ELEG403 COMMUNICATIONS SYSTEMS ENGINEERING

ASSIGNMENT #5 due Monday October 13

1. Haykin Problem 4.19

2. Haykin Problem 4.21

3. Haykin Problem 4.23

4. Haykin Problem 7.1

5. Bandlimited white noise N(t) has spectral density $S_W(f) = 10^{-6}\ V^2/Hz$ over the frequency range -100 kHz to 100 kHz.
   a) What is the rms value of the noise?
   b) Find $R_W(t)$. At what spacings are n(t) and n(t+τ) uncorrelated?

6. The multitone modulating signal $m(t) = K(2\cos(\omega_m t) + \cos(2\omega_m t) + 3\cos(5\omega_m t))$ is the input to an AM system.
   a) Find K so that m(t) is properly normalized (i.e, to prevent overmodulation).
   b) Plot the frequency spectrum of the transmitted signal s(t).
   c) Calculate $P_{side}/P_{carrier}$ and $P_{carrier}/P_{total}$. (P represents power.)

SOLUTIONS TO ASSIGNMENT #5

1. Haykin Problem #4.19

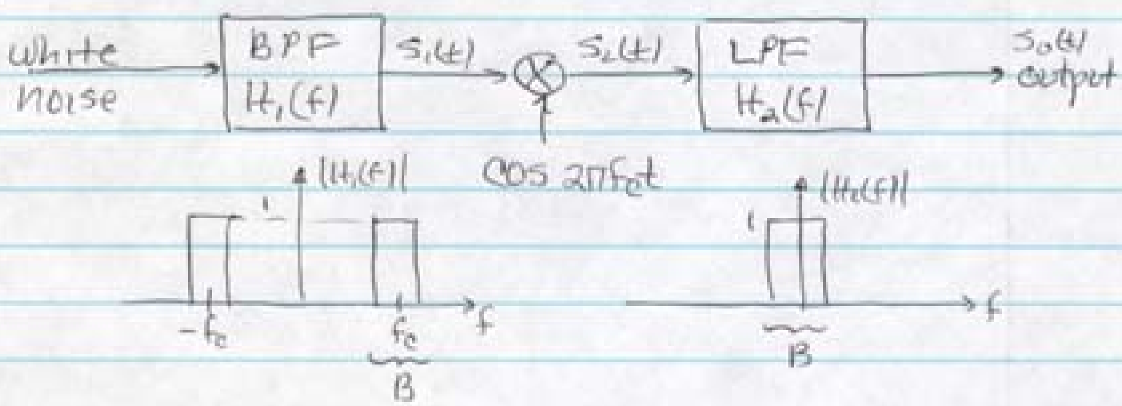
a) The dc power of the random process $X(t)$ is determined by the delta function contained in the power spectral density $S_X(f)$ at $f=0$.

$$P_{dc} = 1$$

b.) The ac power equals the total area under the curve of $S_X(f)$, excluding the delta function at $f=0$.

$$P_{ac} = 2(1)(2) = 4$$

2. Haykin Problem #4.21

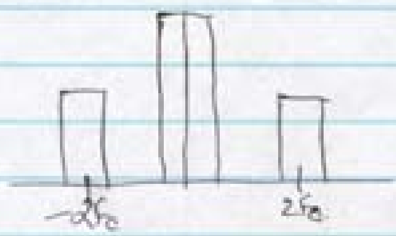


a.) $S_w(f) = \{ N_0/2$ for all f

$$S_1(f) = \begin{cases} N_0/2 & |f-f_c| \le B/2 \\ 0 & \text{elsewhere} \end{cases}$$

$$S_2(f) = \frac{1}{4}\left[S_1(f+f_c) + S_1(f-f_c)\right]$$

$$= \begin{cases} N_0/4\,, & |f| \le B/2 \\ N_0/8\,, & |f-2f_c| \le B/2 \\ 0\,, & \text{elsewhere} \end{cases}$$



2. cont'd) $\therefore \quad S_0(f) = \begin{cases} N_0/4, & -\frac{B}{2} \le f \le \frac{B}{2} \\ 0, & \text{elsewhere} \end{cases}$

b.) $P_{av} = B \cdot \frac{N_0}{4} = \frac{N_0 B}{4}$

3. Haykin Problem # 4.23

Butterworth filter $\quad |H(f)| = \dfrac{1}{\sqrt{1 + (f/f_0)^{2n}}}$

a.) noise equivalent bandwidth

$$B_N = \frac{\int_0^{\infty} |H(f)|^2 df}{H^2(0)}$$

$$= \int_0^{\infty} \frac{1}{1+(f/f_0)^{2n}} df = \frac{\pi f_0}{2n \sin(\pi/2n)}$$

$$= \frac{f_0}{\operatorname{sinc}(1/2n)}$$

b.) $B_N = f_0 \lim_{n \to \infty} \dfrac{1}{\operatorname{sinc}(1/2n)} = f_0$

4. Haykin Problem # 7.1

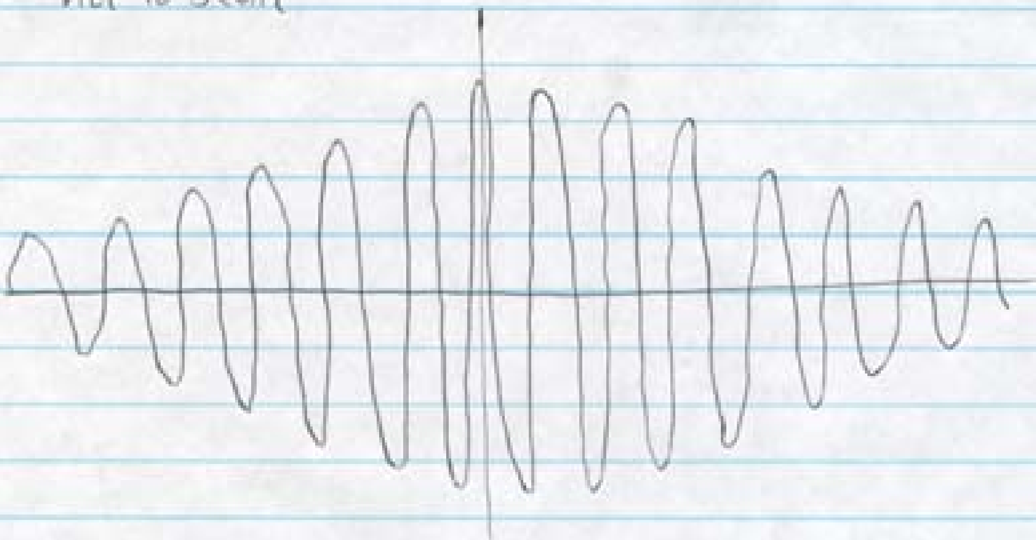
$m(t) = 20 \cos(2\pi t)$ volts

$c(t) = 50 \cos(100\pi t)$ volts

a.) AM wave with 75% modulation

$$s(t) = 50\left[1 + 0.75 \cos(2\pi t)\right] \cos 100\pi t$$

4. cont'd) not to scale



b.) $s(t) = 50\cos(100\pi t) + 18.75\cos(102\pi t) + 18.75\cos(98\pi t)$

Hence, the average power of $s(t)$ is

$$P_{av} = \frac{1}{2}(50)^2 + \frac{1}{2}(18.75)^2 + \frac{1}{2}(18.75)^2$$

$$= 1250 + 351.56 = 1601.56$$

100Ω resistor missing

5. $$S_w(f) = \begin{cases} 10^{-6}\ V^2/Hz, & |f| \leq 100\ kHz \\ 0, & \text{elsewhere} \end{cases}$$

a.) rms value $= \sigma_n$

$$\sigma_n^2 = E[n(t)n(t)] - \cancel{\bar{n}^2}$$

assume = 0 (zero mean) - valid assumption

$$= R_w(0) = \int_{-\infty}^{\infty} S_w(f)\,df = 2kB = 0.2\ V^2$$

rms value $= \sqrt{0.2V^2} = 0.447\ V$

b.) $R_w(\tau) = \mathcal{F}^{-1}[S_w(f)]$

$$= k\int_{-B}^{B} e^{j2\pi f\tau}\,df = \frac{k}{\pi\tau}\sin 2\pi B\tau$$

5. cont'd) $\cdot\ R_W(\tau) = 0.2\ \dfrac{\sin 2\pi \cdot 10^5 \tau}{2\pi \cdot 10^5 \tau}$

$\rightarrow n(t)$ and $n(t+\tau)$ are <u>uncorrelated</u> if $R_W(\tau) = 0$

$$R_W(\tau) = 0 \text{ if } 2\pi B\tau = n\pi,\ n \neq 0$$

$$\tau = \frac{n}{2B} = \frac{n}{2 \cdot 10^5\,Hz} = n \cdot 5\,\mu sec,\ n = 1, 2, \ldots$$

6. $m(t) = K(2\cos\omega_m t + \cos 2\omega_m t + 3\cos 5\omega_m t)$

a.) normalize $m(t)$, i.e., $|m(t)| \leq 1 \Rightarrow |m(t)|_{max} = 1$

$$|m(t)|_{max} = K(2+1+3) = 6K \Rightarrow 1$$

$$K = 1/6 \quad \text{(or more generally } K = 1/6k_a)$$

b.) $s(t) = A_c[1 + k_a m(t)]\cos\omega_c t$

$$= A_c \cos\omega_c t + A_c k_a m(t) \cos\omega_c t$$

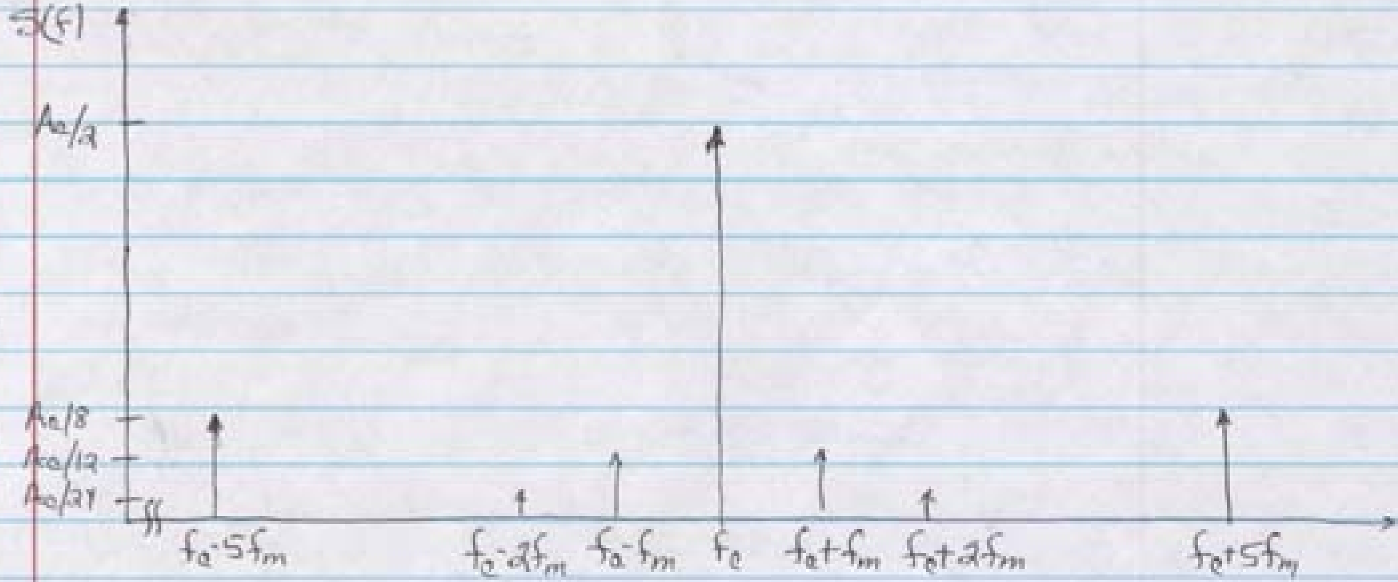
where

$$m(t)\cos\omega_c t = \frac{K}{2}\Big[2\cos(\omega_c+\omega_m)t + 2\cos(\omega_c-\omega_m)t + \cos(\omega_c+2\omega_m)t + \cos(\omega_c-2\omega_m)t + 3\cos(\omega_c+5\omega_m)t + \cos(\omega_c-5\omega_m)t\Big]$$

$$S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$
$$+ \frac{A_c k_a K}{4}\Big[2\delta(f-f_c-f_m) + 2\delta(f-f_c+f_m) + 2\delta(f+f_c-f_m) + 2\delta(f+f_c+f_m)$$
$$+ \delta(f-f_c-2f_m) + \delta(f-f_c+2f_m) + \delta(f+f_c-2f_m) + \delta(f+f_c+2f_m)$$
$$+ 3\delta(f-f_c-5f_m) + 3\delta(f-f_c+5f_m) + 3\delta(f+f_c-5f_m) + 3\delta(f+f_c+5f_m)\Big]$$

$\frac{A_c}{24}$

6.cont'd) – for convenience, only positive frequencies will be plotted



c.) $P_{carrier} = A_c^2/2$

$P_{side} \Rightarrow$ group all sidebands together – only one side
(assume ka = 1 for simplicity)

$$= 2\,\frac{A_c^2 K^2}{16}\left[4+1+9\right] = \frac{14}{8}A_c^2K^2 = \frac{7}{4}A_c^2K^2$$

$$P_{total} = P_{carrier} + 2P_{side}$$
$$= \frac{A_c^2}{2}\left(1+7K^2\right)$$

$$\frac{P_{side}}{P_{carrier}} = \frac{2\,\frac{7}{\cancel{4}}\,\cancel{A_c^2}K^2}{\cancel{A_c^2}/\cancel{2}} = \frac{7}{2}K^2 \leq \frac{7}{72}$$

$$\frac{P_{carrier}}{P_{total}} = \frac{\cancel{A_c^2}/\cancel{2}}{\cancel{A_c^2}/\cancel{2}\,(1+7K^2)} = \frac{1}{1+7K^2} \geq \frac{36}{43} \approx 84\%$$

i.e. most of the transmitted power is in the carrier which provides no information $\Rightarrow$ wasted!

ELEG403 COMMUNICATIONS SYSTEMS ENGINEERING

ASSIGNMENT #6 due Monday October 20

1. Haykin Problem 7.13
2. Haykin Problem 7.14
3. Haykin Problem 7.18
4. Haykin Problem 7.30
5. Haykin Problem 7.38

SOLUTIONS TO ASSIGNMENT #6

1. Haykin Problem #7.13

DSBSC with coherent detection but mismatched frequency

a) Multiplying the DSBSC signal by the local oscillator gives

$$s_1(t) = A_c m(t) \cos(2\pi f_c t) \cos[2\pi(f_c+\Delta f)t]$$
$$= \frac{A_c}{2} m(t) \left[\cos(2\pi \Delta f t) + \cos(2\pi(2f_c+\Delta f)t)\right]$$

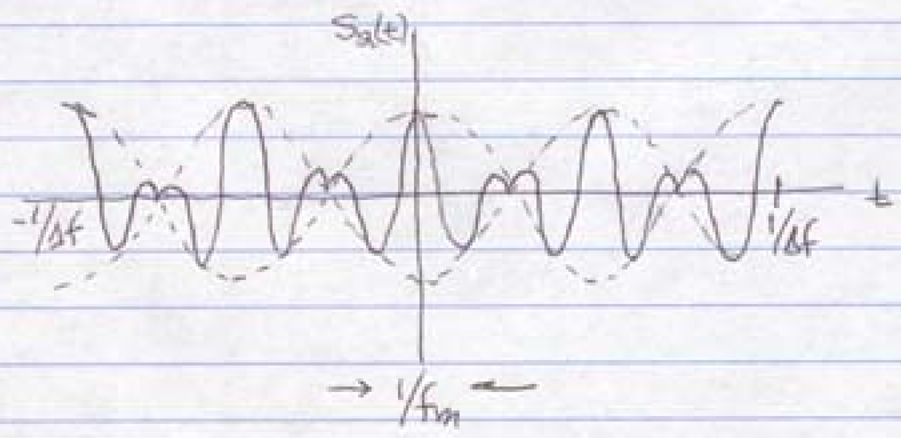
After lowpass filtering,

$$s_2(t) = \frac{A_c}{2} m(t) \cos(2\pi \Delta f t)$$

Thus, the output signal is the message signal modulated by a sinusoid of frequency Δf.

b) $m(t) = \cos(2\pi f_m t)$

$$s_2(t) = \frac{A_c}{4} m(t) [\cos 2\pi(f_m-\Delta f)t + \cos 2\pi(f_m+\Delta f)t]$$



2. Haykin Problem # 7.14

$s(t) = A_c\cos(2\pi f_c t + \phi) + \cos(2\pi f_c t)\, m(t) \longrightarrow$ envelope detector

$$= m(t)\cos(2\pi f_c t) + A_c\cos\phi\cos(2\pi f_c t) - A_c\sin\phi\sin(2\pi f_c t)$$

$$= \underbrace{[m(t) + A_c\cos\phi]}_{\cos 2\pi f_c t} - A_c\sin\phi\sin(2\pi f_c t)$$

$$\text{envelope} = a(t) = \sqrt{(m(t) + A_c\cos\phi)^2 + (A_c\sin\phi)^2}$$

$$= \sqrt{m^2(t) + 2A_c\cos\phi\, m(t) + A_c^2}$$

a) $\phi = 0$

$$a(t) = \sqrt{(m(t) + A_c)^2} = A_c + m(t)$$

b) $\phi \neq 0$ and $|m(t)| \ll A_c/2$

$$a(t) = \sqrt{m^2(t) + 2A_c\cos\phi\, m(t) + A_c^2} \approx \sqrt{2A_c\cos\phi\, m(t) + A_c^2}$$

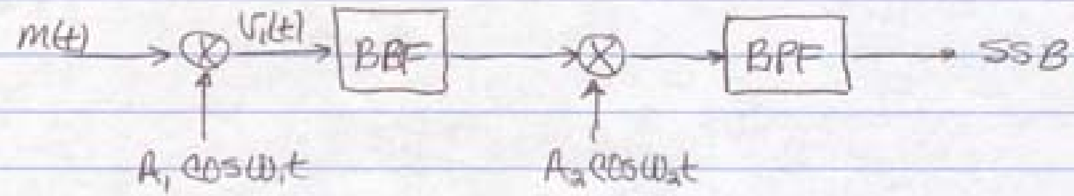
$$= A_c\sqrt{1 + \underbrace{\left(\frac{2}{A_c}m(t)\right)}_{\ll 1}\underbrace{\cos\phi}_{\leq 1}}$$

$$\approx A_c\left(1 + \frac{\cos\phi}{A_c}m(t)\right) = A_c + \cos\phi\, m(t)$$

Here, again, we find that except for the dc component, A_c, the envelope detector output is proportional to $m(t)$ for a constant value of ϕ.

3. Haykin Problem # 7.18

A two-stage modulator for SSB (Fig 7.18)



message signal : voice   0.3 kHz → 3.4 kHz

$f_1 = 100$ kHz

$f_2 = 10$ MHz

a)
b) spectrum of $v_1(t)$ - output of first product modulator
- lower sideband occupies 96.6 to 99.7 kHz ⇒ suppressed by filter
- upper sideband occupies 100.3 to 103.4 kHz ⇒ assume this is passed by filter

spectrum of $v_2(t)$ - output of second product modulator
- lower sideband occupies 9.8966 to 9.8997 MHz
- upper sideband occupies 10.1003 to 10.1034 MHz ⇒ assume this is passed by filter

c.) passband of filter #1   100.3 - 103.4 KHz

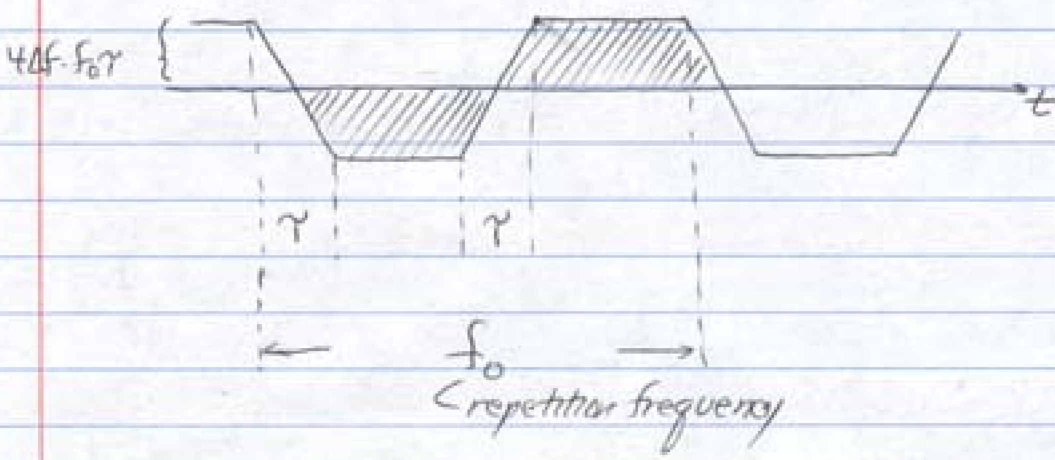
guardband of filter #1   99.7 - 100.3 KHz

passband of filter #2   10.1003 - 10.1034 MHz

guardband of filter #2   9.8997 - 10.1003 MHz

4. Haykin Problem # 7.30

The instantaneous frequency of the mixer output is a shown.



The presence of negative frequency merely indicates that the phasor representing the difference frequency at the mixer output has reversed its direction of rotation.

Let N denote the number of beat cycles in one period. Then, noting that N is equal to the shaded region shown above,

$$N = 2\left[4\Delta f \cdot f_0\tau\left(\frac{1}{2f_0} - \tau\right) + 2\Delta f \cdot f_0\tau^2\right]$$
$$= 4\Delta f \cdot \tau(1 - f_0\tau)$$

Since $f_0\tau \ll 1$, $N \approx 4\Delta f \cdot \tau$

Therefore, the number of beat cycles counted over one second is equal to

$$\frac{N}{1/f_0} = 4\Delta f \cdot f_0\tau$$

5, Haykin Problem # 7.38

$f_c = 100$ MHz
$m(t) = A_m \cos 2\pi f_m t$, $A_m = 20V$
$f_m = 100$ kHz
k_f = freq sensitivity = 25 kHz/V

a.) Δf = frequency deviation
$= k_f A_m = 25 \times 10^3 \times 20$ Hz $= 5 \times 10^5$ Hz

$$\beta = \frac{\Delta f}{f_m} = \frac{5 \times \cancel{10^5} \cancel{\text{Hz}}}{\cancel{10^5} \cancel{\text{Hz}}} = 5$$

$B_T = 2\Delta f (1 + 1/\beta) = 1 \text{ MHz } (6/5) = 1.2$ MHz
Carson's rule

b) Using Fig. 7.41 for $\beta = 5$ gives $B/\Delta f = 3$

$\therefore B = 1.5$ MHz

c.) Double amplitude $\Rightarrow$ $\Delta f = 1$ MHz, $\beta = 10$
Carson's rule $B = 2.2$ MHz
1% rule $B = 2.75$ MHz

d) Double frequency $\Rightarrow$ $\beta = 2.5$
Carson's rule $B = 1.4$ MHz
1% rule $B = 2$ MHz