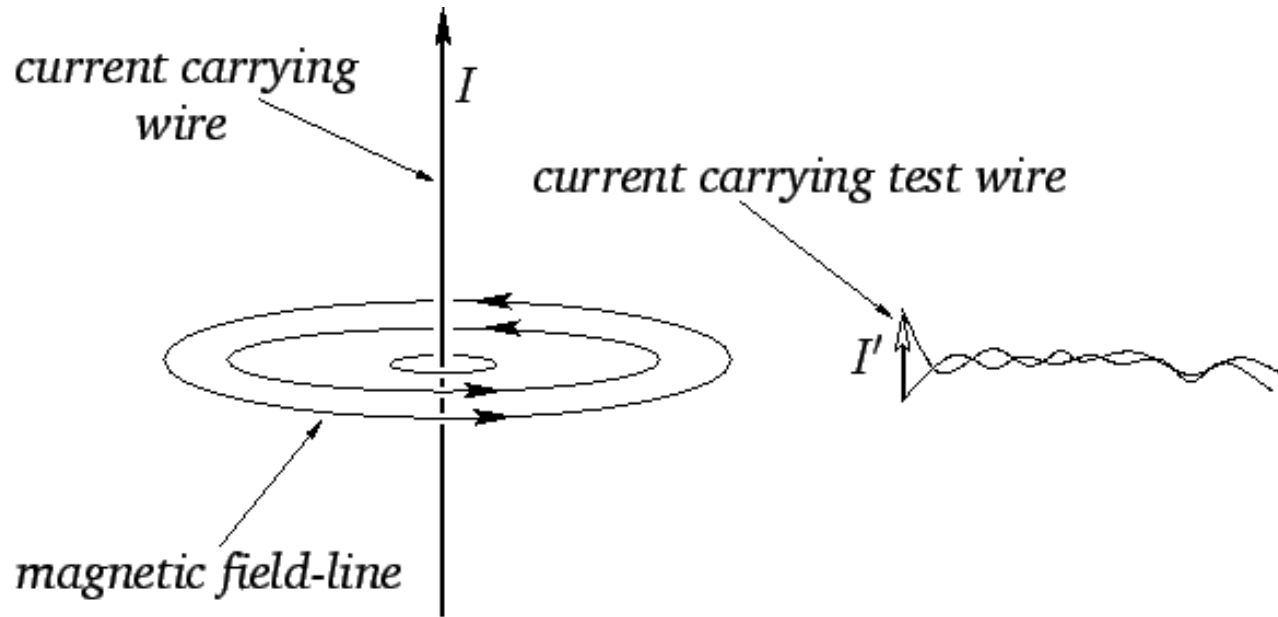
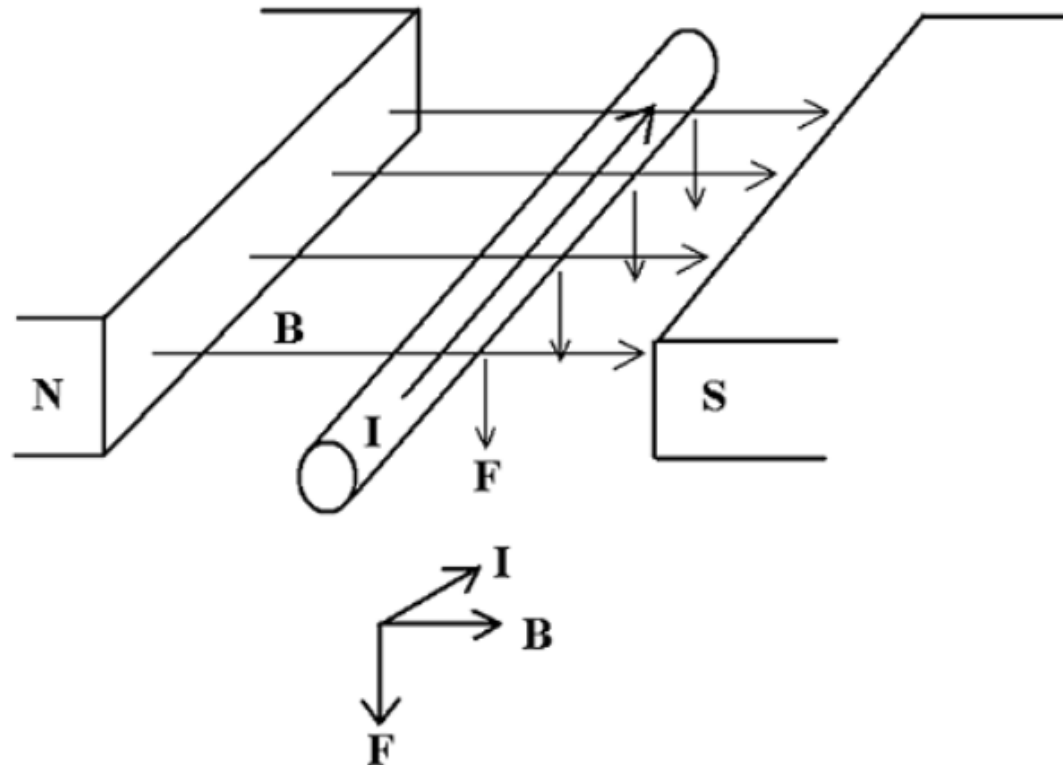


Figure 21: *Ampère's experiment.*

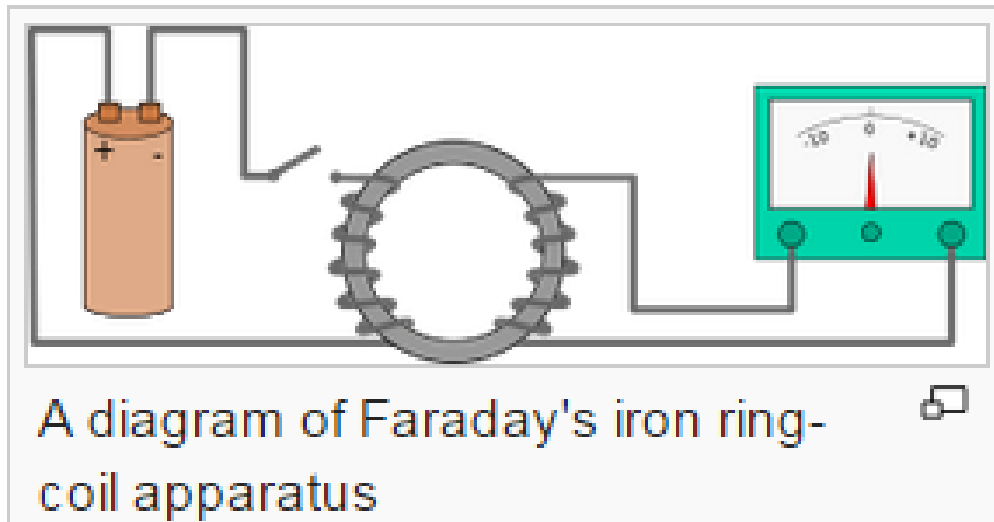
What was known about electromagnetics to Maxwell?

- (3) That an electric current inside a magnetic field produces a force that was at right angles to the direction of the magnetic lines of force and the direction of the current).



# What was known about electromagnetics to Maxwell?

(4) Mutual inductance (Faraday) –also made the first electric motor and generator



What was known about electromagnetics to Maxwell?

Nobody had a good explanation for why these things were happening? Some of it was also a bit spooky!

- Coulomb (1725)

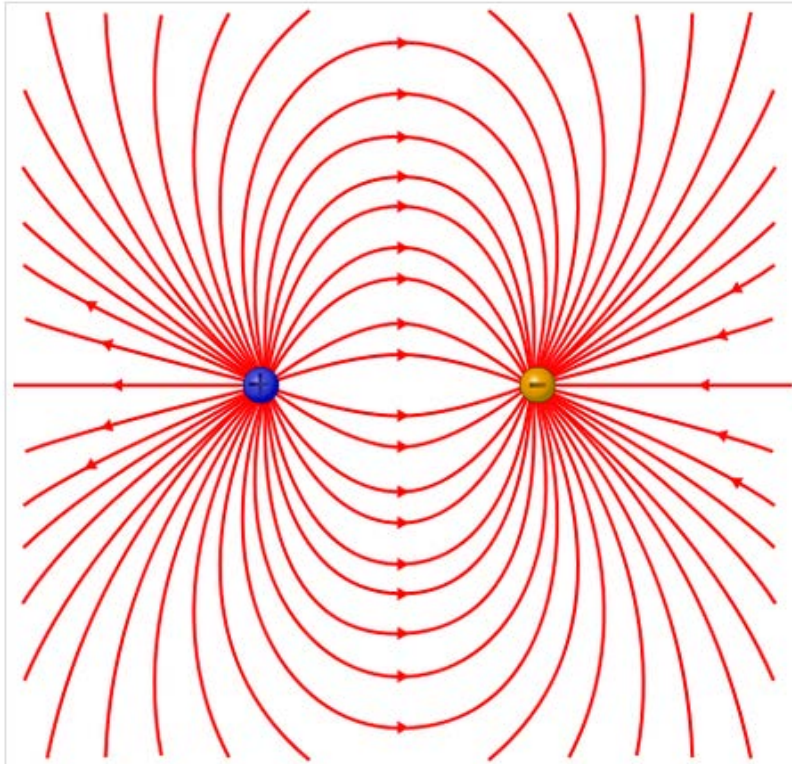
$$F_e = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$



How does  $Q_1$  even know about  $Q_2$ ?

## Faraday had an idea!

Lines of force – first time someone starting thinking about electromagnetism as a field theory.



**Michael Faraday**  
(September 1791 – August 1867)

# Along came Maxwell

He was crazy smart!

- Published his first scientific paper at the age of 14.
- Besides his historic work in electromagnetic theory he also
  - Published the first paper on the theory of feedback control systems
  - Published one of the first papers on the kinetics theory of gases. Led to modern thermodynamic theory.
  - Studied color vision and even printed the first color photograph. We still use the Maxwell color charts to quantify color vision.

James Clerk Maxwell



James Clerk Maxwell (1831–1879)

<b>Born</b>	13 June 1831 <a href="#">Edinburgh, Scotland</a>
<b>Died</b>	5 November 1879 (aged 48) <a href="#">Cambridge, England</a>
<b>Citizenship</b>	British
<b>Nationality</b>	Scottish



James Clerk Maxwell (1831–1879)

Along came Maxwell:

"A Dynamical Theory of the Electromagnetic Field" is the third of James Clerk Maxwell's papers regarding electromagnetism, published in 1865

VIII. *A Dynamical Theory of the Electromagnetic Field.* By J. CLERK MAXWELL, F.R.S.

Received October 27,—Read December 8, 1864.

PART I.—INTRODUCTORY.

(1) THE most obvious mechanical phenomenon in electrical and magnetical experiments is the mutual action by which bodies in certain states set each other in motion while still at a sensible distance from each other. The first step, therefore, in reducing these phenomena into scientific form, is to ascertain the magnitude and direction of the force acting between the bodies, and when it is found that this force depends in a certain way upon the relative position of the bodies and on their electric or magnetic condition, it seems at first sight natural to explain the facts by assuming the existence of something either at rest or in motion in each body, constituting its electric or magnetic state, and capable of acting at a distance according to mathematical laws.

In this way mathematical theories of statical electricity, of magnetism, of the mechanical action between conductors carrying currents, and of the induction of currents have been formed. In these theories the force acting between the two bodies is treated with reference only to the condition of the bodies and their relative position, and without any express consideration of the surrounding medium.

These theories assume, more or less explicitly, the existence of substances the particles of which have the property of acting on one another at a distance by attraction or repulsion. The most complete development of a theory of this kind is that of M. W. WEBER\*, who has made the same theory include electrostatic and electromagnetic phenomena.

In doing so, however, he has found it necessary to assume that the force between two electric particles depends on their relative velocity, as well as on their distance.

This theory, as developed by MM. W. WEBER and C. NEUMANN†, is exceedingly ingenious, and wonderfully comprehensive in its application to the phenomena of statical electricity, electromagnetic attractions, induction of currents and diamagnetic phenomena; and it comes to us with the more authority, as it has served to guide the speculations of one who has made so great an advance in the practical part of electric science, both by introducing a consistent system of units in electrical measurement, and by actually determining electrical quantities with an accuracy hitherto unknown.

\* *Electrodynamische Maassbestimmungen.* Leipzig Trans. vol. i. 1849, and TAYLOR'S Scientific Memoirs, vol. v. art. xiv.

† "Explicare tentatur quomodo fiat ut lucis planum polarizationis per vires electricas vel magneticas declinetur."—Halis Saxonum, 1858.

Along came Maxwell:

**"A Dynamical Theory of the Electromagnetic Field"** is the third of James Clerk Maxwell's papers regarding electromagnetism, published in 1865

In section III of **"A Dynamical Theory of the Electromagnetic Field"**, which is entitled "General Equations of the Electromagnetic Field", Maxwell formulated twenty equations with twenty unknowns which were to become known as **Maxwell's equations**.

In this amazing piece of work Maxwell put forth the foundations of a field theory that could explain all known phenomena regarding electromagnetism. He also added some unknown terms that resulted in the prediction of electromagnetic waves which travel near  $3 \times 10^8$  m/s (speed of light). Also connecting for the first time the fields of optics with electricity and magnetism.

This is how Maxwell's equations looked in his original notation. These are 20 coupled differential equations (Ugh!).

Unfortunately, this work was largely ignored for nearly 20 years. The math was too hard to solve for most.

$e + \frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} = 0$	(1) Gauss' Law
$\begin{aligned} \mu\alpha &= \frac{dH}{dy} - \frac{dG}{dz} \\ \mu\beta &= \frac{dF}{dz} - \frac{dH}{dx} \\ \mu\gamma &= \frac{dG}{dx} - \frac{dF}{dy} \end{aligned}$	(2) Equivalent to Gauss' Law for magnetism
$\begin{aligned} P &= \mu \left( \gamma \frac{dy}{dt} - \beta \frac{dz}{dt} \right) - \frac{dF}{dt} - \frac{d\Psi}{dz} \\ Q &= \mu \left( \alpha \frac{dz}{dt} - \gamma \frac{dx}{dt} \right) - \frac{dG}{dt} - \frac{d\Psi}{dy} \\ R &= \mu \left( \beta \frac{dx}{dt} - \alpha \frac{dy}{dt} \right) - \frac{dH}{dt} - \frac{d\Psi}{dx} \end{aligned}$	(3) Faraday's Law (with the Lorentz Force and Poisson's Law)
$\begin{aligned} \frac{d\gamma}{dy} - \frac{d\beta}{dz} &= 4\pi p' & p' &= p + \frac{df}{dt} \\ \frac{d\alpha}{dz} - \frac{d\gamma}{dx} &= 4\pi q' & q' &= q + \frac{dg}{dt} \\ \frac{d\beta}{dx} - \frac{d\alpha}{dy} &= 4\pi r' & r' &= r + \frac{dh}{dt} \end{aligned}$	(4) Ampère-Maxwell Law
$P = -\xi p \quad Q = -\xi q \quad R = -\xi r$	Ohm's Law
$P = kf \quad Q = kg \quad R = kh$	The electric elasticity equation ( $\mathbf{E} = \mathbf{D}/\epsilon$ )
$\frac{de}{dt} + \frac{dp}{dx} + \frac{dq}{dy} + \frac{dr}{dz} = 0$	Continuity of charge

Maxwell's Equations in his original notation in "A Dynamical Theory of the Electromagnetic Field". The modern and original variables correspond as follows:  $\mathbf{E} \leftrightarrow (P, Q, R)$ ;  $\mathbf{D} \leftrightarrow (f, g, h)$ ;  $\mathbf{H} \leftrightarrow (\alpha, \beta, \gamma)$ ;  $\mathbf{B} \leftrightarrow \mu(\alpha, \beta, \gamma)$ ;  $\mathbf{J} \leftrightarrow (p, q, r)$ ;  $\rho \leftrightarrow e$ ;  $\Psi$  is the electric potential;  $(F, G, H)$  is the magnetic potential. Note that the original set of equations includes Ohm's Law, the Lorentz force, and the continuity equation for charge.

# Enter the Maxwellians

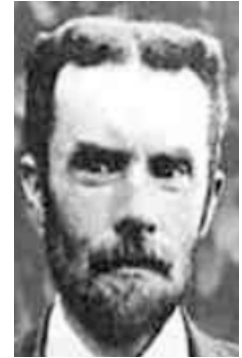
- George Francis FitzGerald
- Oliver Lodge
- Oliver Heaviside
- Heinrich Hertz



These men saw the brilliance of Maxwell's ideas and worked to:

- (1) Validate the existence of EM waves experimentally (Hertz)
- (2) Reformulate Maxwell's 20 equations into a more digestible 4 vector equations (Heaviside and FitzGerald)
- (3) Demonstrated through various solutions to the new Maxwell's equations how they can be used to predict all that is known about electromagnetic phenomenon and some stuff that was yet to be shown.

# Maxwell's Equations in Differential Form (as formulated by Heaviside)



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M} \quad \text{Faraday's Law}$$

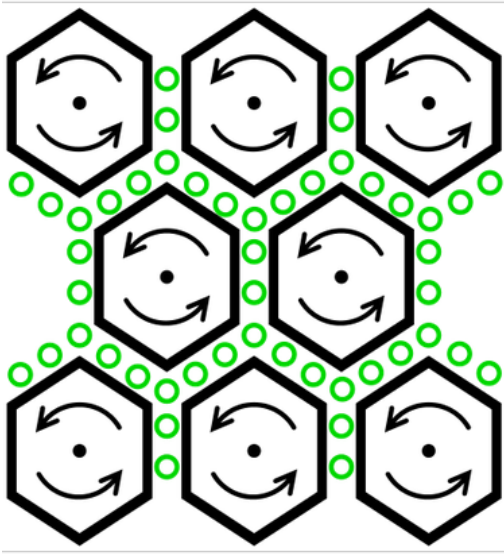
$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_c + \vec{J}_i \quad \text{Ampere's Law}$$

$$\nabla \cdot \vec{D} = \rho \quad \text{Gauss's Law}$$

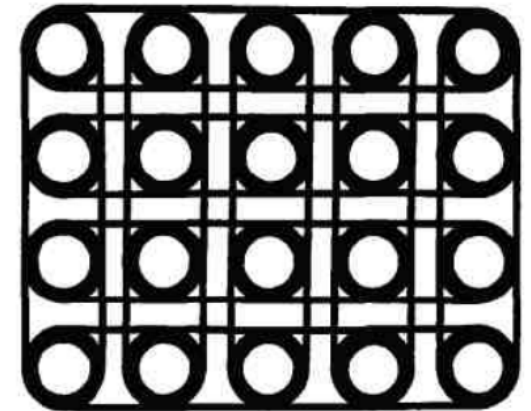
$$\nabla \cdot \vec{B} = \rho_m \quad \text{Gauss's Magnetic Law}$$

# Did Maxwell and his Disciples Have a Good Feeling for What Electric and Magnetic Fields Are?

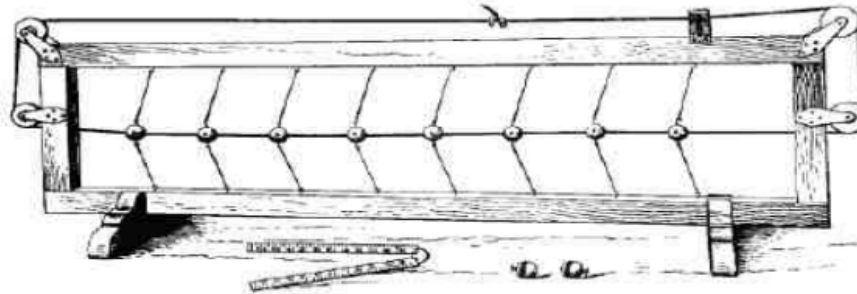
Maxwell's mechanical vortex model



FitzGerald's Wheel and Band Model (1885)



Lodge's string and beads model(1876)



# Vector Analysis Review:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = \rho_m$$

The main mathematics that is needed to understand these equations and solve real problems is vector calculus.

The electric and magnetic fields in Maxwell's equations are vector fields that vary in both time and space.

## The Vector Electromagnetic Fields

$$\vec{E}(x, y, z, t) = E_x(x, y, z, t)\hat{a}_x + E_y(x, y, z, t)\hat{a}_y + E_z(x, y, z, t)\hat{a}_z$$

$$\vec{B}(x, y, z, t) = B_x(x, y, z, t)\hat{a}_x + B_y(x, y, z, t)\hat{a}_y + B_z(x, y, z, t)\hat{a}_z$$

$$\vec{D}(x, y, z, t) = D_x(x, y, z, t)\hat{a}_x + D_y(x, y, z, t)\hat{a}_y + D_z(x, y, z, t)\hat{a}_z$$

$$\vec{H}(x, y, z, t) = H_x(x, y, z, t)\hat{a}_x + H_y(x, y, z, t)\hat{a}_y + H_z(x, y, z, t)\hat{a}_z$$

## The Electromagnetic Sources

$$\vec{J}(x, y, z, t) = J_x(x, y, z, t)\hat{a}_x + J_y(x, y, z, t)\hat{a}_y + J_z(x, y, z, t)\hat{a}_z$$

$$\vec{M}(x, y, z, t) = M_x(x, y, z, t)\hat{a}_x + M_y(x, y, z, t)\hat{a}_y + M_z(x, y, z, t)\hat{a}_z$$

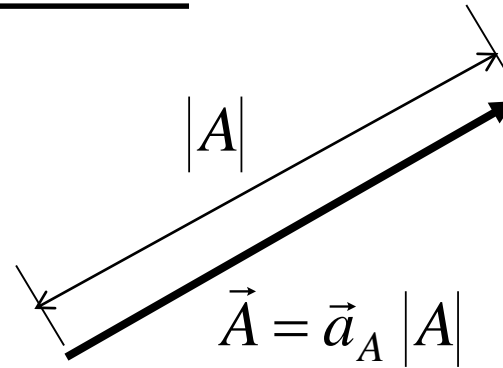
$$\rho(x, y, z, t) = \text{scalar source}$$

$$\rho_m(x, y, z, t) = \text{scalar source}$$

# Vector Analysis Review:

$$\vec{A} = \vec{a}_A |A|$$

$$\vec{a} = \frac{\vec{A}}{|A|}$$



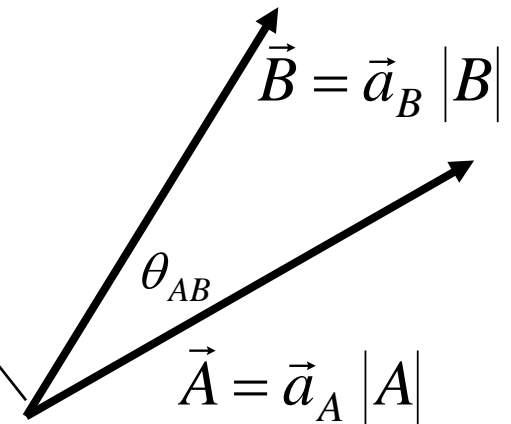
$\vec{a}$  = unit vector

## 1. Dot Product (projection)

$$\vec{A} \cdot \vec{B} = |A||B| \cos(\theta_{AB}) \vec{a}_n$$

## 2. Cross Product

$$\vec{A} \times \vec{B} = \vec{a}_n |A||B| \sin(\theta_{AB})$$



# Orthogonal Coordinate Systems:

$$\vec{A} = \vec{a}_{u_1} A_{u_1} + \vec{a}_{u_2} A_{u_2} + \vec{a}_{u_3} A_{u_3} = |A| \vec{a}_A$$

$$\vec{a}_{u_1} = \vec{a}_{u_2} \times \vec{a}_{u_3}$$

$$\vec{a}_{u_2} = \vec{a}_{u_3} \times \vec{a}_{u_1}$$

$$\vec{a}_{u_3} = \vec{a}_{u_1} \times \vec{a}_{u_2}$$

$$|A| = A_{u_1}^2 + A_{u_2}^2 + A_{u_3}^2$$

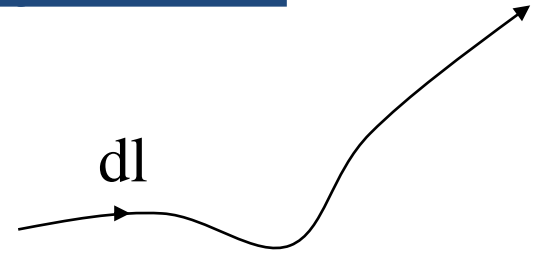
$$\vec{A} \cdot \vec{B} = A_{u_1} B_{u_1} + A_{u_2} B_{u_2} + A_{u_3} B_{u_3}$$

$$\vec{A} \times \vec{B} = \vec{a}_{u_1} (A_{u_2} B_{u_3} - A_{u_3} B_{u_2}) + \vec{a}_{u_2} (A_{u_3} B_{u_1} - A_{u_1} B_{u_3})$$

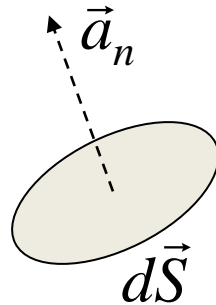
$$+ \vec{a}_{u_3} (A_{u_1} B_{u_2} - A_{u_2} B_{u_1}) \quad \vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_{u_1} & \vec{a}_{u_2} & \vec{a}_{u_3} \\ A_{u_1} & A_{u_2} & A_{u_3} \\ B_{u_1} & B_{u_2} & B_{u_3} \end{vmatrix}$$

# Orthogonal Coordinate Systems:

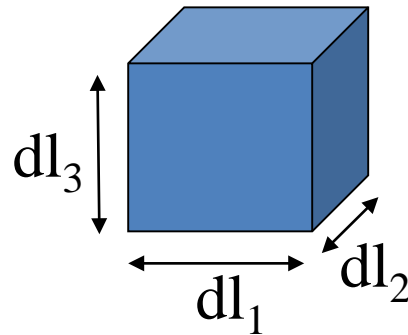
$$d\vec{l} = \vec{a}_{u1} dl_1 + \vec{a}_{u2} dl_2 + \vec{a}_{u3} dl_3$$



$$d\vec{S} = \vec{a}_n dS$$



$$dv = dl_1 dl_2 dl_3$$



# Cartesian Coordinate Systems:

$$\vec{A} = \vec{a}_x A_x + \vec{a}_y A_y + \vec{a}_z A_z = |A| \vec{a}_A$$

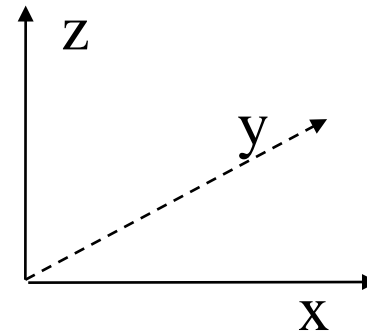
$$\vec{a}_x = \vec{a}_y \times \vec{a}_z$$

$$\vec{a}_y = \vec{a}_x \times \vec{a}_z$$

$$\vec{a}_z = \vec{a}_x \times \vec{a}_y$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



## Cartesian Coordinate Systems (cont):

$$d\vec{l} = \vec{a}_x dx + \vec{a}_y dy + \vec{a}_z dz$$

$$|dl| = dx^2 + dy^2 + dz^2$$

$$ds_x = \vec{a}_x dy dz$$

$$ds_y = \vec{a}_y dx dz$$

$$ds_z = \vec{a}_z dx dy$$

$$dv = dx dy dz$$

# Cylindrical Coordinate Systems:

$$\vec{A} = \vec{a}_r A_r + \vec{a}_\phi A_\phi + \vec{a}_z A_z = |A| \vec{a}_A$$

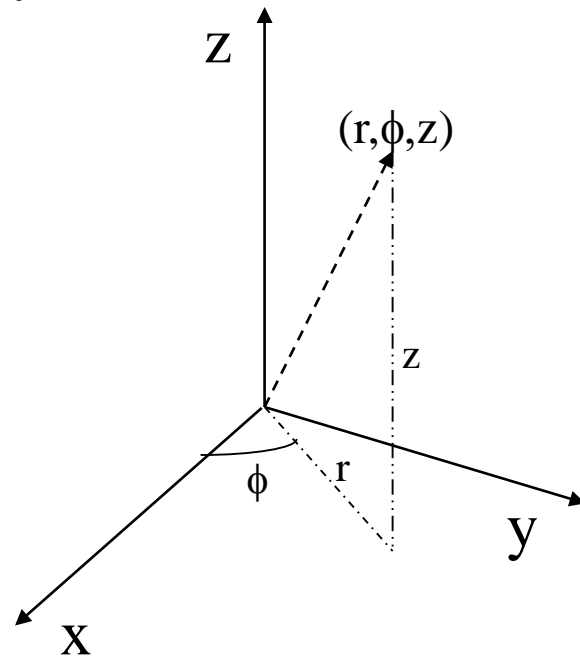
$$d\vec{l} = \vec{a}_r dr + \vec{a}_\phi r d\phi + \vec{a}_z dz$$

$$ds_r = \vec{a}_r r d\phi dz$$

$$ds_\phi = \vec{a}_\phi dr dz$$

$$ds_z = \vec{a}_z r d\phi dz$$

$$dv = r dr d\phi dz$$



# Spherical Coordinate Systems:

$$\vec{A} = \vec{a}_R A_R + \vec{a}_\theta A_\theta + \vec{a}_\phi A_\phi = |A| \vec{a}_A$$

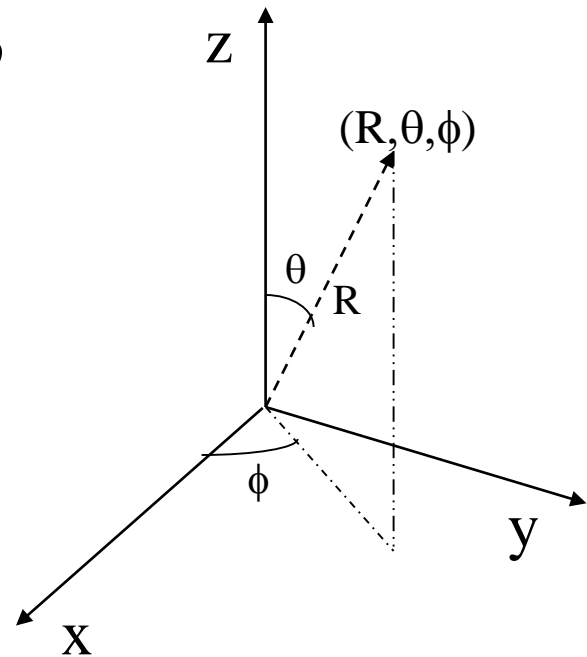
$$d\vec{l} = \vec{a}_R dR + \vec{a}_\theta R d\theta + \vec{a}_\phi R \sin(\theta) d\phi$$

$$ds_R = \vec{a}_R R^2 \sin(\theta) d\theta d\phi$$

$$ds_\theta = \vec{a}_\theta R \sin(\theta) dR d\phi$$

$$ds_\phi = \vec{a}_\phi R dR d\theta$$

$$dv = R^2 \sin(\theta) dR d\theta d\phi$$



## Vector Coordinate Transformation:

$$\begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{vmatrix} A_r \\ A_\phi \\ A_z \end{vmatrix}$$

$$\begin{vmatrix} A_x \\ A_y \\ A_z \end{vmatrix} = \begin{bmatrix} \sin(\theta) \cos(\phi) & \cos(\theta) \cos(\phi) & -\sin(\phi) \\ \sin(\theta) \sin(\phi) & \cos(\theta) \sin(\phi) & \cos(\phi) \\ \cos(\theta) & -\sin(\theta) & 0 \end{bmatrix} \begin{vmatrix} A_R \\ A_\theta \\ A_\phi \end{vmatrix}$$

## Gradient of a Scalar Field:

Assume  $f(x,y,z)$  is a scalar field

The maximum spatial rate of change of  $f$  at some location is a vector given by the gradient of  $f$  denoted by

Grad( $f$ ) or  $\nabla f$

$$\nabla f = \vec{a}_x \frac{\partial f}{\partial x} + \vec{a}_y \frac{\partial f}{\partial y} + \vec{a}_z \frac{\partial f}{\partial z}$$

$$\nabla f = \vec{a}_r \frac{\partial f}{\partial r} + \vec{a}_\phi \frac{\partial f}{r \partial \phi} + \vec{a}_z \frac{\partial f}{\partial z}$$

$$\nabla f = \vec{a}_R \frac{\partial f}{\partial R} + \vec{a}_\theta \frac{\partial f}{r \partial \theta} + \vec{a}_\phi \frac{\partial f}{R \sin(\theta) \partial \phi}$$

## Divergence of a Vector Field:

Assume  $\vec{E}(x,y,z)$  is a vector field. The divergence of  $E$  is defined as the net outward flux of  $E$  in some volume as the volume goes to zero. It is denoted by  $\nabla \cdot \vec{E}$

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\nabla \cdot \vec{E} = \frac{1}{r} \frac{\partial}{\partial r} (rE_r) + \frac{1}{r} \frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z}$$

$$\nabla \cdot \vec{E} = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 E_R) + \frac{1}{R \sin(\theta)} \frac{\partial}{\partial \theta} (\sin(\theta) E_\theta) + \frac{1}{R \sin(\theta)} \frac{\partial E_\phi}{\partial \phi}$$

# Curl of a Vector Field:

Assume  $\vec{E}(x,y,z)$  is a vector field. The curl of E is measure of the circulation of E also called a “vortex” source. It is denoted by  $\nabla \times \vec{E}$

$$\nabla \times \vec{E} = \begin{bmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{bmatrix}$$

$$\nabla \times \vec{E} = \frac{1}{r} \begin{bmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ E_r & rE_\phi & E_z \end{bmatrix}$$

$$\nabla \times \vec{E} = \frac{1}{R^2 \sin(\theta)} \begin{bmatrix} \vec{a}_R & R\vec{a}_\theta & R\sin(\theta)\vec{a}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ E_R & RE_\theta & R\sin(\theta)E_\phi \end{bmatrix}$$

## Laplacian of a Scalar Field:

Assume  $f(x,y,z)$  is a scalar field. The Laplacian is defined as  $\nabla \cdot (\nabla V)$  and denoted by  $\nabla^2 V$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} V \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{R^2} \frac{\partial}{\partial R} \left( R^2 \frac{\partial}{\partial R} V \right) + \frac{1}{R \sin(\theta)} \frac{\partial}{\partial \theta} \left( \sin(\theta) \frac{\partial}{\partial \theta} V \right) + \frac{1}{R^2 \sin^2(\theta)} \frac{\partial^2 V}{\partial \phi^2}$$

# Basic Theorems:

## 1. Divergence Theorem or Gauss's Law

$$\iiint_V \nabla \cdot \vec{E} \, dv = \oiint_S \vec{E} \cdot d\vec{s}$$

## 2. Stokes Theorem

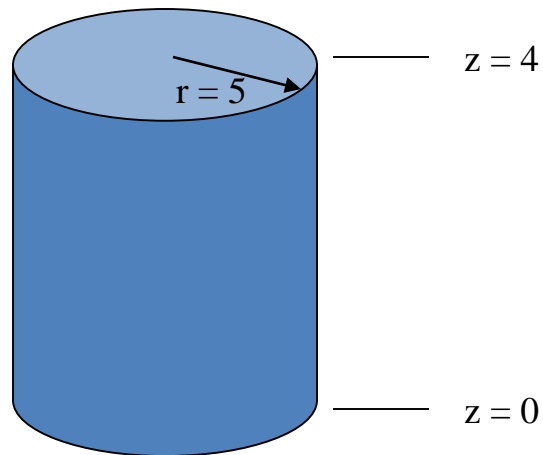
$$\iint_S (\nabla \times \vec{E}) \cdot d\vec{s} = \oint_C \vec{E} \cdot d\vec{l}$$

# Examples:

1. Verify the Divergence Theorem for

$$\vec{E}(r, z) = \vec{a}_r r^2 + \vec{a}_z 2z$$

on a cylindrical region enclosed by  $r=5$ ,  $z=0$  and  $z=4$

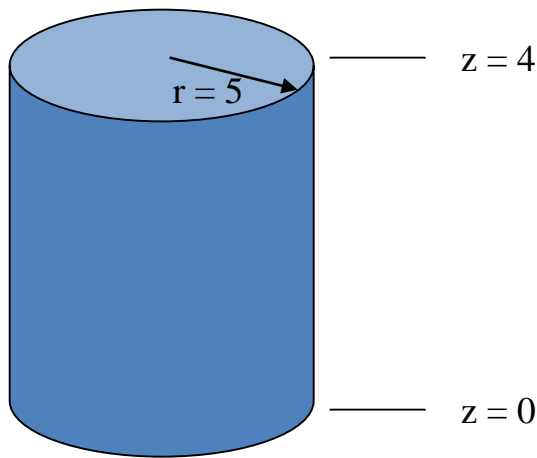


# Examples:

1. Verify the Divergence Theorem for

$$\vec{E}(r, z) = \vec{a}_r r^2 + \vec{a}_z 2z$$

on a cylindrical region enclosed by  $r=5$ ,  $z=0$  and  $z=4$



$$\iiint_V \nabla \cdot \vec{E} \, dv = \oiint_S \vec{E} \cdot d\vec{s}$$

$$\nabla \cdot \vec{E} = \frac{1}{r} \frac{\partial}{\partial r} (rE_r) + \frac{1}{r} \frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z}$$

$$\nabla \cdot \vec{E} = \frac{1}{r} \frac{\partial}{\partial r} (r \cdot r^2) + \frac{1}{r} \frac{\partial 0}{\partial \phi} + \frac{\partial (2z)}{\partial z}$$

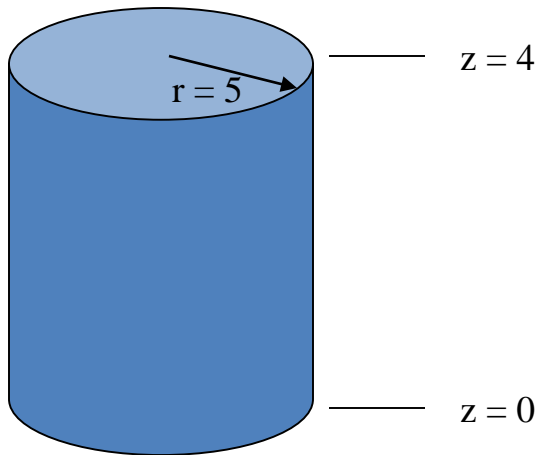
$$\nabla \cdot \vec{E} = \frac{1}{r} (3r^2) + 2 = 3r + 2$$

# Examples:

1. Verify the Divergence Theorem for

$$\vec{E}(r, z) = \vec{a}_r r^2 + \vec{a}_z 2z$$

on a cylindrical region enclosed by  $r=5$ ,  $z=0$  and  $z=4$



$$\iiint_V \nabla \cdot \vec{E} \, dv = \oiint_S \vec{E} \cdot d\vec{s}$$

$$\nabla \cdot \vec{E} = \frac{1}{r} (3r^2) + 2 = 3r + 2$$

$$\begin{aligned} \iiint_V \nabla \cdot \vec{E} \, dv &= \int_{z=0}^{z=4} \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=5} (3r + 2) \cdot r \, dr \, d\theta \, dz \\ &= \int_{z=0}^{z=4} \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=5} (3r^2 + 2r) \cdot dr \, d\theta \, dz \end{aligned}$$

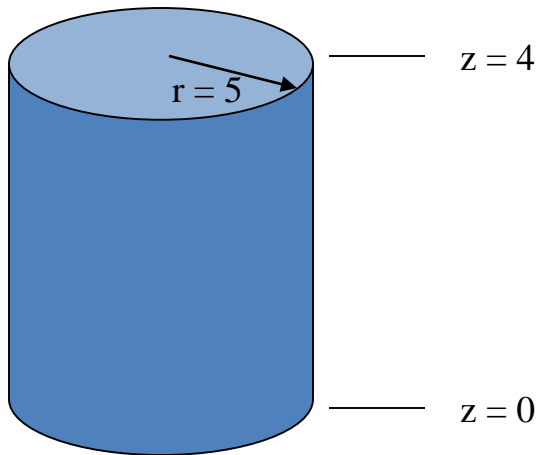
$$\begin{aligned} \iiint_V \nabla \cdot \vec{E} \, dv &= \int_{z=0}^{z=4} \int_{\theta=0}^{\theta=2\pi} (r^3 + r^2) \Big|_0^5 \, d\theta \, dz \\ &= 150 \cdot \int_{z=0}^{z=4} \int_{\theta=0}^{\theta=2\pi} d\theta \, dz \end{aligned}$$

# Examples:

1. Verify the Divergence Theorem for

$$\vec{E}(r, z) = \vec{a}_r r^2 + \vec{a}_z 2z$$

on a cylindrical region enclosed by  $r=5$ ,  $z=0$  and  $z=4$



$$\iiint_V \nabla \cdot \vec{E} \, dv = \oiint_S \vec{E} \cdot d\vec{s}$$

$$\nabla \cdot \vec{E} = \frac{1}{r}(3r^2) + 2 = 3r + 2$$

$$\iiint_V \nabla \cdot \vec{E} \, dv = 150 \cdot \int_{z=0}^{z=4} \int_{\theta=0}^{\theta=2\pi} d\theta dz$$
$$\iiint_V \nabla \cdot \vec{E} \, dv = 150 \cdot 2\pi \cdot 4 = 1200\pi$$

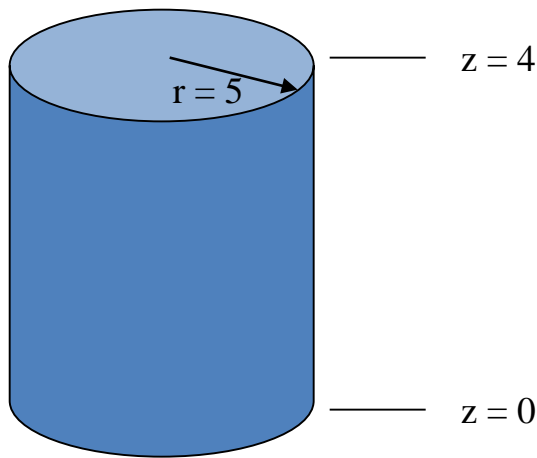
# Examples:

1. Verify the Divergence Theorem for

$$\vec{E}(r, z) = \vec{a}_r r^2 + \vec{a}_z 2z$$

on a cylindrical region enclosed by  $r=5$ ,  $z=0$  and  $z=4$

$$\iiint_V \nabla \cdot \vec{E} \, dv = \oiint_S \vec{E} \cdot d\vec{s}$$



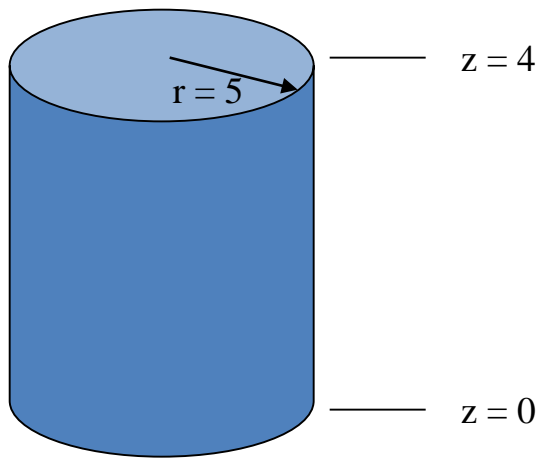
$$\begin{aligned} \oiint_S \vec{E} \cdot d\vec{s} &= \int_{z=0}^{z=4} \int_{\theta=0}^{\theta=2\pi} (\vec{a}_r 5^2 + \vec{a}_z 2z) \cdot \vec{a}_r 5 d\theta dz \\ &+ \int_{r=0}^{r=5} \int_{\theta=0}^{\theta=2\pi} (\vec{a}_r r^2 + \vec{a}_z 2 \cdot 4) \cdot \vec{a}_z r d\theta dr \\ &+ \int_{r=0}^{r=5} \int_{\theta=0}^{\theta=2\pi} (\vec{a}_r r^2 + \vec{a}_z 2 \cdot 0) \cdot \vec{a}_z r d\theta dr \end{aligned}$$

# Examples:

1. Verify the Divergence Theorem for

$$\vec{E}(r, z) = \vec{a}_r r^2 + \vec{a}_z 2z$$

on a cylindrical region enclosed by  $r=5$ ,  $z=0$  and  $z=4$



$$\iiint_V \nabla \cdot \vec{E} \, dv = \oiint_S \vec{E} \cdot d\vec{s}$$

$$\begin{aligned} \oiint_S \vec{E} \cdot d\vec{s} &= \int_{z=0}^{z=4} \int_{\theta=0}^{\theta=2\pi} (\vec{a}_r 5^2 + \vec{a}_z 2z) \cdot \vec{a}_r 5 d\theta dz \\ &+ \int_{r=0}^{r=5} \int_{\theta=0}^{\theta=2\pi} (\vec{a}_r r^2 + \vec{a}_z 2 \cdot 4) \cdot \vec{a}_z r d\theta dr \\ &+ \int_{r=0}^{r=5} \int_{\theta=0}^{\theta=2\pi} (\vec{a}_r r^2 + \vec{a}_z 2 \cdot 0) \cdot \vec{a}_z r d\theta dr \end{aligned}$$

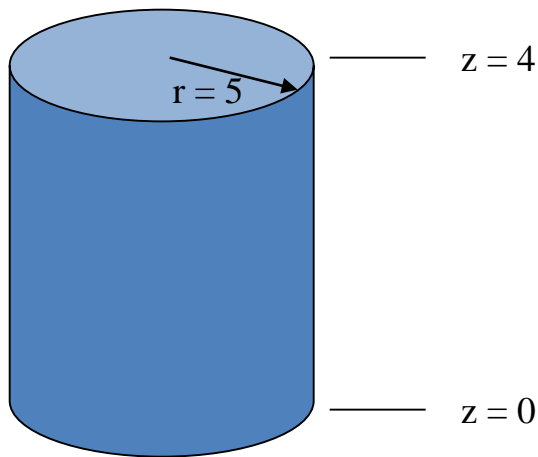
$$\begin{aligned} \oiint_S \vec{E} \cdot d\vec{s} &= 5^3 \cdot 2\pi \cdot 4 + 8 \cdot 2\pi \cdot \frac{1}{2} r^2 \Big|_0^5 \\ &= 5^3 \cdot 2\pi \cdot 4 + 8 \cdot 2\pi \cdot \frac{1}{2} 5^2 = 1200\pi \end{aligned}$$

# Examples:

1. Verify the Divergence Theorem for

$$\vec{E}(r, z) = \vec{a}_r r^2 + \vec{a}_z 2z$$

on a cylindrical region enclosed by  $r=5$ ,  $z=0$  and  $z=4$



$$\iiint_V \nabla \cdot \vec{E} \, dv = \oiint_S \vec{E} \cdot d\vec{s}$$

$$\oiint_S \vec{E} \cdot d\vec{s} = 1200\pi$$

$$\iiint_V \nabla \cdot \vec{E} \, dv = 1200\pi$$

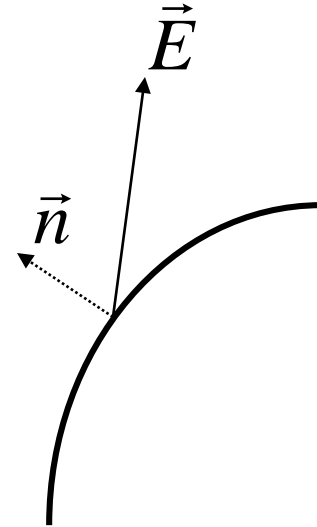
# Odds and Ends:

1. Normal component of field

$$\vec{n} \cdot \vec{E} = E_n$$

2. Tangential component of field

$$\vec{n} \times \vec{E} = E_t$$



# Maxwell's Equations in Differential Form

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{M} \quad \text{Faraday's Law}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_c + \vec{J}_i \quad \text{Ampere's Law}$$

$$\nabla \cdot \vec{D} = \rho \quad \text{Gauss's Law}$$

$$\nabla \cdot \vec{B} = \rho_m \quad \text{Gauss's Magnetic Law}$$

# Maxwell's Equations Field Variables

$\vec{E}$  = Electric Field, V/m

$\vec{D}$  = Electric Displacement, Q/m<sup>2</sup>

$\vec{H}$  = Magnetic Field, A/m

$\vec{B}$  = Magnetic Flux Density, T

# Maxwell's Equations Source Variables

$\vec{J}_c =$  **Conductive Current Density, A/m<sup>2</sup>**

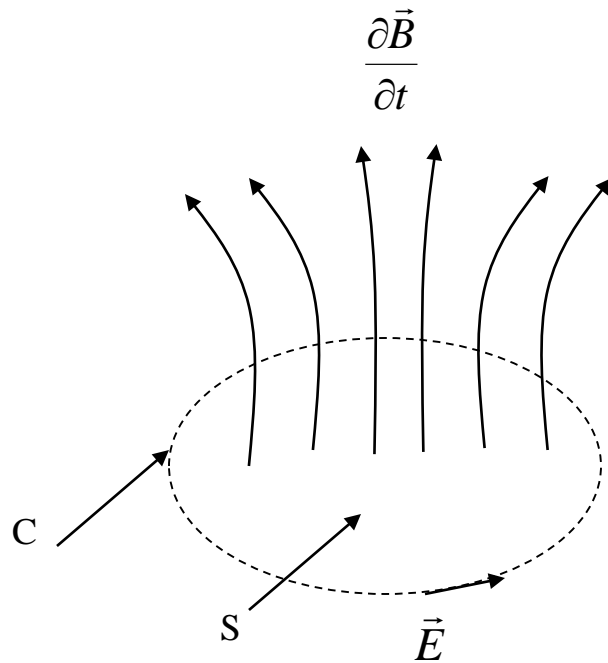
$\vec{J}_i =$  **Impressed Current Density, A/m<sup>2</sup>**

$\vec{M} =$  **Magnetic Current Density, V/m<sup>2</sup>**

$\rho =$  **Electric Charge Density, Q/m<sup>3</sup>**

$\rho_m =$  **Magnetic Charge Density Wb/m<sup>3</sup>**

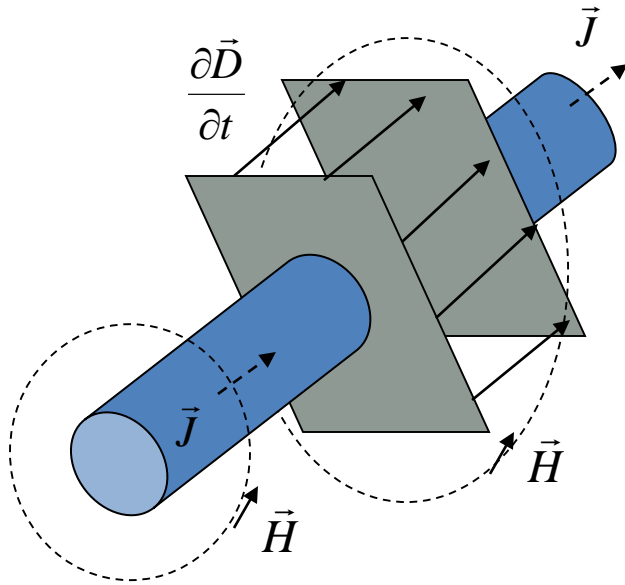
# Faraday's Law



$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \iint_S \vec{B} \cdot d\vec{s}$$

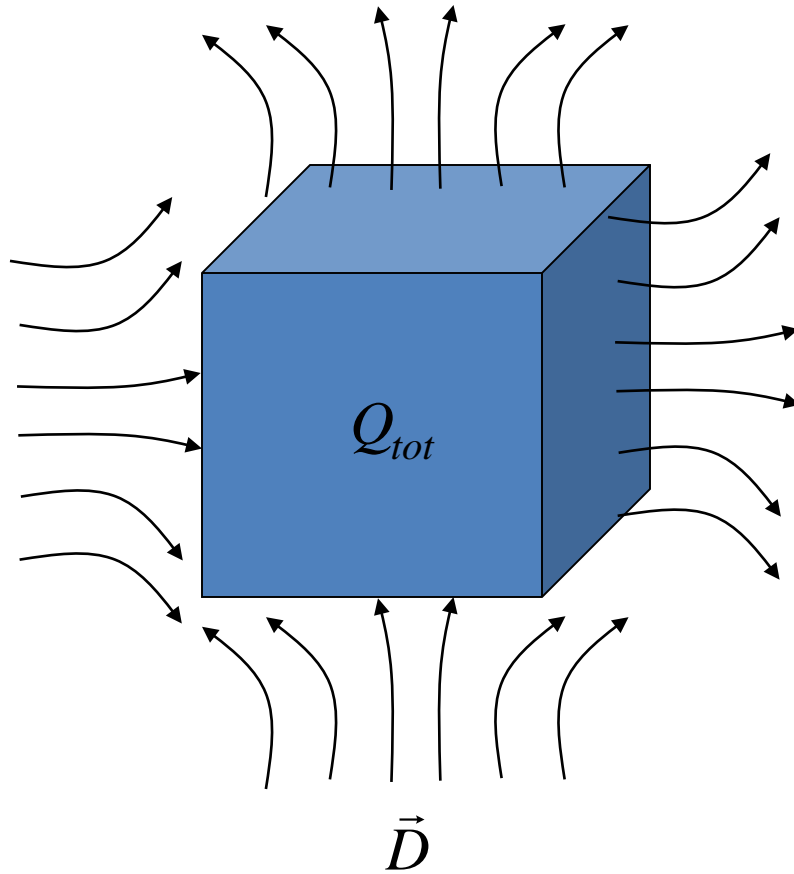
# Ampere's Law



$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\oint_c \vec{H} \cdot d\vec{l} = \frac{\partial}{\partial t} \iint_s \vec{D} \cdot d\vec{s} + \iint_s \vec{J} \cdot d\vec{s}$$

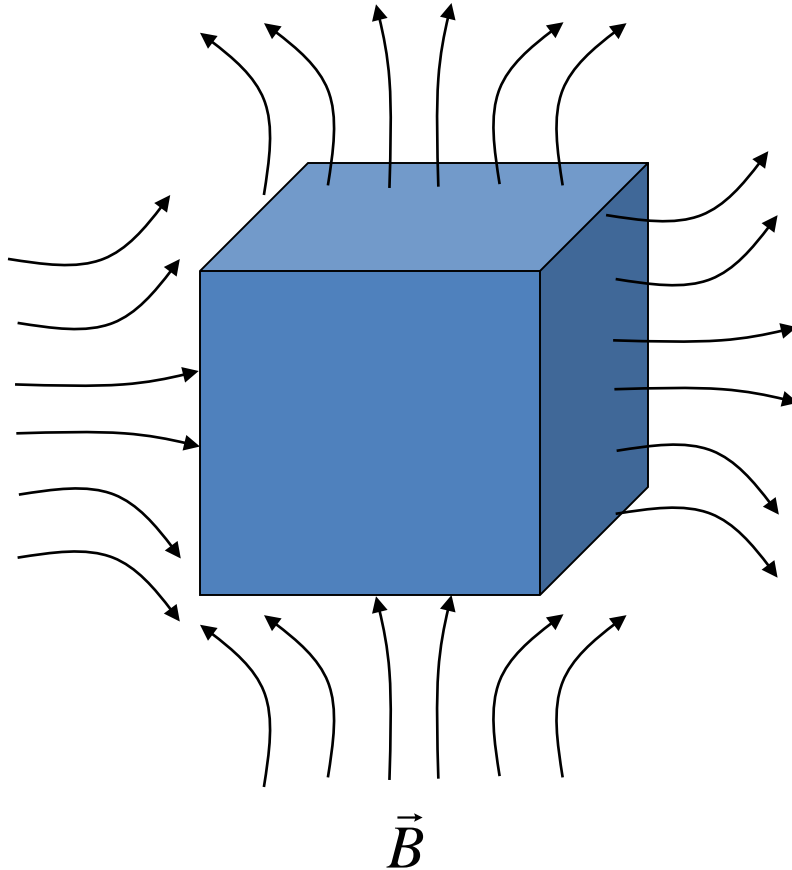
# Gauss's Law



$$\nabla \cdot \vec{D} = \rho$$

$$\iint_S \vec{D} \cdot d\vec{s} = \iiint_V \rho dv = Q_{tot}$$

# Gauss's Magnetic Law



$$\nabla \cdot \vec{B} = 0$$

$$\iint_s \vec{B} \cdot d\vec{s} = 0$$

“all the flow of B entering the volume V must leave the volume”

# Maxwell's Equations in Differential Form: No Magnetic Sources

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

**Faraday's Law**

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_c + \vec{J}_i$$

**Ampere's Law**

$$\nabla \cdot \vec{D} = \rho$$

**Gauss's Law**

$$\nabla \cdot \vec{B} = 0$$

**Gauss's Magnetic Law**

# CONSTITUTIVE RELATIONS

$$\vec{D} = \epsilon \vec{E}$$

$\epsilon = \epsilon_r \epsilon_0 = \text{permittivity (F/m)}$   
 $\epsilon_0 = 8.854 \times 10^{-12} \text{ (F/m)}$

# CONSTITUTIVE RELATIONS

$$\vec{B} = \mu \vec{H}$$

$$\mu = \mu_0 \mu_r$$

$\mu_0$  = permeability of  
free space (H/m)

$$\mu_0 = 4\pi \times 10^{-7} \text{ (H/m)}$$

# CONSTITUTIVE RELATIONS

$$\vec{J}_c = \sigma \vec{E} \quad \sigma = \text{conductivity (S/m)}$$

# Maxwell's Equations in Differential Form: No Magnetic Sources

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

**Faraday's Law**

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} + \vec{J}_i$$

**Ampere's Law**

$$\nabla \cdot \vec{D} = \rho$$

**Gauss's Law**

$$\nabla \cdot \vec{B} = 0$$

**Gauss's Magnetic Law**

# Maxwell's Equations in Differential Form: No Magnetic Sources and No Loss

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{Faraday's Law}$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \vec{J}_i \quad \text{Ampere's Law}$$

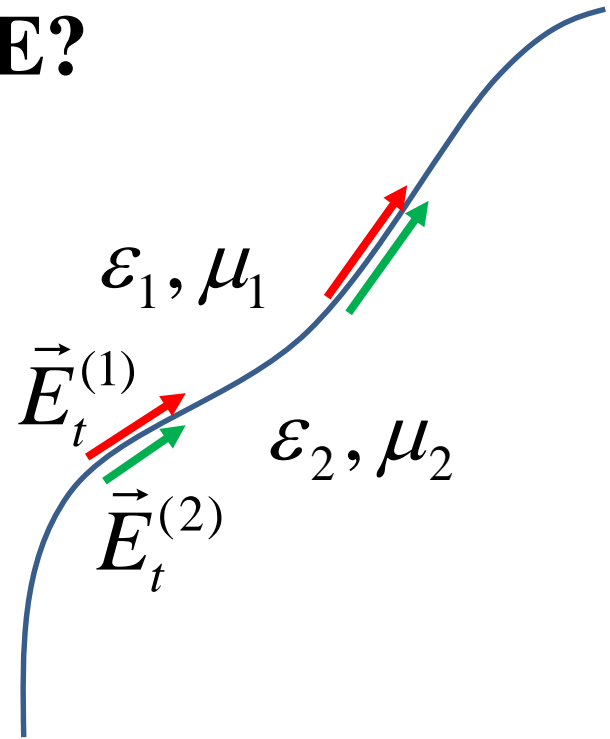
 **Maxwell added this term**

$$\nabla \cdot \vec{D} = \rho \quad \text{Gauss's Law}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{Gauss's Magnetic Law}$$

# Boundary Conditions

## (1) Tangential Component of E?



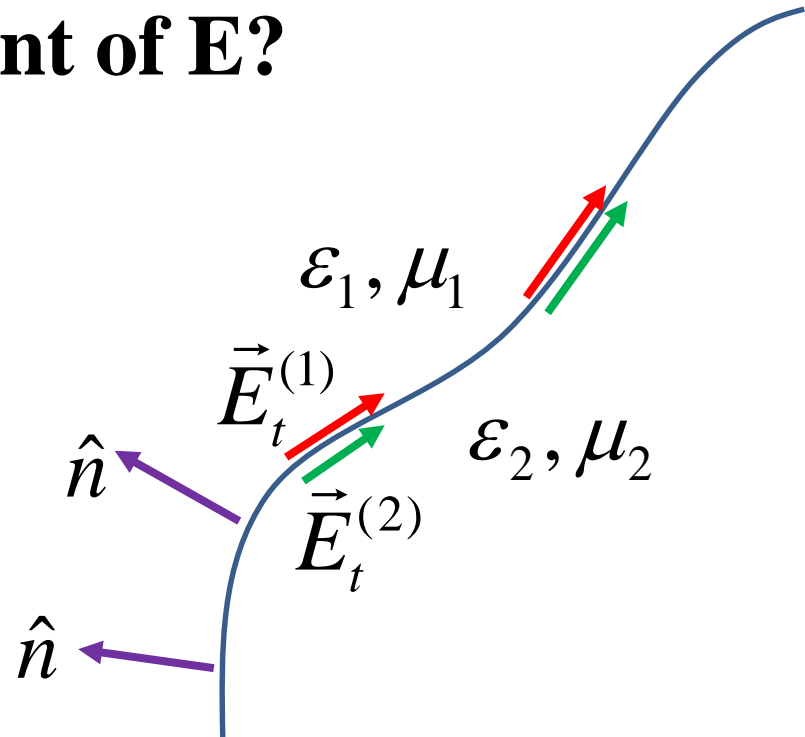
# Boundary Conditions

## (1) Tangential Component of E?

$$\hat{n} \times \vec{E}^{(2)} = \hat{n} \times \vec{E}^{(1)}$$

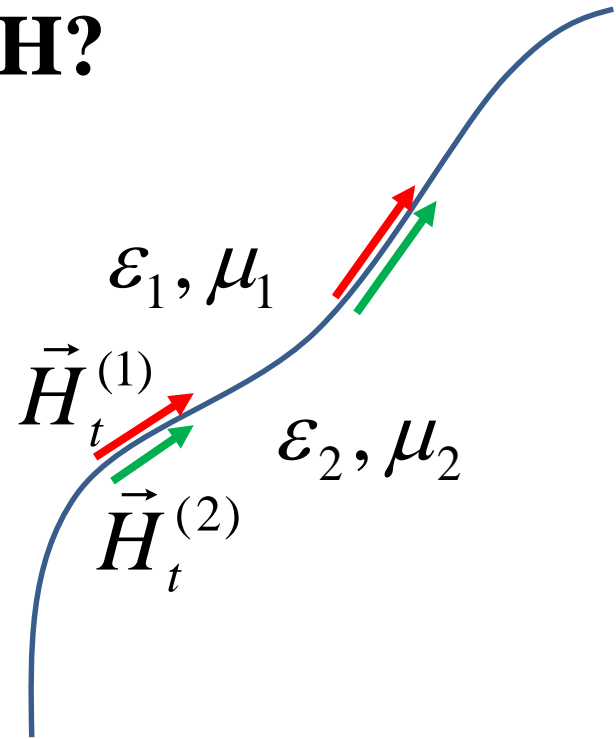
or

$$\hat{n} \times (\vec{E}^{(2)} - \vec{E}^{(1)}) = 0$$



# Boundary Conditions

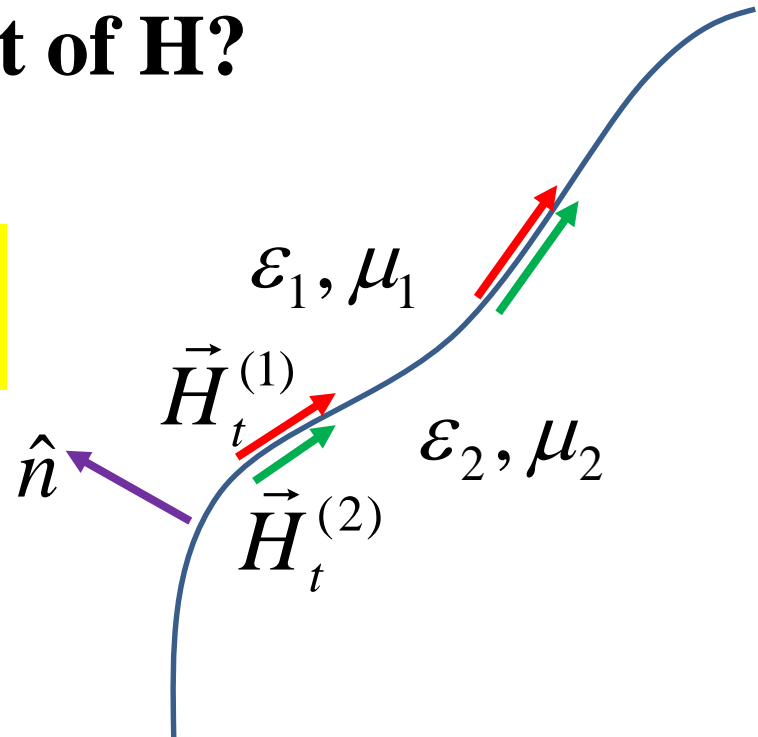
## (2) Tangential Component of H?



# Boundary Conditions

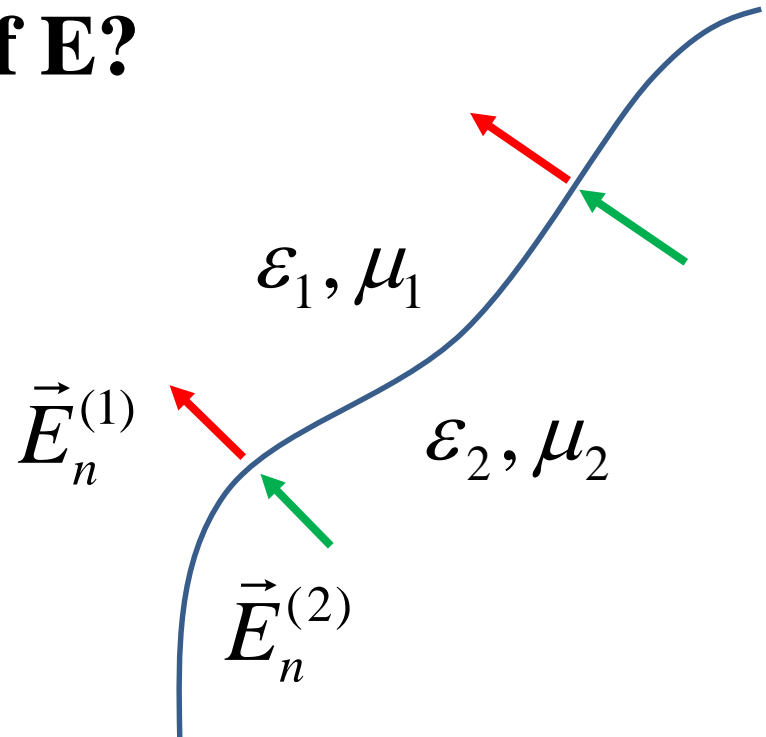
## (2) Tangential Component of H?

$$\hat{n} \times (\vec{H}^{(2)} - \vec{H}^{(1)}) = \vec{J}_s$$



# Boundary Conditions

## (3) Normal Component of $\mathbf{E}$ ?



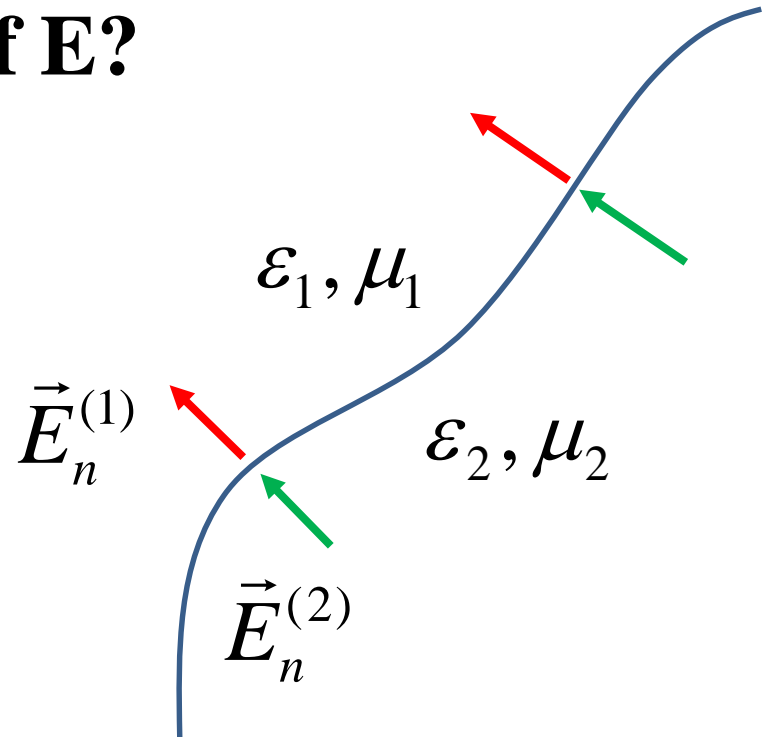
# Boundary Conditions

## (3) Normal Component of $\mathbf{E}$ ?

$$\hat{n} \cdot (\vec{D}^{(2)} - \vec{D}^{(1)}) = \rho_s$$

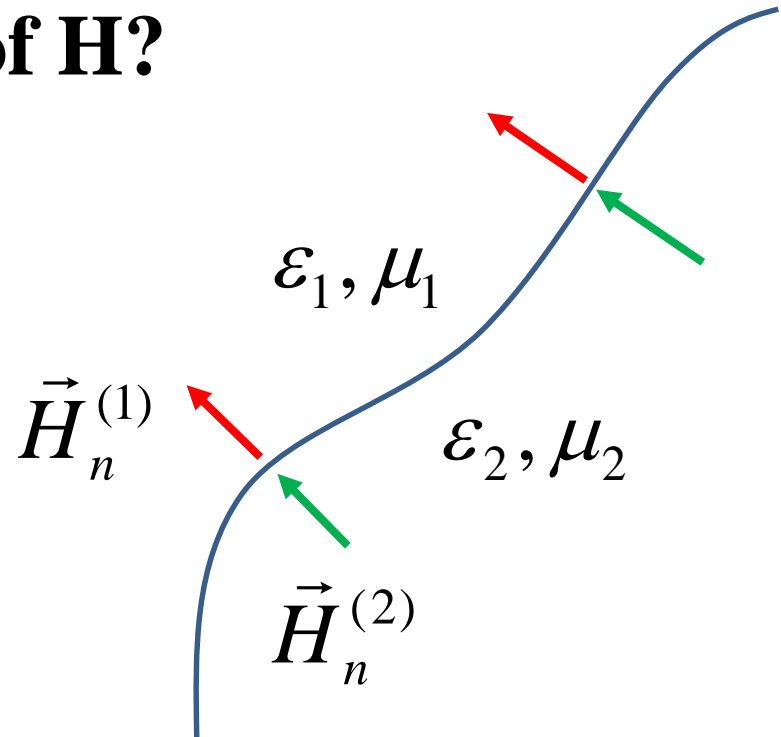
or

$$\hat{n} \cdot (\epsilon_2 \vec{E}^{(2)} - \epsilon_1 \vec{E}^{(1)}) = \rho_s$$



# Boundary Conditions

## (4) Normal Component of $\vec{H}$ ?



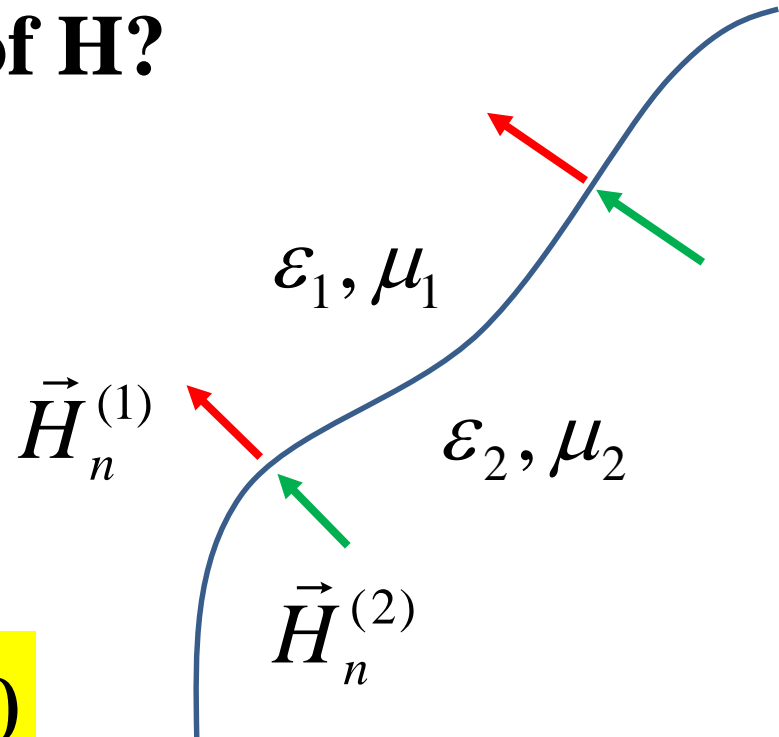
# Boundary Conditions

## (4) Normal Component of H?

$$\hat{n} \cdot (\vec{B}^{(2)} - \vec{B}^{(1)}) = 0$$

or

$$\hat{n} \cdot (\mu_2 \vec{H}^{(2)} - \mu_1 \vec{H}^{(1)}) = 0$$



# Boundary Conditions (ALWAYS TRUE)

$$\hat{n} \times (\vec{E}^{(2)} - \vec{E}^{(1)}) = 0$$

$$\hat{n} \times (\vec{H}^{(2)} - \vec{H}^{(1)}) = \vec{J}_s$$

$$\hat{n} \cdot (\vec{B}^{(2)} - \vec{B}^{(1)}) = 0$$

$$\hat{n} \cdot (\vec{D}^{(2)} - \vec{D}^{(1)}) = \rho_s$$

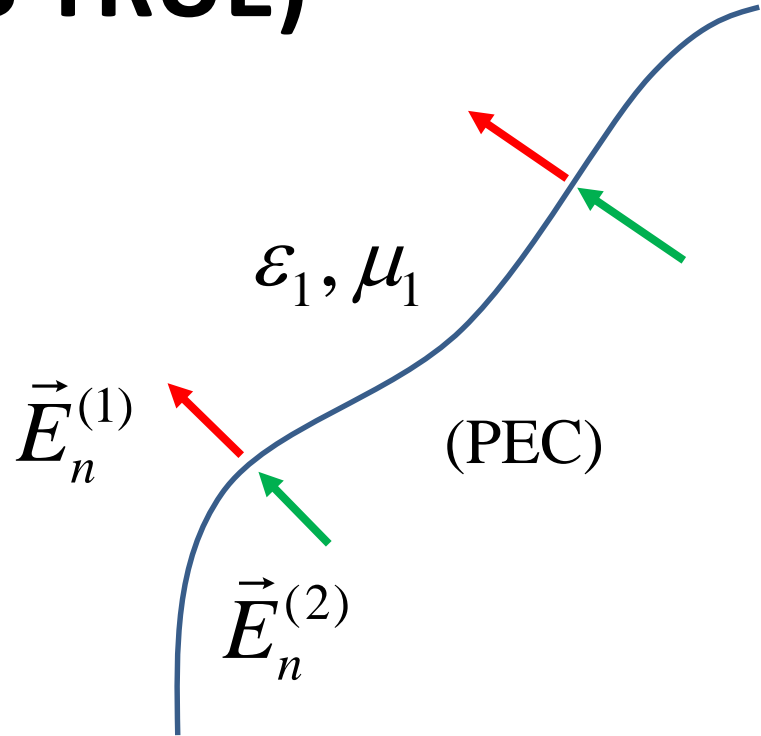
# Boundary Conditions (ALWAYS TRUE)

$$\hat{n} \times (\vec{E}^{(2)} - \vec{E}^{(1)}) = 0$$

$$\hat{n} \times (\vec{H}^{(2)} - \vec{H}^{(1)}) = \vec{J}_s$$

$$\hat{n} \cdot (\vec{B}^{(2)} - \vec{B}^{(1)}) = 0$$

$$\hat{n} \cdot (\vec{D}^{(2)} - \vec{D}^{(1)}) = \rho_s$$



**How do these simplify if one of the materials is a perfect electrical conductor (PEC)?**

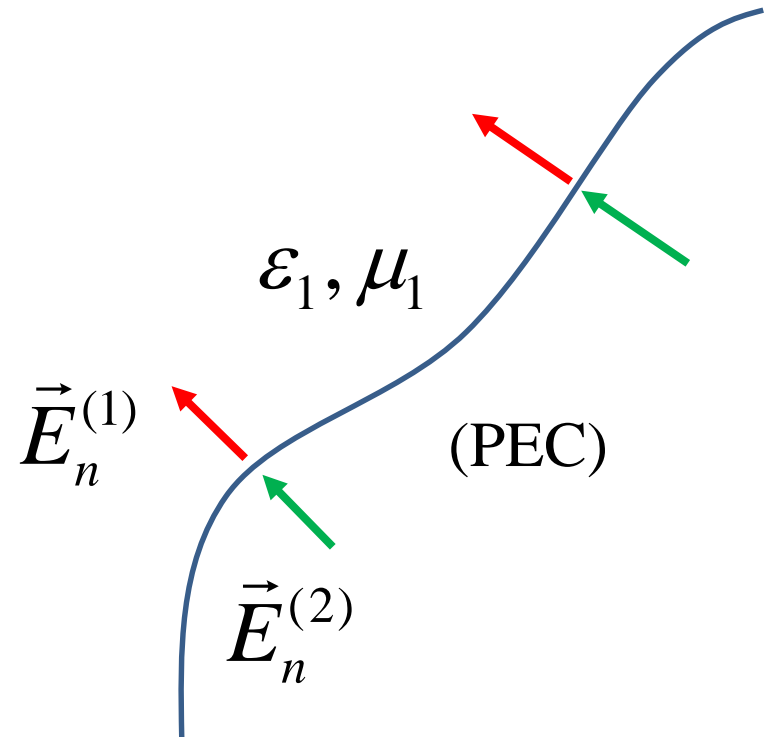
# Boundary Conditions (PEC)

$$\hat{n} \times (\vec{E}^{(2)} - \vec{E}^{(1)}) = 0$$

$$\hat{n} \times (\vec{H}^{(2)} - \vec{H}^{(1)}) = \vec{J}_s$$

$$\hat{n} \cdot (\vec{B}^{(2)} - \vec{B}^{(1)}) = 0$$

$$\hat{n} \cdot (\vec{D}^{(2)} - \vec{D}^{(1)}) = \rho_s$$



**In a PEC all the fields must be zero!!!**

**Why?**

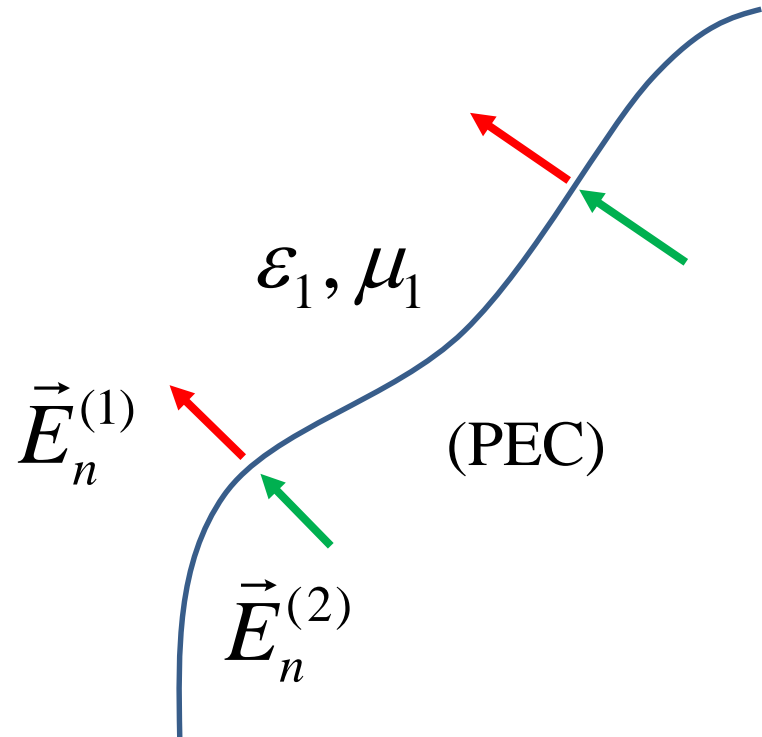
# Boundary Conditions (PEC)

$$\hat{n} \times \vec{E}^{(2)} = 0$$

$$\hat{n} \times \vec{H}^{(2)} = \vec{J}_s$$

$$\hat{n} \cdot \vec{B}^{(2)} = 0$$

$$\hat{n} \cdot \vec{D}^{(2)} = \rho_s$$



**In a PEC all the fields must be zero!!!**

**Why?**

# Boundary Conditions

## NOT A PEC

$$\hat{n} \times (\vec{E}^{(2)} - \vec{E}^{(1)}) = 0$$

$$\hat{n} \times (\vec{H}^{(2)} - \vec{H}^{(1)}) = 0$$

$$\hat{n} \cdot (\vec{B}^{(2)} - \vec{B}^{(1)}) = 0$$

$$\hat{n} \cdot (\vec{D}^{(2)} - \vec{D}^{(1)}) = 0$$

## PEC

$$\hat{n} \times \vec{E}^{(2)} = 0$$

$$\hat{n} \times \vec{H}^{(2)} = \vec{J}_s$$

$$\hat{n} \cdot \vec{B}^{(2)} = 0$$

$$\hat{n} \cdot \vec{D}^{(2)} = \rho_s$$

# THIS IS EVERYTHING

## Constitutive Relations

$$\vec{J}_c = \sigma \vec{E}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

## Boundary Conditions

### NOT A PEC

$$\hat{n} \times (\vec{E}^{(2)} - \vec{E}^{(1)}) = 0$$

$$\hat{n} \times (\vec{H}^{(2)} - \vec{H}^{(1)}) = 0$$

$$\hat{n} \cdot (\vec{B}^{(2)} - \vec{B}^{(1)}) = 0$$

$$\hat{n} \cdot (\vec{D}^{(2)} - \vec{D}^{(1)}) = 0$$

### PEC

$$\hat{n} \times \vec{E}^{(2)} = 0$$

$$\hat{n} \times \vec{H}^{(2)} = \vec{J}_s$$

$$\hat{n} \cdot \vec{B}^{(2)} = 0$$

$$\hat{n} \cdot \vec{D}^{(2)} = \rho_s$$

## Maxwell's equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_c + \vec{J}_i$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

# Example Problem

$$\epsilon_1, \mu_0$$

$$\epsilon_2, \mu_0$$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x \\ + R \cos(1000t + 2\pi z)\vec{a}_x$$

$$\vec{E}_2 = T \cos(1000t - 2\pi z)\vec{a}_x$$

FIND R and T on both sides!

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_c + \vec{J}_i$$

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

z=0

# Example Problem

$$\epsilon_1, \mu_0$$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x \\ + R \cos(1000t + 2\pi z)\vec{a}_x$$

$$\epsilon_2, \mu_0$$

$$\vec{E}_2 = T \cos(1000t - 2\pi z)\vec{a}_x$$

FIND H on both sides!

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$z=0$$

# Example Problem

$$\epsilon_1, \mu_0$$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x + R \cos(1000t + 2\pi z)\vec{a}_x$$

$$\frac{\partial \vec{B}_1}{\partial t} = - \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos(1000t - 2\pi z) & 0 & 0 \\ + R \cos(1000t + 2\pi z) & 0 & 0 \end{vmatrix}$$

$$\epsilon_2, \mu_0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{E}_2 = T \cos(1000t - 2\pi z)\vec{a}_x$$

$$\frac{\partial \vec{B}_2}{\partial t} = - \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ T \cos(1000t - 2\pi z) & 0 & 0 \end{vmatrix}$$

$$z=0$$

# Example Problem

$\epsilon_1, \mu_0$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x - 0.25 \cos(1000t + 2\pi z)\vec{a}_x$$

$$\frac{\partial \vec{B}_1}{\partial t} = - \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos(1000t - 2\pi z) & 0 & 0 \\ + R \cos(1000t + 2\pi z) & 0 & 0 \end{vmatrix}$$

$$\frac{\partial \vec{B}_1}{\partial t} = - \frac{\partial}{\partial z} \left[ \cos(1000t - 2\pi z) + R \cos(1000t + 2\pi z) \right] \hat{a}_y$$

$$\frac{\partial \vec{B}_1}{\partial t} = -2\pi \sin(1000t - 2\pi z) \hat{a}_y$$

$$+ 2\pi \cdot R \sin(1000t + 2\pi z) \hat{a}_y$$

$\epsilon_2, \mu_0$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{E}_2 = 0.75 \cos(1000t - 2\pi z)\vec{a}_x$$

$$\frac{\partial \vec{B}_2}{\partial t} = - \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ T \cos(1000t - 2\pi z) & 0 & 0 \end{vmatrix}$$

$$\frac{\partial \vec{B}_2}{\partial t} = - \frac{\partial}{\partial z} (T \cos(1000t - 2\pi z)) \hat{a}_y$$

$$\frac{\partial \vec{B}_2}{\partial t} = -T \cdot 2\pi \sin(1000t - 2\pi z) \hat{a}_y$$

$z=0$

# Example Problem

$\epsilon_1, \mu_0$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x - 0.25 \cos(1000t + 2\pi z)\vec{a}_x$$

$$\frac{\partial \vec{B}_1}{\partial t} = -2\pi \sin(1000t - 2\pi z)\hat{a}_y + 2\pi \cdot R \sin(1000t + 2\pi z)\hat{a}_y$$

$$\vec{B}_1 = \int \left[ \begin{array}{l} -2\pi \sin(1000t - 2\pi z)\hat{a}_y \\ + 2\pi \cdot R \sin(1000t + 2\pi z)\hat{a}_y \end{array} \right] dt$$

$$\vec{B}_1 = +2\pi \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y - 2\pi \cdot \frac{1}{1000} R \cos(1000t + 2\pi z)\hat{a}_y$$

$\epsilon_2, \mu_0$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{E}_2 = 0.75 \cos(1000t - 2\pi z)\vec{a}_x$$

$$\frac{\partial \vec{B}_2}{\partial t} = -T \cdot 2\pi \sin(1000t - 2\pi z)\hat{a}_y$$

$$\vec{B}_2 = \int -T \cdot 2\pi \sin(1000t - 2\pi z)\hat{a}_y dt$$

$$\vec{B}_2 = T \cdot 2\pi \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$z=0$

# Example Problem

$\epsilon_1, \mu_0$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x \\ - 0.25 \cos(1000t + 2\pi z)\vec{a}_x$$

$$\vec{B}_1 = +2\pi \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y \\ - 2\pi \cdot \frac{1}{1000} R \cos(1000t + 2\pi z)\hat{a}_y$$

$\epsilon_2, \mu_0$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{E}_2 = 0.75 \cos(1000t - 2\pi z)\vec{a}_x$$

$$\vec{B}_2 = T \cdot 2\pi \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$z=0$

# Example Problem

$\epsilon_1, \mu_o$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x \\ - 0.25 \cos(1000t + 2\pi z)\vec{a}_x$$

$$\vec{H}_1 = \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$- \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} R \cos(1000t + 2\pi z)\hat{a}_y$$

$\epsilon_2, \mu_o$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{E}_2 = 0.75 \cos(1000t - 2\pi z)\vec{a}_x$$

$$\vec{H}_2 = T \cdot \frac{2\pi}{\mu_o} \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$z=0$

# Example Problem

$$\epsilon_1, \mu_0$$

$$\epsilon_2, \mu_0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x - 0.25 \cos(1000t + 2\pi z)\vec{a}_x$$

$$\vec{E}_2 = 0.75 \cos(1000t - 2\pi z)\vec{a}_x$$

$$\vec{H}_1 = \frac{2\pi}{\mu_0} \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$\vec{H}_2 = T \cdot \frac{2\pi}{\mu_0} \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$-\frac{2\pi}{\mu_0} \cdot \frac{1}{1000} R \cos(1000t + 2\pi z)\hat{a}_y$$

APPLY BC at Z=0

$$\frac{2\pi}{\mu_0} \cdot \frac{1}{1000} \cos(1000t)\hat{a}_y - \frac{2\pi}{\mu_0} \cdot \frac{1}{1000} R \cos(1000t)\hat{a}_y = T \frac{2\pi}{\mu_0} \cdot \frac{1}{1000} \cos(1000t)\hat{a}_y$$

$$z=0$$

# Example Problem

$$\epsilon_1, \mu_0$$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x - 0.25 \cos(1000t + 2\pi z)\vec{a}_x$$

$$\vec{H}_1 = \frac{2\pi}{\mu_0} \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$- \frac{2\pi}{\mu_0} \cdot \frac{1}{1000} R \cos(1000t + 2\pi z)\hat{a}_y$$

$$\epsilon_2, \mu_0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{E}_2 = 0.75 \cos(1000t - 2\pi z)\vec{a}_x$$

$$\vec{H}_2 = T \cdot \frac{2\pi}{\mu_0} \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

APPLY BC at Z=0

$$\frac{2\pi}{\mu_0} \cdot \frac{1}{1000} \cos(1000t)\hat{a}_y - \frac{2\pi}{\mu_0} \cdot \frac{1}{1000} R \cos(1000t)\hat{a}_y = T \frac{2\pi}{\mu_0} \cdot \frac{1}{1000} \cos(1000t)\hat{a}_y$$

$$1 - R = T$$

# Example Problem

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$\epsilon_1, \mu_0$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x$$

$$+ R \cos(1000t + 2\pi z)\vec{a}_x$$

$$\vec{H}_1 = \frac{2\pi}{\mu_0} \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$- R \frac{2\pi}{\mu_0} \cdot \frac{1}{1000} \cos(1000t + 2\pi z)\hat{a}_y$$

$$\frac{\partial \vec{D}_1}{\partial t} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H_y(z) & 0 \end{vmatrix}$$

$$\frac{\partial \vec{D}_1}{\partial t} = -\frac{\partial}{\partial z} [H_y(z)]\hat{a}_x$$

$\epsilon_2, \mu_0$

$$\vec{E}_2 = T \cos(1000t - 2\pi z)\vec{a}_x$$

$$\vec{H}_2 = T \cdot \frac{2\pi}{\mu_0} \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$\frac{\partial \vec{D}_2}{\partial t} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H_y(z) & 0 \end{vmatrix}$$

$$\frac{\partial \vec{D}_2}{\partial t} = -\frac{\partial}{\partial z} [H_y(z)]\hat{a}_x$$

$z=0$

# Example Problem

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$\epsilon_1, \mu_o$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x$$
$$+ R \cos(1000t + 2\pi z)\vec{a}_x$$

$$\vec{H}_1 = \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$- R \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} \cos(1000t + 2\pi z)\hat{a}_y$$

$$\frac{\partial}{\partial t} \vec{D}_1 = -\frac{(2\pi)^2}{\mu_o} \cdot \frac{1}{1000} \sin(1000t - 2\pi z)\hat{a}_y$$

$$- R \frac{(2\pi)^2}{\mu_o} \cdot \frac{1}{1000} \sin(1000t + 2\pi z)\hat{a}_y$$

$\epsilon_2, \mu_o$

$$\vec{E}_2 = T \cos(1000t - 2\pi z)\vec{a}_x$$

$$\vec{H}_2 = T \cdot \frac{2\pi}{\mu_o} \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$\frac{\partial}{\partial t} \vec{D}_2 = -T \frac{(2\pi)^2}{\mu_o} \cdot \frac{1}{1000} \sin(1000t - 2\pi z)\hat{a}_y$$

$z=0$

# Example Problem

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$\epsilon_1, \mu_o$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x$$
$$+ R \cos(1000t + 2\pi z)\vec{a}_x$$

$$\vec{H}_1 = \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$- \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} R \cos(1000t + 2\pi z)\hat{a}_y$$

$$\vec{D}_1 = \frac{(2\pi)^2}{\mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t - 2\pi z)\hat{a}_y$$

$$+ R \frac{(2\pi)^2}{\mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t + 2\pi z)\hat{a}_y$$

$\epsilon_2, \mu_o$

$$\vec{E}_2 = T \cos(1000t - 2\pi z)\vec{a}_x$$

$$\vec{H}_2 = T \cdot \frac{2\pi}{\mu_o} \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$\vec{D}_2 = \frac{(2\pi)^2}{\mu_o} \cdot \frac{T}{(1000)^2} \cos(1000t - 2\pi z)\hat{a}_y$$

$z=0$

# Example Problem

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$\epsilon_1, \mu_o$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x$$
$$+ R \cos(1000t + 2\pi z)\vec{a}_x$$

$$\vec{H}_1 = \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$- \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} R \cos(1000t + 2\pi z)\hat{a}_y$$

$$\vec{E}_1 = \frac{(2\pi)^2}{\epsilon_1 \epsilon_o \mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t - 2\pi z)\hat{a}_y$$

$$+ R \frac{(2\pi)^2}{\epsilon_1 \epsilon_o \mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t + 2\pi z)\hat{a}_y$$

$\epsilon_2, \mu_o$

$$\vec{E}_2 = T \cos(1000t - 2\pi z)\vec{a}_x$$

$$\vec{H}_2 = T \cdot \frac{2\pi}{\mu_o} \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$\vec{E}_2 = \frac{(2\pi)^2}{\epsilon_2 \epsilon_o \mu_o} \cdot \frac{T}{(1000)^2} \cos(1000t - 2\pi z)\hat{a}_y$$

$z=0$

# Example Problem

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$\epsilon_1, \mu_o$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x$$

$$+ R \cos(1000t + 2\pi z)\vec{a}_x$$

$$\vec{H}_1 = \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$- \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} R \cos(1000t + 2\pi z)\hat{a}_y$$

$$\vec{E}_1 = \frac{(2\pi)^2}{\epsilon_1 \epsilon_o \mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t - 2\pi z)\hat{a}_y$$

$$+ R \frac{(2\pi)^2}{\epsilon_1 \epsilon_o \mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t + 2\pi z)\hat{a}_y$$

$\epsilon_2, \mu_o$

$$\vec{E}_2 = T \cos(1000t - 2\pi z)\vec{a}_x$$

$$\vec{H}_2 = T \cdot \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} \cos(1000t - 2\pi z)\hat{a}_y$$

$$\vec{E}_2 = \frac{(2\pi)^2}{\epsilon_2 \epsilon_o \mu_o} \cdot \frac{T}{(1000)^2} \cos(1000t - 2\pi z)\hat{a}_y$$

APPLY BC at Z=0

$$\frac{(2\pi)^2}{\epsilon_1 \epsilon_o \mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t)\hat{a}_y + R \frac{(2\pi)^2}{\epsilon_1 \epsilon_o \mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t)\hat{a}_y = \frac{(2\pi)^2}{\epsilon_2 \epsilon_o \mu_o} \cdot \frac{T}{(1000)^2} \cos(1000t)\hat{a}_y$$

# Example Problem

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\epsilon_1, \mu_o$$

$$\epsilon_2, \mu_o$$

$$\vec{H}_1 = \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} \cos(1000t - 2\pi z) \hat{a}_y$$

$$- \frac{2\pi}{\mu_o} \cdot \frac{1}{1000} R \cos(1000t + 2\pi z) \hat{a}_y$$

$$\vec{E}_1 = \frac{(2\pi)^2}{\epsilon_1 \epsilon_o \mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t - 2\pi z) \hat{a}_y$$

$$+ R \frac{(2\pi)^2}{\epsilon_1 \epsilon_o \mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t + 2\pi z) \hat{a}_y$$

$$\vec{H}_2 = T \cdot \frac{2\pi}{\mu_o} \frac{1}{1000} \cos(1000t - 2\pi z) \hat{a}_y$$

$$\vec{E}_2 = \frac{(2\pi)^2}{\epsilon_2 \epsilon_o \mu_o} \cdot \frac{T}{(1000)^2} \cos(1000t - 2\pi z) \hat{a}_y$$

APPLY BC at Z=0

$$\frac{(2\pi)^2}{\epsilon_1 \epsilon_o \mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t) \hat{a}_y + R \frac{(2\pi)^2}{\epsilon_1 \epsilon_o \mu_o} \cdot \frac{1}{(1000)^2} \cos(1000t) \hat{a}_y = \frac{(2\pi)^2}{\epsilon_2 \epsilon_o \mu_o} \cdot \frac{T}{(1000)^2} \cos(1000t) \hat{a}_y$$

$$\frac{1}{\epsilon_1} + R \frac{1}{\epsilon_1} = \frac{T}{\epsilon_2}$$

# Example Problem

$$\varepsilon_1, \mu_0 \quad \varepsilon_2, \mu_0$$

APPLY BC at Z=0

$$\frac{1}{\varepsilon_1} + R \frac{1}{\varepsilon_1} = \frac{T}{\varepsilon_2}$$

$$1 - R = T$$


# Example Problem

$$\varepsilon_1, \mu_0 \quad \varepsilon_2, \mu_0$$

APPLY BC at Z=0

$$\frac{1}{\varepsilon_1} + R \frac{1}{\varepsilon_1} = \frac{T}{\varepsilon_2}$$

$$1 - R = T \quad \text{Eq1}$$


$$(1 + R) \frac{1}{\varepsilon_1} = \frac{T}{\varepsilon_2}$$



$$1 + R = \frac{\varepsilon_1}{\varepsilon_2} T \quad \text{Eq2}$$

Eq1+Eq2:

$$2 = \left(1 + \frac{\varepsilon_1}{\varepsilon_2}\right) T$$



$$T = \frac{2}{\left(1 + \frac{\varepsilon_1}{\varepsilon_2}\right)}$$

# Example Problem

$$\varepsilon_1, \mu_0 \quad \varepsilon_2, \mu_0$$

APPLY BC at Z=0

$$\frac{1}{\varepsilon_1} + R \frac{1}{\varepsilon_1} = \frac{T}{\varepsilon_2}$$

$$1 - R = T$$

$$R = 1 - \frac{2}{\left(1 + \frac{\varepsilon_1}{\varepsilon_2}\right)}$$

$$T = \frac{2}{\left(1 + \frac{\varepsilon_1}{\varepsilon_2}\right)}$$

$$R = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

# Example Problem

$$\epsilon_1, \mu_0$$

$$\epsilon_2, \mu_0$$

$$\vec{E}_1 = \cos(1000t - 2\pi z)\vec{a}_x \\ + R \cos(1000t + 2\pi z)\vec{a}_x$$

$$\vec{E}_2 = T \cos(1000t - 2\pi z)\vec{a}_x$$

FIND R and T on both sides!

$$R = \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2}$$

$$T = \frac{2}{\left(1 + \frac{\epsilon_1}{\epsilon_2}\right)}$$

$$z=0$$