ASSIGNMENT #4 due Monday October 6

1. Haykin Problem 3.9

2. Haykin Problem 3.10 (First read pp. 100-102. This is a problem dealing with
equalization, which we did not cover in class.)

3. Haykin Problem 4.1

4. Haykin Problem 4.7

5. Haykin Problems 4.13 and 4.15 (They belong together in one problem.)
SOLUTIONS TO ASSIGNMENT #4

1. Haykin Problem #3.9

a. \( H(f) = |H(f)| \cdot e^{j\phi(f)} \)
   \[ = (a_0 + a_1 \cos(\pi B)) e^{-ja_0 f t_o} \]
   \[ = \left[ a_0 + \frac{a_1}{a} e^{j\pi B} + \frac{a_1}{a} e^{-j\pi B} \right] e^{-ja_0 f t_o} \]
   \[ = a_0 e^{-ja_0 f t_o} + \frac{a_1}{a} e^{-ja_0 f (t_o - \frac{1}{2}B)} \]
   \[ + \frac{a_1}{a} e^{-ja_0 f (t_o + \frac{1}{2}B)} \]

\[ y(t) = a_0 \delta(t-t_o) + \frac{a_1}{a} \delta(t-t_o + \frac{1}{2}B) \]

\[ = \text{Output composed of} \]
- delayed replica of input modified by \( a_0 \)
- two "echoes", one preceding and the other lagging the main response by \( \frac{1}{2}B \) which represents result of distortion

b. \( H(f) = a_0 \exp[-ja_0 f t_o + jB_1 \sin(\pi \frac{f}{B})] \)

Using approximation:
\[ \exp[jB_1 \sin(\pi \frac{f}{B})] \approx 1 + jB_1 \sin(\pi \frac{f}{B}) \]

\[ H(f) = a_0 e^{-ja_0 f t_o} \left[ 1 + jB_1 \sin(\pi \frac{f}{B}) \right] \]
\[ = a_0 e^{-ja_0 f t_o} + \frac{a_1}{a} e^{-ja_0 f (t_o - \frac{1}{2}B)} \]
\[ - \frac{a_1}{a} e^{-ja_0 f (t_o + \frac{1}{2}B)} \]
y(t) = a \chi(t-t_0) + \frac{a_b}{a} \chi(t-t_0 + \frac{t}{a}) - \frac{a_b}{a} \chi(t-t_0 - \frac{t}{a})

Same interpretation as a)

2. Haykin Problem # 3.10 (Multipath Channel)

From the Figure, the channel output is

\[ y(t) = \chi(t-t_0) + a \chi(t-t_0 - \chi) \]

\[ Y(f) = X(f) \left[ 1 + a e^{-j 2\pi ft_0} \right] e^{-j 2\pi ft_0} \]

\[ H(f) = \frac{Y(f)}{X(f)} = \left[ 1 + a e^{-j 2\pi ft_0} \right] e^{-j 2\pi ft_0} \]

\[ H_{eq}(f) \cdot H_{eq}(f) = K e^{-j 2\pi ft_0} \text{ distortionless transmission} \]

\[ H_{eq}(f) = \frac{K}{1 + a e^{-j 2\pi ft_0}} \]

\[ \Rightarrow \text{using approximation} \quad K \left[ 1 - a e^{-j 2\pi ft_0} + a e^{-j 4\pi ft_0} \right] \]

\[ H_{eq}(f) = K - K a e^{-j 2\pi ft_0} + K a^2 e^{-j 4\pi ft_0} \]

For simplicity and without loss of generality, let \( K = 1 \). Then, the three tap weights are

\[ w_0 = 1 \]
\[ w_1 = -a \]
\[ w_2 = a^2 \]
3. Haykin Problem #4.1

\[ g(t) = \begin{cases} 
  e^{-at}, & t > 0 \\
  \frac{1}{a}, & t = 0 \\
  0, & t < 0 
\end{cases} \]

\[ G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt = \frac{1}{a+j2\pi f} \]

Energy spectral density

\[ \psi_g(f) = |G(f)|^2 = \frac{1}{a^2 + (2\pi f)^2} \]

\[ E_w = \int_{-W}^{W} \psi_g(f) df = \text{energy within } [-W, W] \text{ where } W = \frac{a}{2\pi} \]

\[ E_{\text{total}} = \int_{-\infty}^{\infty} \psi_g(f) df = \int_{-\infty}^{\infty} \frac{1}{a^2 + (2\pi f)^2} df \]

\[ E_w = \int_{-W}^{W} \frac{1}{a^2 + (2\pi f)^2} df = \frac{1}{a} \tan^{-1}\left(\frac{2\pi f}{a}\right) \bigg|_{-\frac{a}{2\pi}}^{\frac{a}{2\pi}} = \frac{1}{2a} \pi \]

\[ E_{\text{total}} = \frac{1}{\pi a} \tan^{-1}u \bigg|_{-\infty}^{\infty} = \frac{1}{2\pi a} \pi = \frac{1}{2a} \]

\[ \frac{E_w}{E_{\text{total}}} = \frac{1}{2} \Rightarrow 50\% \]

4. Haykin Problem #4.7

\[ g(t) = A_0 + A_1 \cos(2\pi f_1 t + \theta_1) + A_2 \cos(2\pi f_2 t + \theta_2) \]
b.) \( R_g(t) = A_0^2 + \frac{A_1^2}{\alpha} + \frac{A_2^2}{\alpha} \)

c.) \( R_g(t) \) depends only on the amplitudes \( A_0, A_1, \) and \( A_2 \) of the dc and sinusoidal components and the frequencies \( f_1 \) and \( f_2 \). The phase information contained in the phase angles of the two sinusoidal components is completely lost.
5. Haykin Problems #4.13 and 4.15

#4.13

\[ q_f(t) = A \exp \left( j \pi f_0 t \right) \text{rect} \left( \frac{t}{T} \right) \]

\[ G_T(f) = \int_{-\infty}^{\infty} q_f(t) e^{-j2\pi ft} \, dt = \int_{-T}^{T} A e^{-j2\pi ft} e^{-j2\pi ft} \, dt \]

\[ = \frac{A}{\pi (f_f - f)} \sin \left[ \pi (f_f - f) \right] \]

\[ = 2AT \sin \left[ \pi (f_f - f) \right] \]

\[ S_q(f) \text{ for finite } T = \frac{1}{2T} \left| G_T(f) \right|^2 \]

\[ = 2AT \sin^2 \left[ \pi (f_f - f) \right] \]

As \( T \to \infty, \)

\[ \lim_{T \to \infty} 2AT \sin^2 (\pi ft) = \delta(f) \]

\[ : S_q(f) = \pi \delta(f_f - f) \]

#4.15

\[ R_q(t) = F^{-1} \left[ S_q(f) \right] \]

\[ = A^2 \exp \left[ j \pi f_0 t \right] \]