ELEG 648
Radiation/Antennas I

Mark Mirotznik, Ph.D.
Associate Professor
The University of Delaware

Auxiliary Potential Functions

Summary:

Given: $\vec{J}, \vec{M}$

Solve: $\nabla^2 \vec{A} + k^2 \vec{A} = -\mu \vec{J}$

$$\nabla^2 \vec{F} + k^2 \vec{F} = -\varepsilon \vec{M}$$

Calculate:

$$\vec{E} = \vec{E}_e + \vec{E}_m = -j\omega \vec{A} - \frac{j}{\omega\mu\varepsilon} \nabla(\nabla \cdot \vec{A}) - \frac{1}{\varepsilon} \nabla \times \vec{F}$$

$$\vec{H} = \vec{H}_e + \vec{H}_m = \frac{1}{\mu} \nabla \times \vec{A} - j\omega \vec{F} - \frac{j}{\omega\mu\varepsilon} \nabla(\nabla \cdot \vec{F})$$

The Solution to Maxwell's Equations

$$\vec{A}(r) = \frac{\mu}{4\pi}\iiint \vec{J}(r^{'})\frac{e^{-jk\left|r-r^{'}\right|}}{\left|r-r^{'}\right|}dv^{'}$$

$$\nabla^2\vec{A} + k^2\vec{A} = -\mu\vec{J}$$

$$\vec{E}_e = -j\omega\vec{A} + \frac{1}{j\omega\mu\varepsilon}\nabla(\nabla\cdot\vec{A})$$

$$\vec{H}_e = \frac{1}{\mu}\nabla\times\vec{A}$$

The Solution to Maxwell's Equations

If we have magnetic sources

$$\vec{F}(r) = \frac{\varepsilon}{4\pi}\iiint \vec{M}(r')\frac{e^{-jk|r-r'|}}{|r-r'|}dv'$$

$$\nabla^2\vec{F} + k^2\vec{F} = -\varepsilon\vec{M}$$

$$\vec{E}_m = -\frac{1}{\varepsilon}\nabla\times\vec{F}$$

$$\vec{H}_m = -j\omega\vec{F} - \frac{j}{\omega\mu\varepsilon}\nabla(\nabla\cdot\vec{F})$$

Far-Field Approximation (1)

$$\vec{A} = \frac{\mu}{4\pi} \iiint_V \vec{J}(\vec{r}') \frac{e^{-jkR}}{R} dv' \qquad \vec{F} = \frac{\varepsilon}{4\pi} \iiint_V \vec{M}(\vec{r}') \frac{e^{-jkR}}{R} dv'$$

In the far-field zone:

$$R = \sqrt{r^2 + r'^2 - 2rr'\cos\psi} \approx r - r'\cos\psi$$

$$\vec{A} = \frac{\mu}{4\pi} \iiint_V \vec{J} \frac{e^{-jk(r - r'\cos\psi)}}{r} dv' = \frac{\mu}{4\pi r} e^{-jkr} \iiint_V \vec{J} e^{jkr'\cos\psi} dv'$$

$$= \frac{\mu}{4\pi r} e^{-jkr} \vec{N} \qquad \vec{N} = \iiint_V \vec{J} e^{jkr'\cos\psi} dv'$$

$$\vec{F} = \frac{\varepsilon}{4\pi r} e^{-jkr} \vec{L} \qquad \vec{L} = \iiint_V \vec{M} e^{jkr'\cos\psi} dv'$$

Far-Field Approximation (3)

For $\vec{E}_F$ and $\vec{H}_F$:

$$\begin{cases} H_r \approx 0 \\ H_\theta \approx -j\omega F_\theta \\ H_\varphi \approx -j\omega F_\varphi \end{cases} \qquad \begin{cases} E_r \approx 0 \\ E_\theta \approx \eta H_\varphi = -j\omega\eta F_\varphi \\ E_\varphi \approx -\eta H_\theta = j\omega\eta F_\theta \end{cases}$$

Total fields:

$$E_r \approx 0$$

$$E_\theta \approx -j\omega A_\theta - j\omega\eta F_\varphi = -j\omega(A_\theta + \eta F_\varphi)$$

$$E_\varphi \approx -j\omega A_\varphi + j\omega\eta F_\theta = -j\omega(A_\varphi - \eta F_\theta)$$

Far-Field Approximation (4)

$$H_r \approx 0$$

$$H_\theta \approx \frac{j\omega}{\eta} A_\varphi - j\omega F_\theta = \frac{j\omega}{\eta}(A_\varphi - \eta F_\theta)$$

or

$$H_\varphi \approx -\frac{j\omega}{\eta} A_\theta - j\omega F_\varphi = -\frac{j\omega}{\eta}(A_\theta + \eta F_\varphi)$$

$$E_r \approx 0$$

$$E_\theta \approx -\frac{jke^{-jkr}}{4\pi r}(L_\varphi + \eta N_\theta)$$

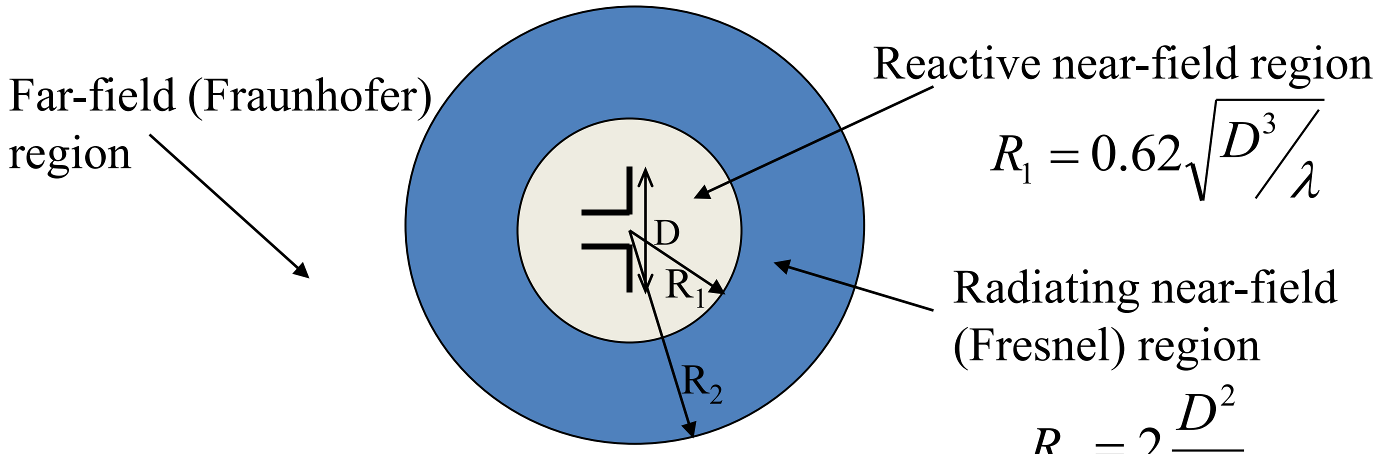
$$E_\varphi \approx \frac{jke^{-jkr}}{4\pi r}(L_\theta - \eta N_\varphi)$$

$$H_r \approx 0$$

$$H_\theta \approx \frac{jke^{-jkr}}{4\pi r}(N_\varphi - \frac{L_\theta}{\eta})$$

$$H_\varphi \approx -\frac{jke^{-jkr}}{4\pi r}(N_\theta + \frac{L_\varphi}{\eta})$$

Field Regions



$$R_1 = 0.62\sqrt{D^3/\lambda}$$

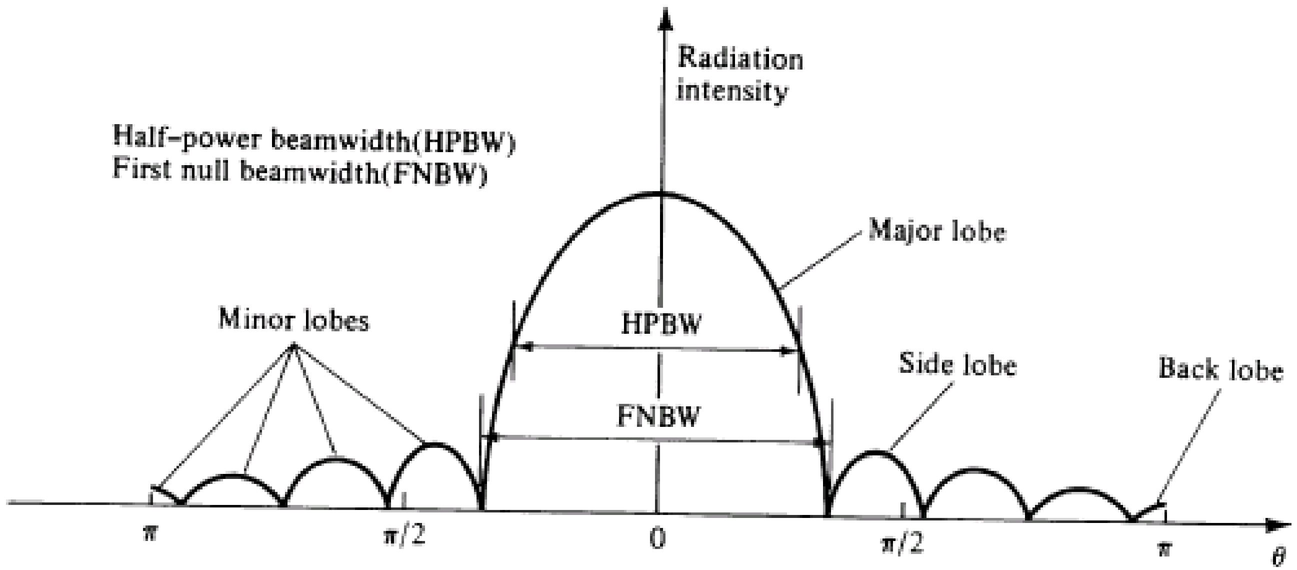
$$R_2 = 2\frac{D^2}{\lambda}$$

Reactive Near Field Region - the region immediately surrounding the antenna where the reactive field (stored energy - standing waves) is dominant.

Near-Field (Fresnel) Region - the region between the reactive near-field and the far-field where the radiation fields are dominant and the field distribution is dependent on the distance from the antenna.

Far-Field (Fraunhofer) Region - the region farthest away from the antenna where the field distribution is essentially independent of the distance from the antenna (propagating waves).

Radiation Pattern Lobes



Half-power beamwidth (HPBW) is the angle between two vectors, originating at the pattern's origin and passing through these points of the major lobe where the radiation intensity is half its maximum.

First-null beamwidth (FNBW) is the angle between two vectors, originating at the pattern's origin and tangent to the main beam at its base. It very often approximately true that $FNBW \approx 2 \cdot HPBW$.

Radiation Intensity

Radiation intensity in a given direction is the power per unit solid angle radiated in this direction by the antenna.

$$P_{rad}^{tot} = \int_0^{2\pi}\int_0^{\pi} P_{rad}(\theta,\phi,r)\, r^2 \sin(\theta)\, d\theta\, d\phi$$

define $U(\theta,\phi) = r^2 P_{rad}(\theta,\phi,r)$

This assumes we are in the far field and the power varies as 1/r^2

$$P_{rad}^{tot} = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\, \sin(\theta)\, d\theta\, d\phi = \int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\, d\Omega$$

$$U_{ave} = \frac{\int_0^{2\pi}\int_0^{\pi} U(\theta,\phi)\, d\Omega}{4\pi} = \frac{P_{rad}^{tot}}{4\pi}$$

The average radiation intensity for a given antenna represents the radiation intensity of a point source (or isotropic antenna) that produces the same amount of radiate power as the actual antenna.

The power pattern is a trace of the function $|U(\theta,\varphi)|$ usually normalized to its maximum value. The normalized pattern will be denoted as $\overline{U}(\theta,\varphi)$.

Radiation Intensity

The power pattern is a trace of the function $|U(\theta,\varphi)|$ usually normalized to its maximum value. The normalized pattern will be denoted as $\overline{U}(\theta,\varphi)$.

$$P_{rad}(\theta,\varphi,r)=\frac{1}{2}\tilde{E}\times\tilde{H}^{*}=\frac{1}{2\eta}\left|\tilde{E}\right|^{2}=\frac{1}{2\eta}\left|E_{\theta}{}^{2}+E_{\varphi}{}^{2}\right|$$

$$U(\theta,\varphi)=\frac{r^{2}}{2\eta}\left|E_{\theta}{}^{2}+E_{\varphi}{}^{2}\right|$$

$$\overline{U}(\theta,\varphi)=\frac{U(\theta,\varphi)}{U_{\max}}$$

Directive Gain

Directivity of an antenna in a given direction is the ratio of the radiation intensity in this direction and the radiation intensity averaged over all directions. The radiation intensity averaged over all directions is equal to the total power radiated by the antenna divided by 4π. If a direction is not specified, then the direction of maximum radiation is implied.

$$D(\theta,\varphi) = \frac{U(\theta,\varphi)}{U_{ave}} = \frac{U(\theta,\varphi)}{P_{rad}^{tot}/4\pi} = 4\pi\frac{U(\theta,\varphi)}{P_{rad}^{tot}}$$

$$D_{\max} = D_o = 4\pi\frac{U_{\max}}{P_{rad}^{tot}} \geq 1 \quad (directivity)$$

Directive in dB

$$D(\theta,\varphi)[dB] = 10\log_{10} D(\theta,\varphi)$$

Antenna Gain

The gain G of an antenna is the ratio of the radiation intensity U in a given direction and the radiation intensity that would be obtained, if the power fed to the antenna were radiated isotropically.

$$G(\theta,\varphi) = 4\pi \frac{U(\theta,\varphi)}{P_{input}}$$

DIRECTIVITY ⇒ **POWER DENSITY IN A CERTAIN DIRECTION DIVIDED BY THE TOTAL POWER RADIATED**

GAIN ⇒ **POWER DENSITY IN A CERTAIN DIRECTION DIVIDED BY THE TOTAL INPUT POWER TO THE ANTENNA TERMINALS (FEED POINTS)**

IF ANTENNA HAS OHMIC LOSS…
THEN, GAIN < DIRECTIVITY

Antenna Gain

Sources of Antenna System Loss

1. losses due to impedance mismatches
2. losses due to the transmission line
3. conductive and dielectric losses in the antenna
4. losses due to polarization mismatches

According to IEEE standards the antenna gain does not include losses due to impedance or polarization mismatches. Therefore the antenna gain only accounts for dielectric and conductive losses found in the antenna itself. However Balanis and others have included impedance mismatch as part of the antenna gain.

The antenna gain relates to the directivity through a coefficient called the radiation efficiency (e_t)

conduction losses dielectric losses

impedance mismatch

$$G(\theta,\varphi) = e_t \cdot D(\theta,\varphi) = e_p e_r e_c e_d \cdot D(\theta,\varphi) \qquad e_t \leq 1$$

Polarization losses

Overall Antenna Efficiency

The overall antenna efficiency is a coefficient that accounts for all the different losses present in an antenna system.

$$e = e_p \overbrace{e_r e_c e_d}^{e_t} = e_p \cdot e_r e_{cd}$$

$$e_p = polarization \quad mismatches$$

$$e_r = reflection \quad efficiency \;\; (impedance\ mismatch)$$

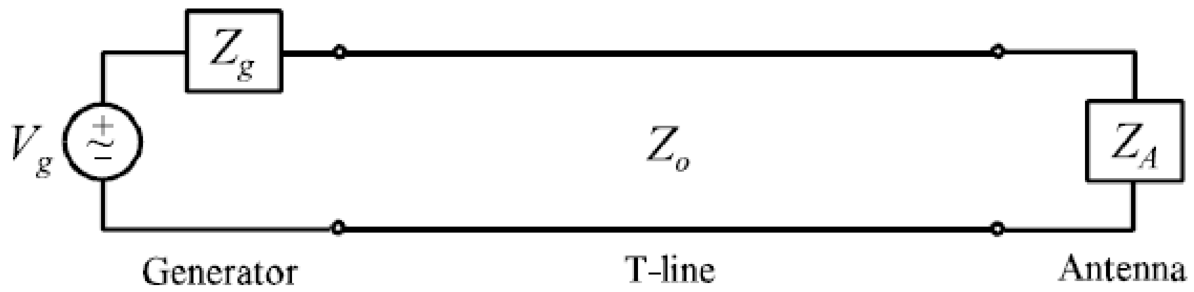
$$e_c = conduction \quad losses$$

$$e_d = dielectric \quad losses$$

$$e_{cd} = conductor \,\&\, dielectric \quad losses$$

Typical Antenna System for Transmission

The typical transmitting system can be defined by a generator, transmission line and transmitting antenna as shown below.



The generator is modeled by a complex source voltage V_g and a complex source impedance Z_g.

Reflection Efficiency

The reflection efficiency through a reflection coefficient (Γ) at the input (or feed) to the antenna.

$$e_r = 1 - |\Gamma|^2$$

$$\Gamma = \frac{Z_{input} - Z_{generator}}{Z_{input} + Z_{generator}}$$

$$Z_{input} = antenna \quad input \quad impedance\ (\Omega)$$

$$Z_{output} = generator \quad output \quad impedance\ (\Omega)$$

Radiation Resistance

The radiation resistance is one of the few parameters that is relatively straight forward to calculate.

$$R_{rad} = \frac{2P_{rad}^{total}}{\left|I_o\right|^2} = \frac{2\oiint_{4\pi} U(\theta,\varphi)\,d\Omega}{\left|I_o\right|^2}$$

Antenna Radiation Efficiency

Conduction and dielectric losses of an antenna are very difficult to separate and are usually lumped together to form the e_{cd} efficiency. Let R_{cd} represent the actual losses due to conduction and dielectric heating. Then the efficiency is given as

$$e_{cd} = \frac{P_{rad}}{P_{total}} = \frac{P_{rad}}{P_{rad} + P_{ohmic}} = \frac{R_{rad}}{R_{cd} + R_{rad}}$$

For wire antennas (without insulation) there is no dielectric losses only conductor losses from the metal antenna. For those cases we can approximate R_{cd} by:

$$R_{cd} = \frac{l}{2\pi b}\sqrt{\frac{\omega\mu_o}{2\sigma}}$$

where b is the radius of the wire, ω is the angular frequency, σ is the conductivity of the metal and l is the antenna length

Antenna Radiation Efficiency

For wire antennas (without insulation) there is no dielectric losses only conductor losses from the metal antenna. For those cases we can approximate R_{cd} by:

$$R_{cd} = \frac{l}{2\pi b}\sqrt{\frac{\omega\mu_o}{2\sigma}}$$

where b is the radius of the wire, ω is the angular frequency, σ is the conductivity of the metal and l is the antenna length

f	δ	R
0	∞	$R_{DC} = 0.818\ \text{m}\Omega$
1 kHz	2.09 mm	~
10 kHz	0.661 mm	$R_{HF} = 1.60\ \text{m}\Omega$
100 kHz	0.209 mm	$R_{HF} = 5.07\ \text{m}\Omega$
1 MHz	0.0661 mm	$R_{HF} = 16.0\ \text{m}\Omega$

Resistance of 1 m of #10 AWG (a = 2.59 mm) copper wire.

Polarization Loss Factor

Assume the polarization of the electric field incident on an antenna is directed in the ρ_w direction

$$\tilde{E}_{inc} = \hat{\rho}_w E_{inc}$$

Also assume the polarization of perferred by the antenna is directed in the ρ_a direction

$$\tilde{E}_a = \hat{\rho}_a E_a$$

The polarization loss factor is defined as

$$PLF = \left| \hat{\rho}_w \cdot \hat{\rho}_a^{\,*} \right|^2 = \left| \cos(\psi_p) \right|^2$$

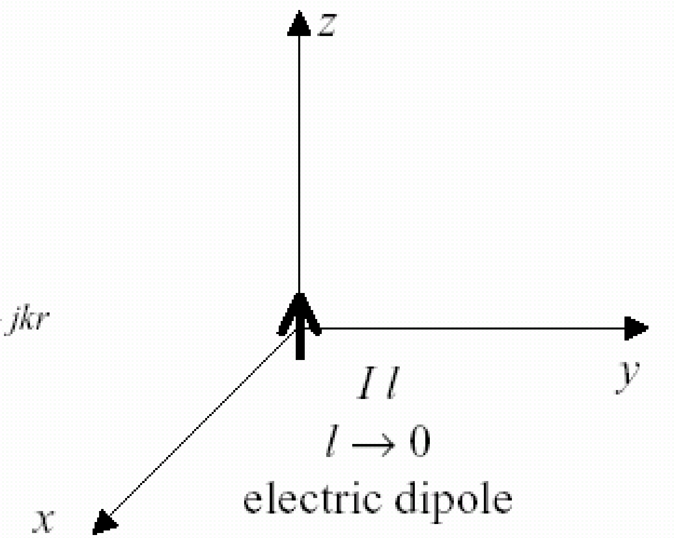
tells us the amount of incident power lost by any mismatches in polarization between the incident field and the antenna.

Hertzian Dipole

Example #1:

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \iiint_V \vec{J}(\vec{r}') \frac{e^{-jkR}}{R} dv'$$

$$= \hat{z} \frac{\mu}{4\pi} \int_{-l/2}^{l/2} \frac{Ie^{-jkR}}{R} dz' = \hat{z} \frac{\mu Il}{4\pi r} e^{-jkr}$$



In spherical coordinate system:

$$A_r = A_z \cos\theta = \frac{\mu Il}{4\pi r} e^{-jkr} \cos\theta$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu Il}{4\pi r} e^{-jkr} \sin\theta$$

$$A_\varphi = 0$$

$$\vec{H} = \frac{1}{\mu}\nabla \times \vec{A}$$

$$= \frac{1}{\mu}\left\{ \frac{\hat{r}}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(A_\varphi \sin\theta\right) - \frac{\partial A_\theta}{\partial\varphi}\right] + \frac{\hat{\theta}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}\left(rA_\varphi\right)\right] + \frac{\hat{\varphi}}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right] \right\}$$

$$= \hat{\varphi}\frac{1}{\mu r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$$

$$\vec{H} = \hat{\varphi}\,\frac{jkIl\sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

$$\vec{E} = \frac{1}{j\omega\varepsilon}\nabla \times \vec{H}$$

$$E_r = \eta \frac{Il\cos\theta}{2\pi r^2}\left(1+\frac{1}{jkr}\right)e^{-jkr}$$

$$E_\theta = \eta \frac{jkIl\sin\theta}{4\pi r}\left(1+\frac{1}{jkr}\right)e^{-jkr}$$

$$E_\varphi = 0$$

For $r >> \lambda$: **(far-field zone)**

$$E_\theta = \eta \frac{jkIl\sin\theta}{4\pi r}e^{-jkr} = \eta H_\varphi$$

$$H_\varphi = \frac{jkIl\sin\theta}{4\pi r}e^{-jkr} = \frac{E_\theta}{\eta}$$

Far field

$$P_e = \frac{1}{2}\iint\left(\vec{E}\times\vec{H}^*\right)\cdot d\vec{s}$$

$$= \frac{1}{2}\int_0^{2\pi}\int_0^{\pi} E_\theta H_\varphi^* r^2 \sin\theta d\theta d\varphi$$

$$= \eta\frac{\pi}{3}\left|\frac{Il}{\lambda}\right|^2\left[1-\frac{j}{(kr)^3}\right]$$

$$\mathrm{Re}(P_e) = \eta\frac{\pi}{3}\left|\frac{Il}{\lambda}\right|^2$$

Time average radiated power

$$P_{rad} = \frac{1}{2}|I|^2 R_{rad}$$

$$\eta\frac{\pi}{3}\left|\frac{Il}{\lambda}\right|^2 = \frac{1}{2}|I|^2 R_{rad}$$

$$R_{rad} = \eta\frac{2\pi}{3}\left|\frac{l}{\lambda}\right|^2$$

Radiation Resistance

Directivity and Effective Aperture

$$D(\theta,\phi) = 4\pi \frac{U(\theta,\phi)}{P_{rad}}$$

$$U(\theta,\phi) = \frac{r^2}{2\eta}\left(\left|E_\theta(\theta,\phi)\right|^2 + \left|E_\phi(\theta,\phi)\right|^2\right)$$

$$U(\theta,\phi) = \frac{r^2}{2\eta}\left(\left|j\eta \frac{kIle^{-jkr}}{4\pi r}\sin(\theta)\right|^2\right) = \frac{\eta}{2}\left(\frac{kIl}{4\pi}\sin(\theta)\right)^2 = \frac{\eta(kIl)^2}{32\pi^2}\sin^2(\theta)$$

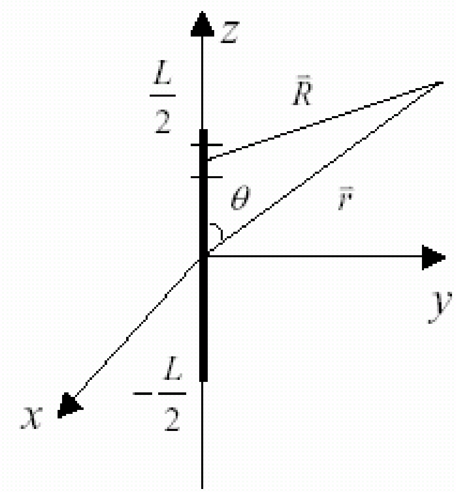
$$D(\theta,\phi) = 4\pi \frac{\frac{\eta(\frac{2\pi}{\lambda}Il)^2}{32\pi^2}\sin^2(\theta)}{\frac{\eta\pi}{3}(\frac{Il}{\lambda})^2} = \frac{3}{2}\sin^2(\theta)$$ ⟸ Directive gain

$$D_o = \max(D(\theta,\phi)) = \frac{3}{2}$$ ⟸ Directivity

Finite Length Dipole

Example #2:

$$I(z) = I_m \sin\left[k\left(\frac{L}{2} - |z|\right)\right]$$



$$\vec{A} = \frac{\mu}{4\pi}\int_C \vec{I}\,\frac{e^{-jkR}}{R}\,dl'$$

$$= \hat{z}\,\frac{\mu}{4\pi}\int_{-L/2}^{L/2} I_m \sin\left[k\left(\frac{L}{2} - |z|\right)\right]\frac{e^{-jkR}}{R}\,dz'$$

Consider $r >> z'$:

$$R = \sqrt{r^2 + z'^2 - 2rz'\cos\theta}$$

$$R \approx r\left(1 - \frac{2z'\cos\theta}{r}\right)^{1/2} = r\left[1 - \frac{z'\cos\theta}{r} - \cdots\right] \approx r - z'\cos\theta$$

Finite Length Dipole

$$\vec{A} = \hat{z}\frac{\mu}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}} I_m \sin\left[k\left(\frac{L}{2} - |z'|\right)\right]\frac{e^{-jk(r-z'\cos\theta)}}{r}dz'$$

$$= \hat{z}\frac{\mu I_m}{4\pi r}e^{-jkr}\int_{-\frac{L}{2}}^{\frac{L}{2}} \sin\left[k\left(\frac{L}{2} - |z'|\right)\right]e^{jkz'\cos\theta}dz'$$

Consider

$$I = \int_{-\frac{L}{2}}^{\frac{L}{2}} \sin\left[k\left(\frac{L}{2} - |z'|\right)\right]e^{jkz'\cos\theta}dz'$$

$$= \int_{0}^{\frac{L}{2}} \sin\left[k\left(\frac{L}{2} - z'\right)\right]e^{jkz'\cos\theta}dz' + \int_{-\frac{L}{2}}^{0} \sin\left[k\left(\frac{L}{2} + z'\right)\right]e^{jkz'\cos\theta}dz'$$

Finite Length Dipole

$$= \int_0^{\frac{L}{2}} \sin\left[k\left(\frac{L}{2}-z'\right)\right] e^{jkz'\cos\theta} dz' + \int_0^{\frac{L}{2}} \sin\left[k\left(\frac{L}{2}-z'\right)\right] e^{-jkz'\cos\theta} dz'$$

$$= 2\int_0^{\frac{L}{2}} \sin\left[k\left(\frac{L}{2}-z'\right)\right] \cos(kz'\cos\theta) dz'$$

$$= \frac{2}{k\cos\theta}\int_0^{\frac{L}{2}} \sin\left[k\left(\frac{L}{2}-z'\right)\right] d\sin(kz'\cos\theta)$$

$$= \frac{2}{k\cos\theta}\left\{\sin\left[k\left(\frac{L}{2}-z'\right)\right]\sin(kz'\cos\theta)\Big|_0^{\frac{L}{2}} + k\int_0^{\frac{L}{2}} \cos\left[k\left(\frac{L}{2}-z'\right)\right]\sin(kz'\cos\theta) dz'\right\}$$

$$= \frac{2}{\cos\theta}\int_0^{\frac{L}{2}} \cos\left[k\left(\frac{L}{2}-z'\right)\right]\sin(kz'\cos\theta) dz'$$

Finite Length Dipole

$$= -\frac{2}{k\cos^2\theta}\int_0^{\frac{L}{2}}\cos\left[k\left(\frac{L}{2}-z'\right)\right]d\cos(kz'\cos\theta)$$

$$= -\frac{2}{k\cos^2\theta}\left\{\cos\left[k\left(\frac{L}{2}-z'\right)\right]\cos(kz'\cos\theta)\Big|_0^{\frac{L}{2}}\right.$$

$$\left.-k\int_0^{\frac{L}{2}}\sin\left[k\left(\frac{L}{2}-z'\right)\right]\cos(kz'\cos\theta)dz'\right\}$$

$$= -\frac{2}{k\cos^2\theta}\left[\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)\right]+\frac{1}{\cos^2\theta}I$$

$$I = \frac{2\left[\cos\left(k\frac{L}{2}\cos\theta\right)-\cos\left(k\frac{L}{2}\right)\right]}{k\sin^2\theta}$$

Finite Length Dipole

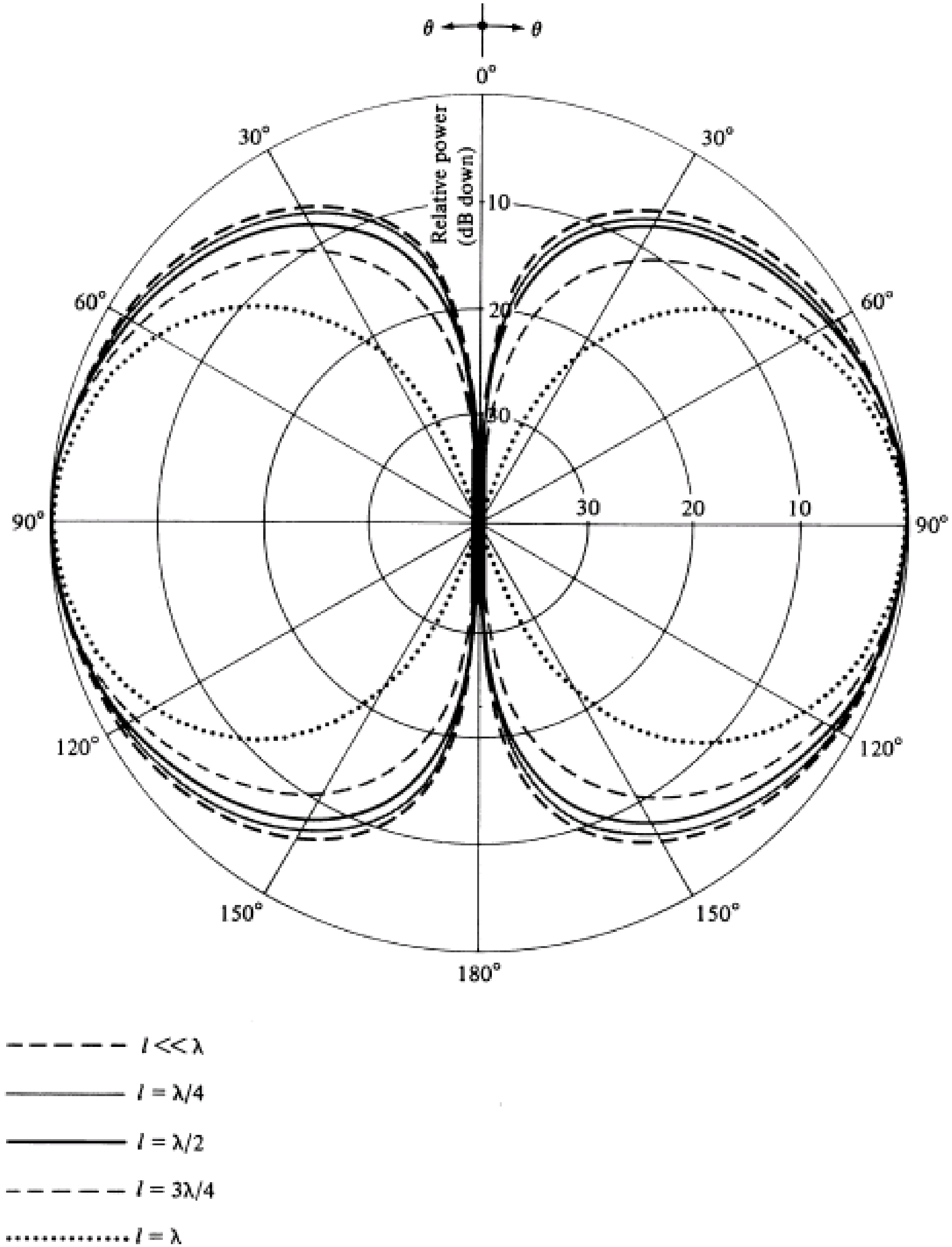
Vector potential:

$$A_z = \frac{\mu I_m}{4\pi r} e^{-jkr} \frac{2\left[\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)\right]}{k \sin^2\theta}$$

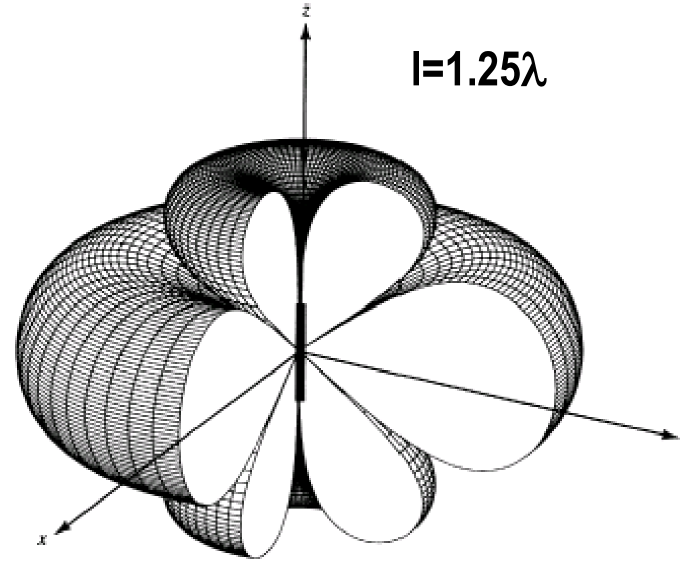
Far field:

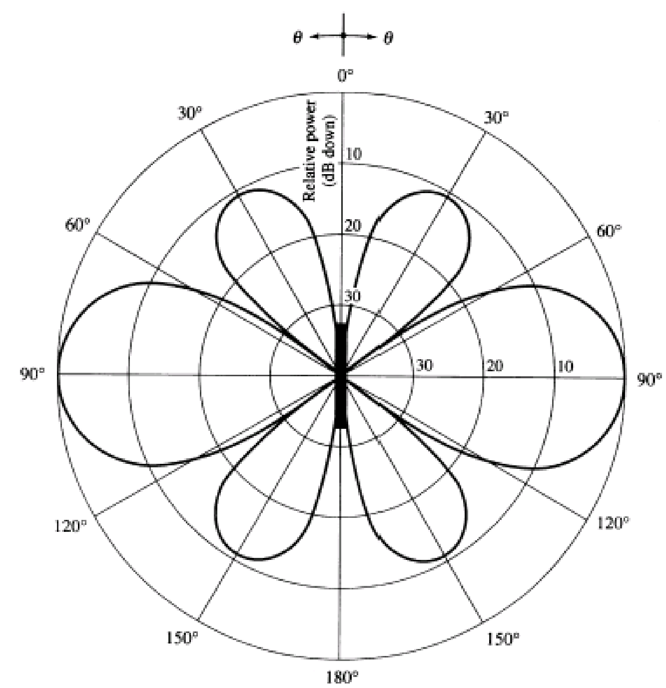
$$E_\theta = \frac{j\eta I_m e^{-jkr}}{2\pi r} \frac{\left[\cos\left(k\frac{L}{2}\cos\theta\right) - \cos\left(k\frac{L}{2}\right)\right]}{\sin\theta}$$

$$H_\varphi = E_\theta / \eta$$



l=1.25λ





(b) Two-dimensional

3. Half-wavelength dipole

This is a classical and widely used thin wire antenna: $l = \frac{\lambda}{2}$

$$E_\theta \simeq j\eta \frac{I_0 e^{-j\beta r}}{2\pi r} \cdot \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \qquad H_\varphi = \frac{E_\theta}{\eta} \tag{8.35}$$

Radiated power flow density:

$$P = \frac{\eta}{2}|E_\theta|^2 = \eta \frac{|I_0|^2}{8\pi^2 r^2} \underbrace{\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2}_{F(\theta)\text{ - normalized power pattern}} \simeq \eta \frac{|I_0|^2}{8\pi^2 r^2}\sin^3\theta \tag{8.36}$$

Radiation intensity:

$$U = r^2 P = \eta \frac{|I_0|^2}{8\pi^2} \underbrace{\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2}_{F(\theta)\text{ - normalized power pattern}} \simeq \eta \frac{|I_0|^2}{8\pi^2}\sin^3\theta \tag{8.37}$$

Radiated power

The radiated power of the half-wavelength dipole is, of course, a special case of the integral in (8.26).

$$\Pi = \eta \frac{|I_0|^2}{4\pi} \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$\Pi = \eta \frac{|I_0|^2}{8\pi} \underbrace{\int_0^{2\pi} \frac{1-\cos y}{y} dy}_{\Im}, \qquad (8.38)$$

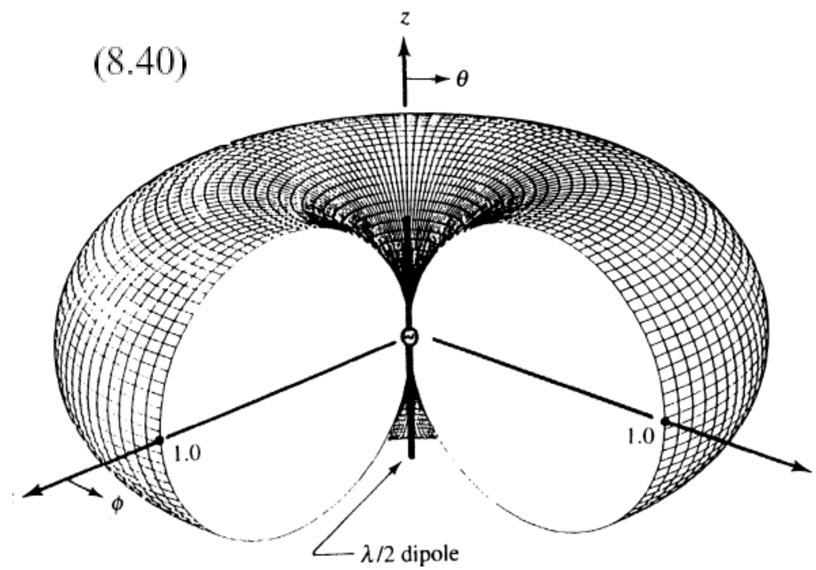
$$\Im = 0.5772 + \ln(2\pi) - C_i(2\pi) \simeq 2.435$$

$$\Rightarrow \Pi = 2.435 \frac{\eta}{8\pi} |I_0|^2 = 36.525 |I_0|^2 \qquad (8.39)$$

Radiation resistance

$$R_r = \frac{2\Pi}{|I_0|^2} \simeq 73 \quad \Omega \qquad (8.40)$$

$$D_0 = 4\pi \frac{U_{\max}}{\Pi} = 4\pi \frac{U_{/\theta=90^\circ}}{\Pi} = \frac{4}{\Im} = \frac{4}{2.435} = 1.643$$



Maximum effective area

$$A_e = \frac{\lambda^2}{4\pi} D_0 \simeq 0.13\lambda^2$$

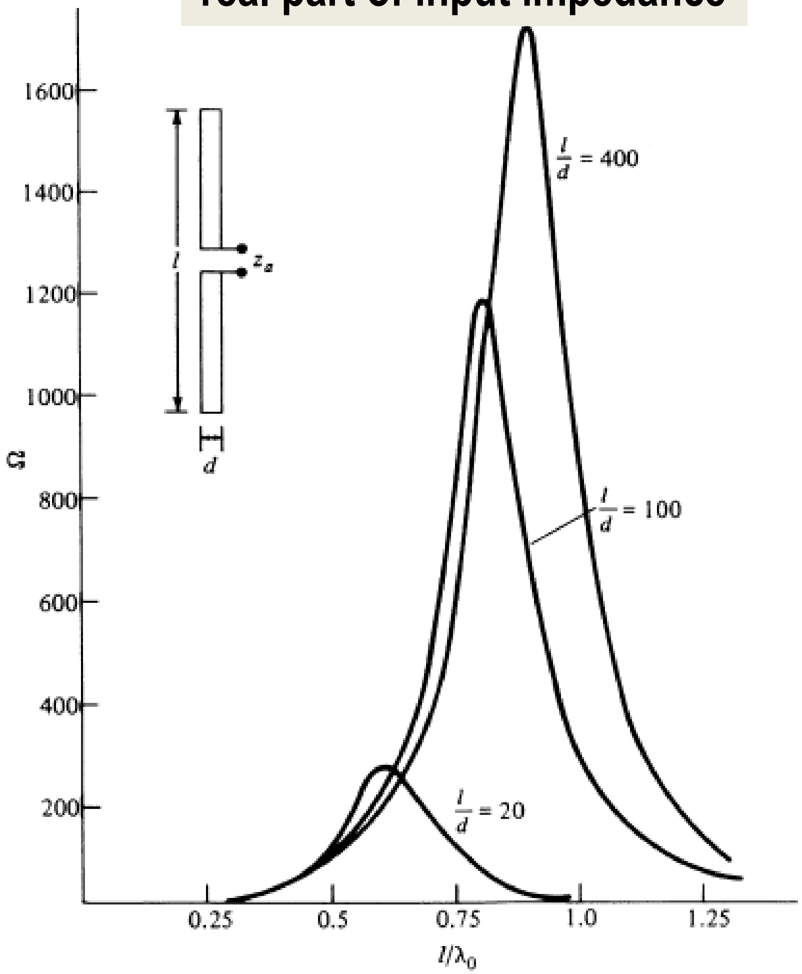
Input resistance

Since $l = \lambda/2$,

$$R_{in} = R_r \simeq 73 \quad \Omega$$

Finite Length Dipole
Does the wire thickness matter?

real part of input impedance



imaginary part of input impedance