ASSIGNMENT #5 due Monday October 13

1. Haykin Problem 4.19
2. Haykin Problem 4.21
3. Haykin Problem 4.23
4. Haykin Problem 7.1
5. Bandlimited white noise N(t) has spectral density $S_W(f) = 10^{-6}$ V$^2$/Hz over the frequency range -100 kHz to 100 kHz.
   a) What is the rms value of the noise?
   b) Find $R_W(t)$. At what spacings are $n(t)$ and $n(t+\tau)$ uncorrelated?
6. The multitone modulating signal $m(t) = K(2\cos(\omega_m t) + \cos(2\omega_m t) + 3 \cos(5\omega_m t))$ is the input to an AM system.
   a) Find $K$ so that $m(t)$ is properly normalized (i.e., to prevent overmodulation).
   b) Plot the frequency spectrum of the transmitted signal $s(t)$.
   c) Calculate $P_{side}/P_{carrier}$ and $P_{carrier}/P_{total}$ (P represents power.)
SOLUTIONS TO ASSIGNMENT #5

1. Haykin Problem #4.19
   
   a) The dc power of the random process $X(t)$ is determined by the delta function contained in the power spectral density $S_x(f)$ at $f = 0$.
   
   $$ P_{dc} = 1 $$
   
   b) The ac power equals the total area under the curve of $S_x(f)$, excluding the delta function at $f = 0$.
   
   $$ P_{ac} = 2 \times (1) \times (2) = 4 $$

2. Haykin Problem #4.21

   ![Diagram](image)

   a) $S_w(f) = \{ \begin{array}{ll} \frac{N_0}{2} & \text{for all} f \\ \frac{N_0}{2} & |f - f_0| < \frac{B}{2} \\ 0 & \text{elsewhere} \end{array} \}

   b) $S_{i}(f) = \{ \begin{array}{ll} \frac{N_0}{4} & 1f \leq \frac{B}{2} \\ \frac{N_0}{8} & 1f - 2f_0 \leq \frac{B}{2} \\ 0 & \text{elsewhere} \end{array} \}

   c) $S_{a}(f) = \frac{4}{\pi} [ S_{i}(f+f_0) + S_{i}(f-f_0) ]$

   $$ = \{ \begin{array}{ll} \frac{N_0}{4} & 1f \leq \frac{B}{2} \\ \frac{N_0}{8} & 1f - 2f_0 \leq \frac{B}{2} \\ 0 & \text{elsewhere} \end{array} \} $$
2. cont'd) \[
S_0(f) = \begin{cases} 
N_0/4, & -B/2 \leq f \leq B/2 \\
0, & \text{elsewhere} 
\end{cases}
\]

b) \[
P_{av} = B \cdot \frac{N_0}{4} = \frac{N_0 B}{4}
\]

3. Haykin Problem #4.23

Butterworth filter \[
|H(f)|^2 = \frac{1}{1 + (f/f_o)^{2n}}
\]

a) noise equivalent bandwidth \[
B_N = \sum |H(f)|^2 df = \frac{H^2(0)}{2n \sin(\pi/2n)}
\]
\[
= \int_{0}^{\infty} \frac{1}{1 + (f/f_o)^{2n}} df = \frac{\pi f_o}{2n} \frac{\sin(\pi/2n)}{\sin(\pi/2n)}
\]
\[
= \frac{f_o}{\sin(\pi/2n)}
\]

b) \[
B_N = f_o \lim_{n \to \infty} \frac{1}{\sin(\pi/2n)} = f_o
\]

4. Haykin Problem #7.1

\[
M(t) = 20 \cos(2\pi t) \text{ volts}
\]
\[
c(t) = 50 \cos(100\pi t) \text{ volts}
\]

a) AM wave with 75% modulation \[
S(t) = 50 \left[1 + 0.75 \cos(2\pi t)\right] \cos(100\pi t)
\]
4. cont'd) 

\[ s(t) = 50 \cos(100\pi t) + 18.75 \cos(102\pi t) + 18.75 \cos(98\pi t) \]

Hence, the average power of \( s(t) \) is

\[
P_{\text{av}} = \frac{1}{a} (50)^2 + \frac{1}{a} (18.75)^2 + \frac{1}{a} (18.75)^2
\]

\[
= 1250 + 351.56 = 1601.56
\]

5.

\[ S_w(f) = \begin{cases} 
10^{-6} \text{V}^2/\text{Hz}, & |f| \leq 100 \text{ kHz} \\
0, & \text{elsewhere}
\end{cases} \]

a) \text{rms value} = \sigma_n

\[
\sigma_n^2 = \mathbb{E}[n(t)n(t)] - \mathbb{E}^2 - \text{valid assumption}
\]

\[
= R_w(0) = \int_{-\infty}^{\infty} S_w(f) df = 2kB = 0.2 \text{ V}^2
\]

\text{rms value} = \sqrt{0.2} = 0.447 \text{ V}

b) \[ R_w(x) = \mathcal{F}^{-1}[S_w(f)] \]

\[
= k \int_{-B}^{B} e^{j2\pi f} df = \frac{k}{\pi} \sin \pi B T
\]
\[ r_w(t) = 0.2 \sin \frac{2\pi \cdot 10^5 t}{2\pi \cdot 10^3 t} \]

\( n(t) \) and \( n(t+\tau) \) are uncorrelated if \( r_w(t) = 0 \)

\[ r_w(t) = 0 \text{ if } 2\pi B \tau = n\pi, n \neq 0 \]

\[ \tau = \frac{n}{2B} = \frac{n}{2 \cdot 10^5 \text{Hz}} = n \cdot 5 \mu \text{sec}, n = 1, 2, \ldots \]

\[ M(t) = K(2 \cos \omega_m t + \cos 3\omega_m t + 3 \cos 5\omega_m t) \]

a.) Normalize \( m(t) \), i.e., \( |m(t)| \leq 1 \Rightarrow |m(t)|_{max} = 1 \)

\[ |m(t)|_{max} = K(2+1+3) = 6K \Rightarrow 1 \]

\[ K = \frac{1}{6} \] (or more generally \( K = \frac{1}{6k_0} \))

b.) \( S(t) = A_e [1 + k_a m(t)] \cos \omega_w t \)

\[ = A_e \cos \omega_w t + A_e k_a m(t) \cos \omega_w t \]

where

\[ m(t) \cos \omega_w t = \frac{K}{2} \left( a \cos (\omega_w + \omega_m) t + a \cos (\omega_w - \omega_m) t \right) + \cos (\omega_w + 2\omega_m) t + \cos (\omega_w - 2\omega_m) t + 3 \cos (\omega_w + 5\omega_m) t + 3 \cos (\omega_w - 5\omega_m) t \]

\[ S(f) = \frac{A_e}{2} \left[ S(f - f_e) + S(f + f_e) \right] \]

\[ + \frac{A_e k_0 K}{4} \left[ a S(f - f_e - f_m) + a S(f - f_e + f_m) + 2 S(f + f_e - f_m) + 2 S(f + f_e + f_m) \\
+ S(f - f_e - 2f_m) + S(f - f_e + 2f_m) + S(f + f_e - 2f_m) + S(f + f_e + 2f_m) + 3 S(f - f_e - 5f_m) + 3 S(f - f_e + 5f_m) + 3 S(f + f_e - 5f_m) + 3 S(f + f_e + 5f_m) \right] \]
- for convenience, only positive frequencies will be plotted

\[ S(f) \]

\[ a_c/a \]

\[ f_a - s_f_m \]
\[ f_a - d_f_m \]
\[ f_a - f_m \]
\[ f_e \]
\[ f_e + f_m \]
\[ f_e + s_f_m \]

\( p_{e\text{mer}} = a_c^2 / 2 \)

\[ p_{\text{side}} = \text{group all sidebands together - only one side} \]

\[ \text{assume } k_a = 1 \text{ for simplicity} \]

\[ = 2 \left( \frac{a_c^2 k_a^2}{k_0} \right) \left( 4 + 1 + 9 \right) = \frac{14}{8} a_c^2 k_a^2 = \frac{7}{4} a_c^2 k_a^2 \]

\[ p_{\text{total}} = p_{e\text{mer}} + a p_{\text{side}} \]

\[ = \frac{a_c^2}{2} \left( 1 + 7 k_a^2 \right) \]

\[ \frac{p_{\text{side}}}{p_{e\text{mer}}} = 2 \frac{\frac{3}{2} a_c^2 k_a^2}{a_c^2 / 2} = \frac{7}{2} k_a^2 \leq \frac{7}{72} \]

\[ \frac{p_{e\text{mer}}}{p_{\text{total}}} = \frac{a_c^2 / 2}{a_c^2 / 2 \left( 1 + 7 k_a^2 \right)} = \frac{1}{1 + 7 k_a^2} \geq \frac{36}{43} \approx 84\% \]

i.e. most of the transmitted power is in the carrier which provides no information \( \Rightarrow \) wasted!
ELEG403 COMMUNICATIONS SYSTEMS ENGINEERING

ASSIGNMENT #6 due Monday October 20

1. Haykin Problem 7.13
2. Haykin Problem 7.14
3. Haykin Problem 7.18
4. Haykin Problem 7.30
5. Haykin Problem 7.38
1. Haykin Problem #7.13

DSBSC with coherent detection but mismatched frequency

(a) Multiplying the DSBSC signal by the local oscillator gives

\[ S_v(t) = A_v m(t) \cos(\omega_c t) \cos(\omega_m t + \omega_c t) \]

\[ = \frac{A_v}{2} m(t) \left[ \cos(\omega_m t + \omega_c t) + \cos(\omega_m t - \omega_c t) \right] \]

After lowpass filtering,

\[ S_a(t) = \frac{A_v}{2} m(t) \cos(\omega_m t) \]

Thus, the output signal is the message signal modulated by a sinusoid of frequency \( \omega_m \).

(b) \( m(t) = \cos(\omega_m t) \)

\[ S_a(t) = \frac{A_v}{2} \cos(\omega_m t) \left[ \cos(\omega_m - \omega_c) t + \cos(\omega_m + \omega_c) t \right] \]

\[ \rightarrow 1/\omega_m \]
2. Haykin Problem # 7.14

\[ s(t) = A_c \cos(\pi f_c t + \phi) + \cos(\pi f_0 t) \; m(t) \rightarrow \text{envelope detector} \]

\[ = m(t) \cos(\pi f_0 t) + A_c \cos \phi \cos(\pi f_0 t) \]

\[ - A_c \sin \phi \sin(\pi f_0 t) \]

\[ = \left[ m(t) + A_c \cos \phi \right] - A_c \sin \phi \sin(\pi f_0 t) \]

envelope \( = a(t) = \sqrt{(m(t) + A_c \cos \phi)^2 + (A_c \sin \phi)^2} \)

\[ = \sqrt{m^2(t) + 2A_c \cos \phi m(t) + A_c^2} \]

a) \( \phi = 0 \)

\[ a(t) = \sqrt{(m(t) + A_c)^2} = A_c + m(t) \]

b) \( \phi \neq 0 \) and \( |m(t)| \ll A_c/2 \)

\[ a(t) = \sqrt{m^2(t) + 2A_c \cos \phi m(t) + A_c^2} \approx \sqrt{2A_c \cos \phi m(t) + A_c^2} \]

\[ = A_c \sqrt{1 + \left( \frac{2}{A_c} m(t) \right) \cos \phi} \]

\[ \approx A_c \left( 1 + \frac{\cos \phi}{A_c} m(t) \right) = A_c + \cos \phi m(t) \]

Here, again, we find that except for the dc component, \( A_c \), the envelope detector output is proportional to \( m(t) \) for a constant value of \( \phi \).
3. Haykin Problem #7.18

A two-stage modulator for SSB (Fig. 7.18)

\[ m(t) \rightarrow V(t) \rightarrow BBF \rightarrow X \rightarrow BPF \rightarrow SSB \]

\[ A_1 \cos(w)+ \rightarrow A_2 \cos(w) \]

Message signal: Voice 0.3 kHz → 3.4 kHz

\[ f_1 = 100 \text{ kHz} \]

\[ f_2 = 10 \text{ MHz} \]

a) Spectrum of \( V_1(t) \) - output of first product modulator

b) - Lower sideband occupies 96.6 to 99.7 kHz → suppressed by filter
- Upper sideband occupies 100.3 to 103.4 kHz → assume this is passed by filter

Spectrum of \( V_2(t) \) - output of second product modulator

- Lower sideband occupies 9.896 to 9.897 MHz
- Upper sideband occupies 10.103 to 10.104 MHz → assume filter

a) Passband of filter #1 100.3 - 103.4 kHz

Guardband of filter #1 99.7 - 100.3 kHz

Passband of filter #2 10.1003 - 10.1034 MHz

Guardband of filter #2 9.8997 - 10.1003 MHz
4. Haykin Problem #7.30

The instantaneous frequency of the mixer output is shown.

The presence of negative frequency merely indicates that the phasor representing the difference frequency at the mixer output has reversed its direction of rotation.

Let $N$ denote the number of beat cycles in one period. Then, noting that $N$ is equal to the shaded region shown above,

$$N = 2 \left[ 4\Delta f \cdot f_0 \gamma \left( \frac{f_0}{2f_0} - \gamma \right) + 2\Delta f \cdot f_0 \gamma^2 \right]$$

$$= 4\Delta f \cdot \gamma \left( 1 - f_0 \gamma \right)$$

Since $f_0 \gamma \ll 1$, $N \approx 4\Delta f \cdot \gamma$

Therefore, the number of beat cycles counted over one second is equal to

$$\frac{N}{1/f_0} = 4\Delta f \cdot f_0 \gamma$$
5. Haykin Problem # 7.38

\[ f_a = 100 \text{ MHz} \]
\[ m(t) = A_m \cos 2\pi f_m t, \quad A_m = 20 \text{ V} \]
\[ f_m = 100 \text{ kHz} \]
\[ k_f = \text{ freq. sensitivity} = 25 \text{ kHz/V} \]

(a) \[ \Delta f = \text{ frequency deviation} \]
\[ = k_f A_m = 25 \times 10^3 \times 20 \text{ Hz} = 5 \times 10^5 \text{ Hz} \]

\[ \beta = \frac{\Delta f}{f_m} = \frac{5 \times 10^5 \text{ Hz}}{10^5 \text{ Hz}} = 5 \]

\[ \beta_f = 2 \Delta f \left(1 + \frac{1}{\beta}\right) = 1 \text{ MHz} \left(6/5\right) = 1.2 \text{ MHz} \]
Carson's rule.

(b) Using Fig. 7.41 for \( \beta = 5 \) gives \( \frac{B}{\Delta f} = 3 \)

\[ B = 1.5 \text{ MHz} \]

(a) Double amplitude \( \Rightarrow \Delta f = 1 \text{ MHz}, \beta = 10 \)
Carson's rule \( B = 3.2 \text{ MHz} \)

190 rule \( B = 2.75 \text{ MHz} \)

(b) Double frequency \( \Rightarrow \beta = 2.5 \)
Carson's rule \( B = 1.4 \text{ MHz} \)

190 rule \( B = 1.3 \text{ MHz} \)