ELEG403 COMMUNICATIONS SYSTEMS ENGINEERING

ASSIGNMENT #5 due Monday October 13

1. Haykin Problem 4.19

2. Haykin Problem 4.21

3. Haykin Problem 4.23

4. Haykin Problem 7.1

5. Bandlimited white noise N(t) has spectral density $S_W(f) = 10^{-6}$ $V^2$/Hz over the frequency range -100 kHz to 100 kHz.
   a) What is the rms value of the noise?
   b) Find $R_W(t)$. At what spacings are n(t) and n(t+τ) uncorrelated?

6. The multitone modulating signal $m(t) = K(2\cos(\omega_m t) + \cos(2\omega_m t) + 3\cos(5\omega_m t))$ is the input to an AM system.
   a) Find K so that m(t) is properly normalized (i.e, to prevent overmodulation).
   b) Plot the frequency spectrum of the transmitted signal s(t).
   c) Calculate $P_{side}/P_{carrier}$ and $P_{carrier}/P_{total}$. (P represents power.)

SOLUTIONS TO ASSIGNMENT #5

1. Haykin Problem #4.19

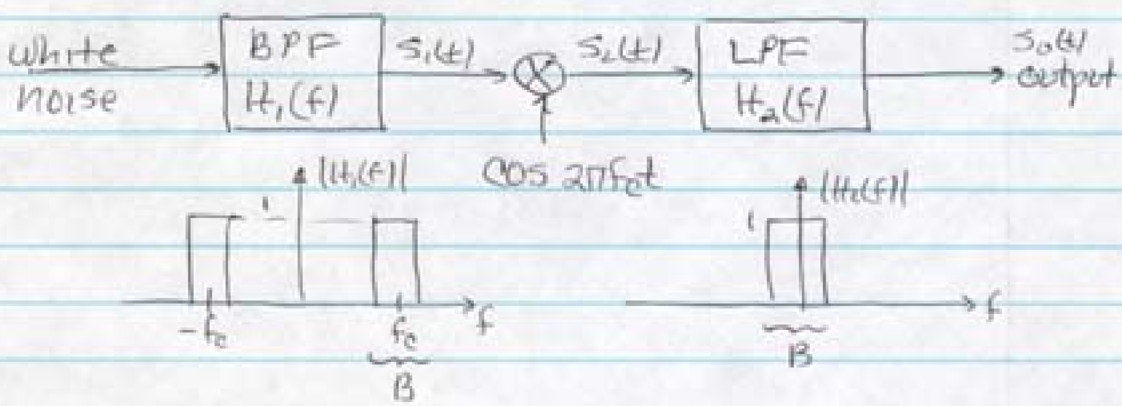
a) The dc power of the random process $X(t)$ is determined by the delta function contained in the power spectral density $S_X(f)$ at $f=0$.

$$P_{dc} = 1$$

b.) The ac power equals the total area under the curve of $S_X(f)$, excluding the delta function at $f=0$.

$$P_{ac} = 2(1)(2) = 4$$

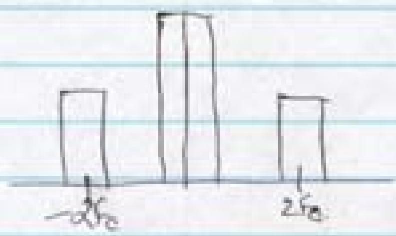
2. Haykin Problem #4.21




a.) $S_W(f) = \{ N_0/2$ for all $f$

$$S_1(f) = \begin{cases} N_0/2 & |f-f_c| \le B/2 \\ 0 & \text{elsewhere} \end{cases}$$

$$S_2(f) = \frac{1}{4}\left[S_1(f+f_c) + S_1(f-f_c)\right]$$
$$= \begin{cases} N_0/4, & |f| \le B/2 \\ N_0/8, & |f-2f_c| \le B/2 \\ 0, & \text{elsewhere} \end{cases}$$



2. cont'd) $\therefore \quad S_o(f) = \begin{cases} N_o/4 \, , & -\frac{B}{2} \le f \le \frac{B}{2} \\ 0 \, , & \text{elsewhere} \end{cases}$

b.) $P_{av} = B \cdot \frac{N_o}{4} = \frac{N_o B}{4}$

3. Haykin Problem #4.23

Butterworth filter $\quad |H(f)| = \dfrac{1}{\sqrt{1 + (f/f_o)^{2n}}}$

a.) noise equivalent bandwidth

$$B_N = \frac{\int_0^\infty |H(f)|^2 df}{H^2(0)}$$

$$= \int_0^\infty \frac{1}{1+(f/f_o)^{2n}} df = \frac{\pi f_o}{2n \sin(\pi/2n)}$$

$$= \frac{f_o}{\text{sinc}(1/2n)}$$

b.) $B_N = f_o \lim_{n \to \infty} \dfrac{1}{\text{sinc}(1/2n)} = f_o$

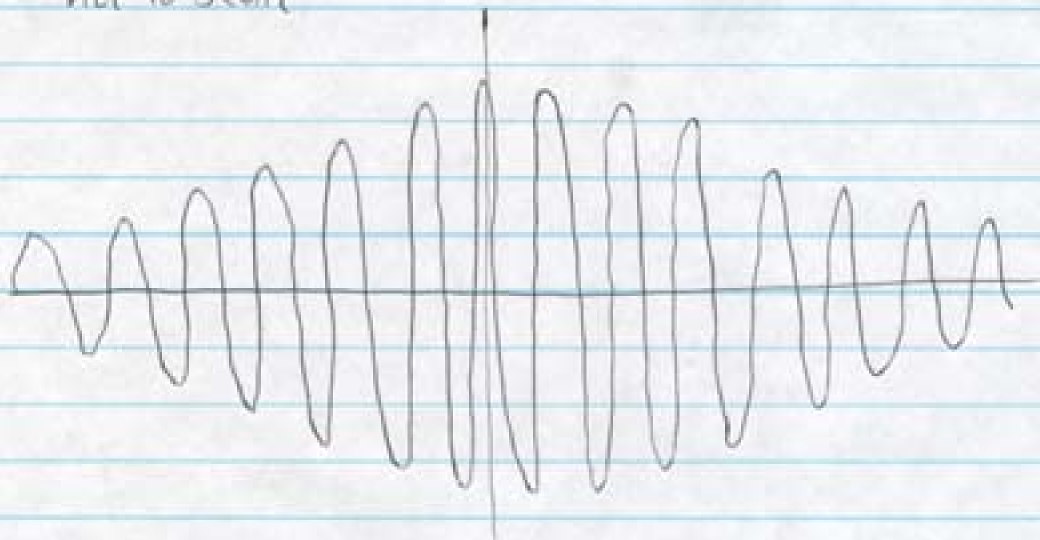
4. Haykin Problem #7.1

$$m(t) = 20 \cos(2\pi t) \text{ volts}$$
$$c(t) = 50 \cos(100\pi t) \text{ volts}$$

a.) AM wave with 75% modulation

$$s(t) = 50 \left[1 + 0.75 \cos(2\pi t)\right] \cos 100\pi t$$

4. cont'd) not to scale



b.) $s(t) = 50\cos(100\pi t) + 18.75\cos(102\pi t) + 18.75\cos(98\pi t)$

Hence, the average power of $s(t)$ is

$$P_{av} = \frac{1}{2}(50)^2 + \frac{1}{2}(18.75)^2 + \frac{1}{2}(18.75)^2$$

$$= 1250 + 351.56 = 1601.56$$ 100Ω resistor missing

5. $$S_w(f) = \begin{cases} 10^{-6}\ V^2/Hz, & |f| \leq 100\ kHz \\ 0, & \text{elsewhere} \end{cases}$$

a.) rms value $= \sigma_n$

$$\sigma_n^2 = E[n(t)n(t)] - \cancel{\bar{n}^2}$$

assume = 0 (zero mean) - valid assumption

$$= R_w(0) = \int_{-\infty}^{\infty} S_w(f)\,df = 2KB = 0.2\ V^2$$

rms value $= \sqrt{0.2V^2} = 0.447$ V

b.) $$R_w(\tau) = \mathcal{F}^{-1}[S_w(f)]$$

$$= K\int_{-B}^{B} e^{j2\pi f\tau}\,df = \frac{K}{\pi\tau}\sin 2\pi B\tau$$

5. cont'd) $\cdot\ R_W(\tau) = 0.2\ \dfrac{\sin 2\pi \cdot 10^5 \tau}{2\pi \cdot 10^5 \tau}$

$\rightarrow n(t)$ and $n(t+\tau)$ are <u>uncorrelated</u> if $R_W(\tau) = 0$

$$R_W(\tau) = 0 \text{ if } 2\pi B\tau = n\pi,\ n \neq 0$$

$$\tau = \frac{n}{2B} = \frac{n}{2 \cdot 10^5 \text{Hz}} = n \cdot 5\,\mu\text{sec},\ n = 1, 2, \ldots$$

6. $m(t) = K(2\cos\omega_m t + \cos 2\omega_m t + 3\cos 5\omega_m t)$

a.) normalize $m(t)$, i.e., $|m(t)| \leq 1 \Rightarrow |m(t)|_{max} = 1$

$$|m(t)|_{max} = K(2+1+3) = 6K \Rightarrow 1$$

$$K = 1/6 \quad (\text{or more generally } K = 1/6k_a)$$

b.) $s(t) = A_c[1 + k_a m(t)]\cos\omega_c t$

$= A_c \cos\omega_c t + A_c k_a m(t) \cos\omega_c t$

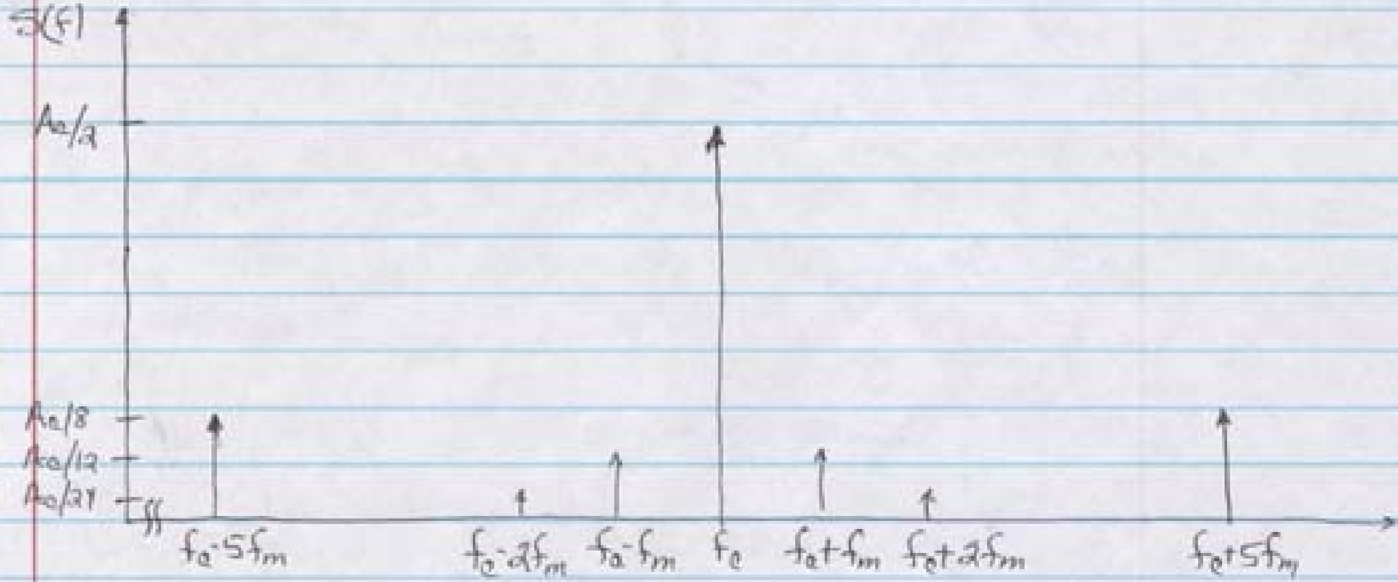
where

$$m(t)\cos\omega_c t = \frac{K}{2}\left[2\cos(\omega_c+\omega_m)t + 2\cos(\omega_c-\omega_m)t + \cos(\omega_c+2\omega_m)t + \cos(\omega_c-2\omega_m)t + 3\cos(\omega_c+5\omega_m)t + \cos(\omega_c-5\omega_m)t\right]$$

$$S(f) = \frac{A_c}{2}[\delta(f-f_c) + \delta(f+f_c)]$$
$$+ \frac{A_c k_a K}{4}\Big[2\delta(f-f_c-f_m) + 2\delta(f-f_c+f_m)$$
$$+ 2\delta(f+f_c-f_m) + 2\delta(f+f_c+f_m)$$
$$+ \delta(f-f_c-2f_m) + \delta(f-f_c+2f_m)$$
$$+ \delta(f+f_c-2f_m) + \delta(f+f_c+2f_m)$$
$$+ 3\delta(f-f_c-5f_m) + 3\delta(f-f_c+5f_m)$$
$$+ 3\delta(f+f_c-5f_m) + 3\delta(f+f_c+5f_m)\Big]$$

$\frac{A_c}{24}$

6.cont'd) – for convenience, only positive frequencies will be plotted




c.) $P_{carrier} = A_c^2/2$

$P_{side} \Rightarrow$ group all sidebands together – only one side
(assume $k_a = 1$ for simplicity)

$$= 2\,\frac{A_c^2 K^2}{16}\left[4+1+9\right] = \frac{14}{8} A_c^2 K^2 = \frac{7}{4} A_c^2 K^2$$

$$P_{total} = P_{carrier} + 2P_{side}$$
$$= \frac{A_c^2}{2}\left(1 + 7K^2\right)$$

$$\frac{P_{side}}{P_{carrier}} = \frac{2\,\frac{7}{4} A_c^2 K^2}{A_c^2/2} = \frac{7}{2} K^2 \leq \frac{7}{72}$$

$$\frac{P_{carrier}}{P_{total}} = \frac{A_c^2/2}{A_c^2/2\,(1+7K^2)} = \frac{1}{1+7K^2} \geq \frac{36}{43} \approx 84\%$$

i.e. most of the transmitted power is in the carrier which provides no information $\Rightarrow$ wasted!