ELEG403 COMMUNICATIONS SYSTEMS ENGINEERING

ASSIGNMENT #1 due Monday September 15

1. Determine whether each of the following signals is periodic and, if so, find the fundamental period $T_0$.
   a) $j\exp(j10t)$
   b) $\exp((-1+j)t)$
   c) $\cos(10t+1) - \sin(4t-1)$
   d) $(\cos(2t-x/3))^2$

2. Consider a filter with input $x(t)$ and output $y(t)$. For each of the following input-output relationships, determine whether the corresponding filter is linear, time-invariant, or both.
   a) $y(t) = t^2x(t-1)$
   b) $y(t) = x^2(t-2)$
   c) $y(t) = x(t+1) + x(t-1)$
   d) $y(t) = x(t\sin(t))$

3. Haykin Problem P2.1 #1

4. Haykin Problem P2.1 #2
SOLUTIONS TO ASSIGNMENT #1

#1 a) periodic
\[ \chi(t) = e^{-j10t} = e^{j(\pi + 10t)} \]
- repeats such that \( 10t = 2k\pi \)
  \( t = \frac{k\pi}{5}, \ k = 1, 2, 3, ... \)
  \( T_0 = \text{fundamental period} = \frac{\pi}{5} \)

b) not periodic
\[ \chi(t) = e^{-(1+j)t} = e^{-t} e^{jt} \]
monotonically decreasing with \( t \)

c) periodic
\[ \chi(t) = \cos(10t+1) - \sin(4t-1) \]
\( \leq \) sum of two sinusoids with
fundamental periods
\[ T_{0e} = \frac{2\pi}{10} = \frac{\pi}{5} \]
\[ T_{0s} = \frac{2\pi}{4} = \frac{\pi}{2} \]
The fundamental period of the composite signal
is the least common multiple of \( T_{0e} \) and \( T_{0s} \)
which is equal to \( T_0 = \pi \)

d) periodic
\[ \chi(t) = \left[ \cos(2t - \frac{\pi}{3}) \right]^2 \]
\[ = \frac{1}{2} \left[ 1 + \cos(4t - \frac{2\pi}{3}) \right] \]
constant periodic
The fundamental period \( T_0 = \frac{2\pi}{4} = \frac{\pi}{2} \)
#2. a) Linear/time-varying

To test for linearity, we calculate the output when the input is equal to \( x(t) = \alpha x_1(t) + \beta x_2(t) \). In this case,

\[
\hat{y}(t) = \alpha x_1(t-1) + \beta x_2(t-1)
\]

\[
= \alpha y_1(t) + \beta y_2(t) \Rightarrow \text{linear}
\]

To check for time invariance, we apply the signal \( x_d(t) = x(t-\tau) \) to the system input. The output is then

\[
y_d(t) = t^2 x_d(t-1)
\]

\[
= t^2 x(t-\tau) \neq (t-\tau)^2 x(t-1-\tau) = y(t-\tau)
\]

\[
\Rightarrow \text{time varying}
\]

b) Not linear/time-invariant

\[
\hat{y}(t) = \hat{x}^2(t) = \left[ \alpha x_1(t) + \beta x_2(t) \right]^2
\]

\[
\neq \alpha x_1^2(t) + \beta x_2^2(t) = \alpha y_1(t) + \beta y_2(t)
\]

\[
\Rightarrow \text{not linear}
\]

\[
y_d(t) = x_d(t-\tau) = x^2(t-2-\tau) = y(t-\tau)
\]

\[
\Rightarrow \text{time invariant}
\]

c) Linear/time-invariant

\[
\hat{y}(t) = \alpha x_1(t+1) + \beta x_2(t+1) + \alpha x_1(t-1) + \beta x_2(t-1)
\]

\[
= \alpha y_1(t) + \alpha y_2(t) + \beta y_2(t+1) + \beta y_2(t-1)
\]

\[
= y_1(t) + y_2(t) \Rightarrow \text{linear}
\]

\[
y_d(t) = x_d(t+1) + x_d(t-1)
\]

\[
= x(t+1-\tau) + x(t-1-\tau) = y(t-\tau) \Rightarrow \text{time invariant}
\]
#2 (cont.

\[ \dot{y}(t) = x_n(t) + \beta x_n(t) = \alpha y(t) + \beta y(t) \]

\[ y(t) = \chi(t) - \chi(t - \tau) \]

\[ \Rightarrow \text{time varying} \]

#3: Hoykin Problem Part 1

**Q_p(t) = periodic sequence of raised cosine pulses**

\[ Q_p(t) = \alpha_0 + 2 \sum_{n=1}^{\infty} \alpha_n \cos(nt) \quad T_0 = 2 \text{ sec} \]

\[ \alpha_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} Q_p(t) \, dt = \frac{1}{2} \int_{-\pi/2}^{\pi/2} (1 + \cos(2\pi t)) \, dt = \frac{1}{2} \]

\[ \alpha_1 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (1 + \cos(2\pi t)) \cos(\pi t) \, dt \]

\[ = \frac{1}{2} \left[ \frac{\sin(2\pi t) + \frac{1}{2} \sin(3\pi t) + \frac{1}{2} \sin(2\pi t)}{2\pi} \right]^{\pi/2}_{-\pi/2} \]

\[ = \frac{3}{2\pi} - \frac{1}{6\pi} = \frac{4}{3\pi} \]

\[ \alpha_2 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (1 + \cos(2\pi t)) \cos(2\pi t) \, dt = \frac{1}{4} \]


Haykin Problem 2A.1 4.2

Periodic pulsed RF waveform \((f_0 T_o \gg 1 \Rightarrow \text{center freq.} \gg \text{fund. freq.})\)

\[ g_{p(t)} = A_p(t) \cos(2\pi f_0 t) \]

where

\[ A_p(t) = \begin{cases} A, & -\frac{T_0}{4} \leq t \leq \frac{T_0}{4} \\ 0, & \text{remainder of period} \end{cases} \]

This just represents shift to right and left

Only need spectrum of \(g_{p(t)}\) and then shift

\[ A_p(t) = \sum_{n=-\infty}^{\infty} C_n' e^{j2\pi n f_0 t} \]

where

\[ C_n' = \frac{A}{n\pi} \sin \left( \frac{n\pi f_0}{2} \right) \]

(eq 2.17) (p Haykin)

\[ = \frac{A}{2} \sin \frac{n\pi f_0}{2} \text{ for this waveform} \]

\[ = \frac{A}{2} \text{sinc} \left( \frac{n}{2} \right) \]

\[ |C_n'| \text{ is this spectrum shifted up to } f_0 \text{ and down to } -f_0 \]