ELEG 205
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Lecture #8

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Chapter 5: Useful Circuit Analysis Techniques

1. Voltage and Current Divider
2. Linear Superposition
3. Thevenin Equivalent Circuits
4. Maximum Power Transfer
From the terminals (a) and (b) you cannot tell the difference between these two circuits.
Step #1: $R_{th}$ is the equivalent resistance looking into terminals (a) and (b) after all the sources have been killed.
Step #2: $V_{th}$ is equal to the “open circuit” voltage across terminals (a) and (b)
Maximum Power Transfer

Question: What value of $R_L$ will absorb the maximum amount of power from the rest of the circuit?
Maximum Power Transfer

\[ V_L = \frac{R_L}{R_{th} + R_L} \cdot V_{th} \]
Maximum Power Transfer

How do we find the value of $R_L$ that will maximize $P_L$?
Maximum Power Transfer

\[
\begin{align*}
\frac{dP_L}{dR_L} &= 0 \\
\frac{d}{dR_L} \left[ \frac{R_L}{(R_{th} + R_L)^2} V_{th}^2 \right] &= V_{th}^2 \cdot \frac{(R_{th} + R_L)^2 - 2R_L(R_{th} + R_L)}{(R_{th} + R_L)^4} = 0
\end{align*}
\]
Maximum Power Transfer

\[ V_{th}^2 \cdot \frac{(R_{th} + R_L)^2 - 2R_L(R_{th} + R_L)}{(R_{th} + R_L)^4} = 0 \]

\[ (R_{th} + R_L)^2 - 2R_L(R_{th} + R_L) = 0 \]

\[ R_{th}^2 + 2R_{th}R_L + R_L^2 - 2R_{th}R_L - 2R_L^2 = 0 \]

\[ R_{th}^2 - R_L^2 = 0 \quad \Rightarrow \quad R_L = R_{th} \]
For maximum power transfer the load resistor \( R_L \) should equal the Thevenin resistance \( R_{th} \).
In this circuit find the resistance \( R_L \) that will absorb the maximum power from the rest of the circuit and find what that maximum power is.
Step #1: Find the Thevenin Equivalent circuit looking into the circuit from the resistor $R_L$. 

**Maximum Power Transfer: Example**
Step #2: Find the Thevenin resistance
Step #2: Find the Thevenin resistance

\[ R_{th} = \frac{300 \cdot 300}{300 + 300} = 150 \, \Omega \]
Step #3: Find the Thevenin voltage
Maximum Power Transfer: Example

KCL #2: By inspection $V_2 = 10$ V

KCL #1: \[ \frac{V_1 - 20}{300} + \frac{V_1 - V_2}{300} = 0 \]
Maximum Power Transfer: Example

KCL #2: By inspection $V_2 = 10$ V

KCL #1: $\frac{V_1 - 20}{300} + \frac{V_1 - V_2}{300} = 0$  $\Rightarrow$  $\frac{V_1 - 20}{300} + \frac{V_1 - 10}{300} = 0$  $\Rightarrow$  $V_1 = 15$ V
KCL #2: By inspection $V_2 = 10\, V$

KCL #1: \[
\frac{V_1 - 20}{300} + \frac{V_1 - V_2}{300} = 0 \quad \Rightarrow \quad \frac{V_1 - 20}{300} + \frac{V_1 - 10}{300} = 0 \quad \Rightarrow \quad V_1 = 15\, V
\]

\[
V_{oc} = V_1 - V_2 = 15 - 10 = 5\, V
\]
Step #4: For maximum power transfer $R_L = R_{th} = 150 \, \Omega$

How much power is transferred?
Step #4: For maximum power transfer $R_L = R_{th} = 150 \, \Omega$

How much power is transferred?

$$P_L = \frac{V_L^2}{R_L} = \frac{2.5^2}{150} = 0.042 \, W = 42 \, mW$$
In this circuit find the resistance $R_L$ that will absorb the maximum power from the rest of the circuit and find what that maximum power is.
In this circuit find the resistance $R_L$ that will absorb the maximum power from the rest of the circuit and find what that maximum power is.
Step #1: Find the Thevenin Equivalent circuit looking into the circuit form the resistor $R_L$
Maximum Power Transfer: Example

Step #1: Find the Thevenin Equivalent circuit looking into the circuit from the resistor $R_L$
Maximum Power Transfer: Example

\[ R_{th} = 600 \parallel 600 \parallel 300 \parallel 150 = 75 \, \Omega \]
Maximum Power Transfer: Example

Step #2: Find the Thevenin voltage
Step #2: Find the Thevenin voltage
Simplify circuit by combining resistors
Step #2: Find the Thevenin voltage
Nodal analysis
Maximum Power Transfer: Example

KCL #1: By inspection $V_1 = 10$ V

KCL #2: \[
\frac{V_2 + 20 - V_1}{150} + \frac{V_2 - V_1}{200} + \frac{V_2 + 20}{600} = 0
\]
KCL #1: By inspection $V_1 = 10$ V

KCL #2: \[
\frac{V_2 + 20 - V_1}{150} + \frac{V_2 - V_1}{200} + \frac{V_2 + 20}{600} = 0
\]

\[
\Rightarrow \frac{V_2 + 20 - 10}{150} + \frac{V_2 - 10}{200} + \frac{V_2 + 20}{600} = 0
\]

\[
\Rightarrow V_2 = -3.75 \text{ V}
\]
Maximum Power Transfer: Example

\[ V_2 = -3.75 \, V \]

KVL outer loop: \[-V_{oc} + 20 + V_2 - 10 = 0 \implies -V_{oc} + 20 - 3.75 - 10 = 0 \implies V_{oc} = 6.25 \, V\]
Maximum Power Transfer: Example

Step #4: For maximum power transfer $R_L = R_{th} = 75 \, \Omega$

How much power is transferred?
Maximum Power Transfer: Example

Step #4: For maximum power transfer $R_L = R_{th} = 75 \, \Omega$

How much power is transferred?

$$P_L = \frac{V_L^2}{R_L} = \frac{3.125^2}{75} = 0.13 \, W = 130 \, mW$$