Quiz (10 minutes)

Simplify the following complex expressions into a single Phasor (i.e. a single complex number in polar notation)

\[ \hat{V} = \frac{1 - j}{j} \cdot e^{-j \frac{\pi}{2}} \]
Quiz

Simplify the following complex expressions into a single Phasor (i.e. a single complex number in polar notation)

\[ \tilde{V} = \frac{1 - j}{j} \cdot e^{-j\frac{\pi}{2}} = \sqrt{2e^{-j\frac{\pi}{4}}} \cdot e^{-j\frac{\pi}{2}} \]

\[ \tilde{V} = \sqrt{2e^{-j\frac{\pi}{4}}} \cdot e^{-j\frac{\pi}{2}} = \sqrt{2e^{j\left(-\frac{\pi}{4} - \frac{\pi}{2} - \frac{\pi}{2}\right)}} \]

\[ \tilde{V} = \sqrt{2e^{\frac{-j5\pi}{4}}} \]
Chapter 10: Steady-State
Sinusoidal Analysis

\[ V(t) = V_m \cos(\omega t + \phi) \]
Solving Sinusoidal Steady State Circuits

STEP #1: Transform all sources to the phasor domain using the phasor transform

\[ V(t) = V_m \cos(\omega t + \phi) \]  \[ \tilde{V} = V_m \cdot e^{j\phi} \]
Solving Sinusoidal Steady State Circuits

**STEP #2:** Transform all components (i.e. resistors, inductors and capacitors) into their complex impedance values.

<table>
<thead>
<tr>
<th>Component</th>
<th>Impedance, $Z, \Omega$</th>
<th>Admittance, $Y, \text{sieman}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>$Z = \frac{\tilde{V}}{\tilde{I}} = R$</td>
<td>$Y = \frac{\tilde{I}}{\tilde{V}} = \frac{1}{R}$</td>
</tr>
<tr>
<td>Inductor</td>
<td>$Z = \frac{\tilde{V}}{\tilde{I}} = j\omega L$</td>
<td>$Y = \frac{\tilde{I}}{\tilde{V}} = \frac{1}{j\omega L}$</td>
</tr>
<tr>
<td>Capacitor</td>
<td>$Z = \frac{\tilde{V}}{\tilde{I}} = \frac{1}{j\omega C}$</td>
<td>$Y = \frac{\tilde{I}}{\tilde{V}} = j\omega C$</td>
</tr>
</tbody>
</table>
Solving Sinusoidal Steady State Circuits

STEP #3: Using Kirchhoff's laws and Ohm’s law for impedances solve for the unknown phasor quantities.

\[ \tilde{V} = 15 \cdot e^{j \frac{\pi}{2}} \]

\[ -15.9j \Omega \]

\[ 10 \Omega \]

\[ 62.8j \Omega \]

\[ 31.4j \Omega \]

\[ \tilde{I} \]
Solving Sinusoidal Steady State Circuits

STEP #4: Transform phasor results back into the time domain.

\[ i(t) = \text{Re}\{I e^{j\omega t}\} \]
Solving Sinusoidal Steady State Circuits

Examples

\[ v(t) = A \cos(\omega t) \]

Find \( v_R(t) \)
Examples

STEPS #1 and #2: Transform all sources to the phasor domain using the phasor transform and all components to impedances
Examples

STEPS #3: Solve for the phasor output voltage using KCL, KVL, .....
Examples

STEPS #3: Solve for the phasor output voltage using KCL, KVL, .....

\[
\begin{align*}
A \cdot e^{j\theta} & = \frac{1}{j\omega C} \left( j\omega L \cdot \frac{1}{j\omega C} \right) \\
Z_{eq} & = \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}}
\end{align*}
\]
Examples

STEPS #3: Solve for the phasor output voltage using KCL, KVL, .....
Examples

STEPS #3: Solve for the phasor output voltage using KCL, KVL, ..... 

What does this equivalent impedance look like as a function of frequency?

\[ Z_{eq} = \frac{j\omega L}{1-\omega^2 LC} \]
**Examples**

**STEPS #3:** Solve for the phasor output voltage using KCL, KVL, ..... 

\[ Z_{eq} = \frac{j\omega L}{1 - \omega^2 LC} \]

At \( \omega = 0 \) what is \( Z_{eq} = \)?

At \( \omega = \infty \) what is \( Z_{eq} = \)?
Examples

STEPS #3: Solve for the phasor output voltage using KCL, KVL, .....
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Examples

STEPS #4: Convert back to the time domain

\[ \tilde{V}_R = \frac{A \cdot R}{\sqrt{R^2 + \left(\frac{\omega L}{1 - \omega^2 LC}\right)^2}} e^{-j \tan^{-1}\left(\frac{R}{\omega L}\right)} \]

\[ v_R(t) = \frac{A \cdot R}{\sqrt{R^2 + \left(\frac{\omega L}{1 - \omega^2 LC}\right)^2}} \cos(\omega t - \tan^{-1}\left(\frac{R \cdot (1 - \omega^2 LC)}{\omega L}\right)) \]
Examples

Band Reject Filter

\[ A = \frac{A \cdot R}{\sqrt{R^2 + \left(\frac{\omega L}{1 - \omega^2 LC}\right)^2}} \]

Resonant frequency

\[ \omega_o = \frac{1}{\sqrt{LC}} \]

\( \omega = 0 \)

\( \omega = \text{big} \)
Examples: \( L=C=1, \ R=1 \)
Examples: $L=C=1$, $R=10$
Examples

**STEPS #3:** Solve for the phasor output voltage using KCL, KVL, ..... 

\[ \tilde{V}_{LC} = A \frac{j\omega L}{R + \frac{j\omega L}{1 - \omega^2 LC}} = A \frac{R}{1 - j \frac{R \cdot (1 - \omega^2 LC)}{\omega L}} = \frac{A}{\sqrt{1 + \left(\frac{R \cdot (1 - \omega^2 LC)}{\omega L}\right)^2}} e^{j \tan^{-1}\left(\frac{R \cdot (1 - \omega^2 LC)}{\omega L}\right)} \]
Examples

STEPS #4: Convert back to the time domain

\[
\tilde{V}_{LC} = \frac{A}{\sqrt{1 + \left(\frac{R(1 - \omega^2 LC)}{\omega L}\right)^2}} e^{j \tan^{-1}\left(\frac{R(1 - \omega^2 LC)}{\omega L}\right)}
\]

\[
v_{LC}(t) = \frac{A}{\sqrt{1 + \left(\frac{R(1 - \omega^2 LC)}{\omega L}\right)^2}} \cos(\omega t + \tan^{-1}\left(\frac{R(1 - \omega^2 LC)}{\omega L}\right))
\]
Examples

\[ A = \frac{1}{\sqrt{1 + \left( \frac{R \cdot (1 - \omega^2 LC)}{\omega L} \right)^2}} \]

- **Resonant frequency**
  \[ \omega_o = \frac{1}{\sqrt{LC}} \]

- **Bandpass filter**
Examples: $L=C=1$, $R=10$
Solve for the time varying steady-state voltage $i_x(t)$
STEPS #1 and #2: Transform all sources to the phasor domain using the phasor transform and all components to impedances.

[Diagram showing a circuit transformation]
STEPS #3: Solve for the phasor output current using mesh analysis
STEPS #3: Solve for the phasor output current using mesh analysis

By inspection: \( \tilde{I}_1 = 0.02 \)

By inspection: \( \tilde{I}_2 = 0.01 \)

KVL Loop #3: \(-200j \cdot (\tilde{I}_3 - \tilde{I}_2) + 100\tilde{I}_3 - 2 + 100 \cdot (\tilde{I}_3 - \tilde{I}_1) = 0\)
STEPS #3: Solve for the phasor output current using mesh analysis

By inspection: \( \tilde{I}_1 = 0.02 \)

By inspection: \( \tilde{I}_2 = 0.01 \)

KVL Loop #3: \(-200j \cdot (\tilde{I}_3 - \tilde{I}_2) + 100\tilde{I}_3 - 2 + 100 \cdot (\tilde{I}_3 - \tilde{I}_1) = 0\)

\[-200j \cdot (\tilde{I}_3 - 0.01) + 100\tilde{I}_3 - 2 + 100 \cdot (\tilde{I}_3 - 0.02) = 0\]

\[\tilde{I}_3 \cdot (200 - 200j) = -4 + 2j\]

\[\tilde{I}_3 = \frac{-4 + 2j}{200 - 200j} = \frac{-2 + 1j}{100(1 - j)} = \frac{\sqrt{5} \cdot e^{j \tan^{-1}(\frac{1}{2})}}{100\sqrt{2} \cdot e^{j \tan^{-1}(\frac{1}{2})}} = 0.0158 \cdot e^{j0.3218}\]
STEPS #4: Convert back to the time domain

\[ \tilde{I}_3 = \frac{-4 + 2j}{200 - 200j} = \frac{-2 + 1j}{100(1 - j)} = \frac{\sqrt{5} \cdot e^{j\tan^{-1}\left(\frac{1}{-2}\right)}}{100\sqrt{2} \cdot e^{j\tan^{-1}\left(\frac{1}{-1}\right)}} = 0.0158 \cdot e^{j0.3218} \]

\[ i_x(t) = 0.0158 \cdot \cos(1000t + 0.3218) \]
Examples

\[ v(t) = V_{in} \cos(\omega t) \]

\[ \omega = \frac{1}{\sqrt{L C}} \]

\[ v_{out}(t) \]
Examples

STEPS #1 and #2: Transform all sources to the phasor domain using the phasor transform and all components to impedances.
Examples

STEPS #3: Solve the circuit in the Phasor domain using KCL, KVL, Ohm’s law, laws of the op-amp ...
Examples

STEPS #3: Solve the circuit in the Phasor domain using KCL, KVL, Ohm’s law, laws of the op-amp ...

\[ \tilde{V}_+ = 0 \]

\[ \tilde{V}_- = \tilde{V}_+ = 0 \]

\[ \tilde{I}_1 = \frac{\tilde{V}_{\text{in}} - \tilde{V}_-}{R_1} = \frac{\tilde{V}_{\text{in}}}{R_1} \]

\[ \tilde{I}_{f1} = \frac{\tilde{V}_{\text{out}} - \tilde{V}_-}{R_f} = \frac{\tilde{V}_{\text{out}}}{R_1} \]

\[ \tilde{I}_{f2} = \frac{\tilde{V}_{\text{out}} - \tilde{V}_-}{1} = j\omega C \tilde{V}_{\text{out}} \]
Examples

STEPS #3: Solve the circuit in the Phasor domain using KCL, KVL, Ohm’s law, laws of the op-amp ...

\[
\begin{align*}
\tilde{I}_1 + \tilde{I}_{f1} + \tilde{I}_{f2} &= 0_{out} \\
\frac{\tilde{V}_{in}}{R_1} + \frac{\tilde{V}_{out}}{R_f} + j\omega C\tilde{V}_{out} &= 0 \\
\tilde{V}_{out}\left(\frac{1}{R_f} + j\omega C\right) &= -\frac{\tilde{V}_{in}}{R_1}
\end{align*}
\]
Examples

STEPS #3: Solve the circuit in the Phasor domain using KCL, KVL, Ohm's law, laws of the op-amp ...

\[ \tilde{V}_{out} \left( \frac{1}{R_f} + j \omega C \right) = -\frac{\tilde{V}_{in}}{R_1} \]

\[ \tilde{V}_{out} \left( \frac{1 + j \omega R_f C}{R_f} \right) = -\frac{\tilde{V}_{in}}{R_1} \]

\[ \tilde{V}_{out} = -\frac{R_f}{R_1} \cdot \frac{1}{1 + j \omega R_f C} \tilde{V}_{in} \]

\[ \frac{\tilde{V}_{out}}{\tilde{V}_{in}} = -\frac{R_f}{R_1} \cdot \frac{1}{\sqrt{1 + (\omega R_f C)^2}} e^{-j \tan^{-1}(\omega R_f C)} \]
Examples

STEPS #4: Convert back to the time domain

\[ \frac{\tilde{V}_{out}}{\tilde{V}_{in}} = -\frac{R_f}{R_1} \cdot \frac{1}{\sqrt{1 + (\omega R_f C)^2}} e^{-j\tan^{-1}(\omega R_f C)} \]

\[ v_{out}(t) = -\frac{R_f}{R_1} \cdot \frac{V_{in}}{\sqrt{1 + (\omega R_f C)^2}} \cos(\omega t - \tan^{-1}(\omega R_f C)) \]

What kind of filter is this?
Examples

\[ V_{in} \frac{R_f}{R_1} \]

\[ V_{in} \frac{R_f}{R_1} \cdot \frac{1}{\sqrt{2}} \]

\[ \frac{R_f}{R_1} \cdot \frac{V_{in}}{\sqrt{1 + (\omega R_f C)^2}} \]

\[ \omega = 0 \quad \omega_c = \frac{1}{R_f C} \quad \omega = \text{big} \]