Self-similar Distributions

Sir John Tenniel; *Alice's Adventures in Wonderland,* Lewis Carroll

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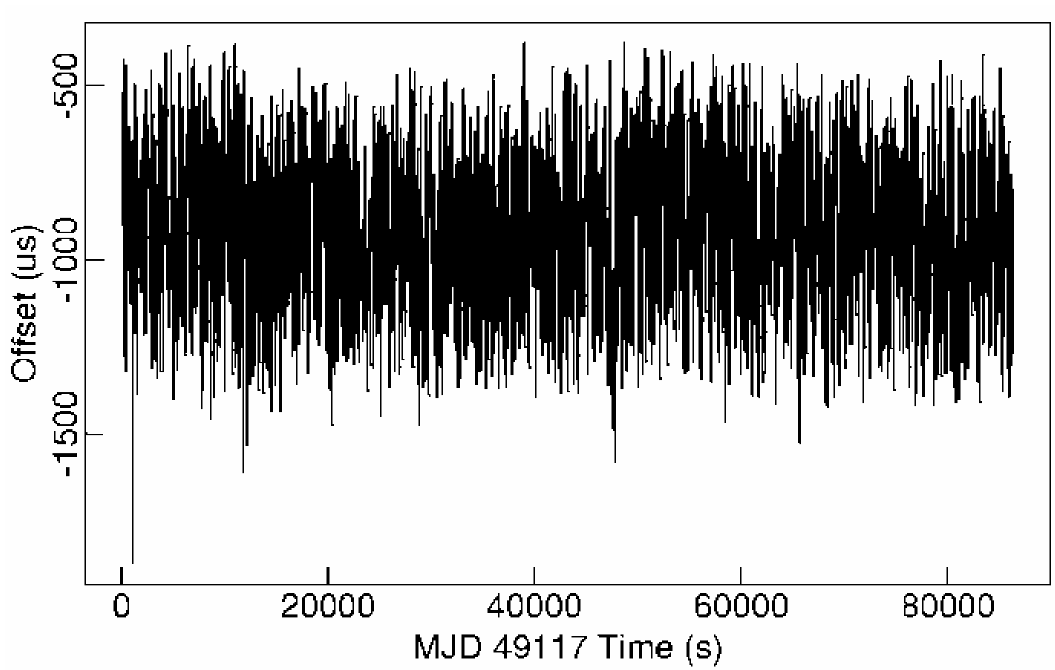
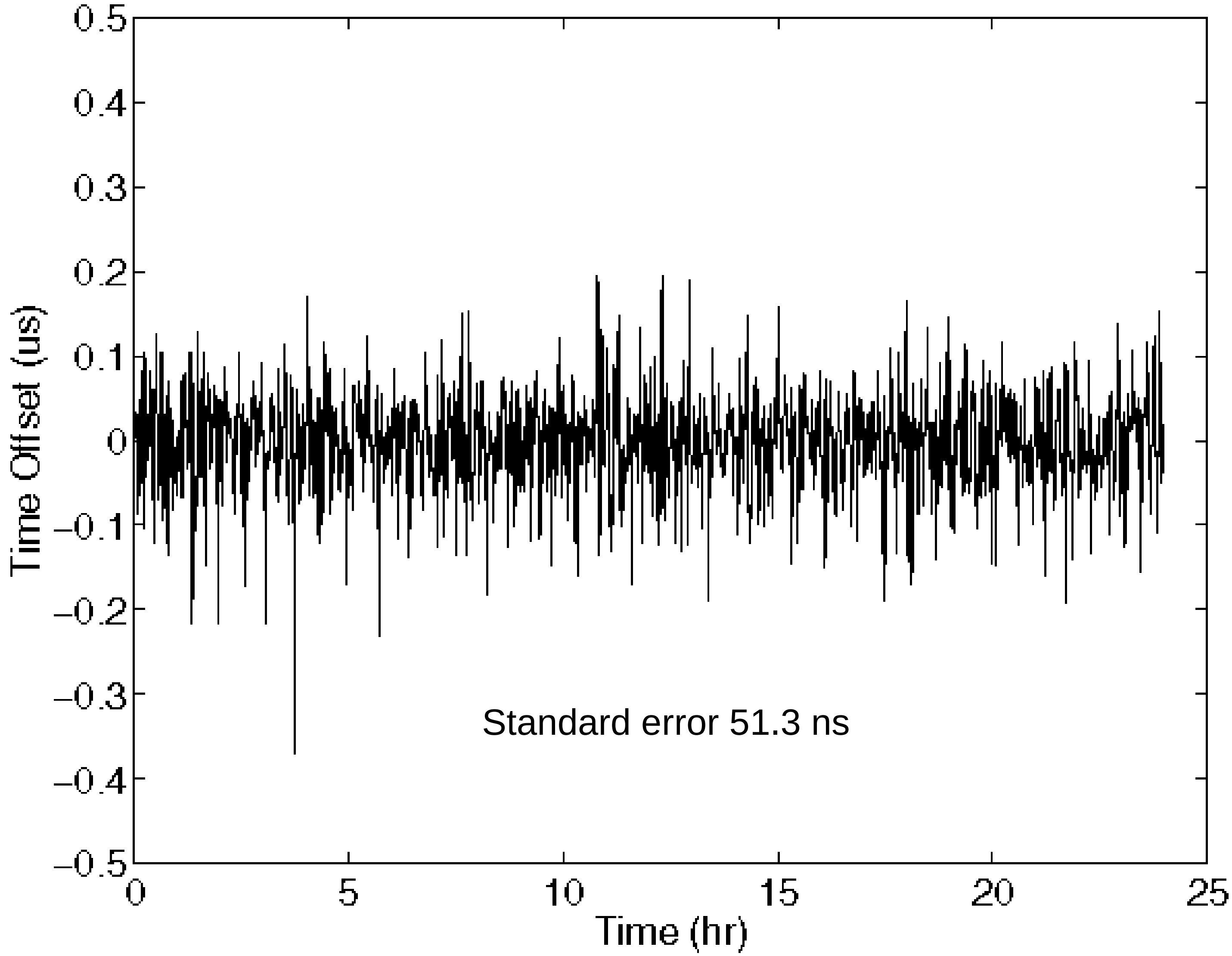
Minimize effects of serial port hardware and driver jitter

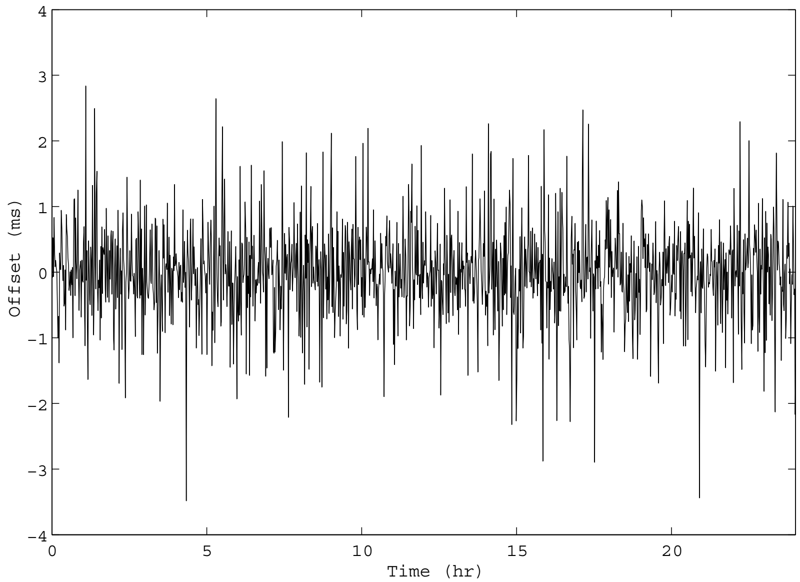


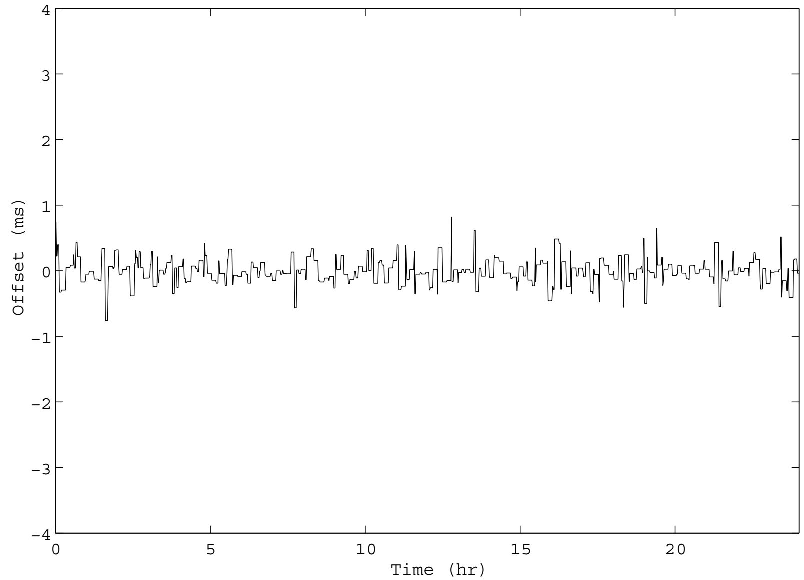
- Graph shows raw jitter of millisecond timecode and 9600-bps serial port
  - Additional latencies from 1.5 ms to 8.3 ms on SPARC IPC due to software driver and operating system; rare latency peaks over 20 ms
  - Latencies can be minimized by capturing timestamps close to the hardware
  - Jitter is reduced using median filter of 60 samples
  - Using on-second format and median filter, residual jitter is less than 50 μs

Measured PPS time error for Alpha 433



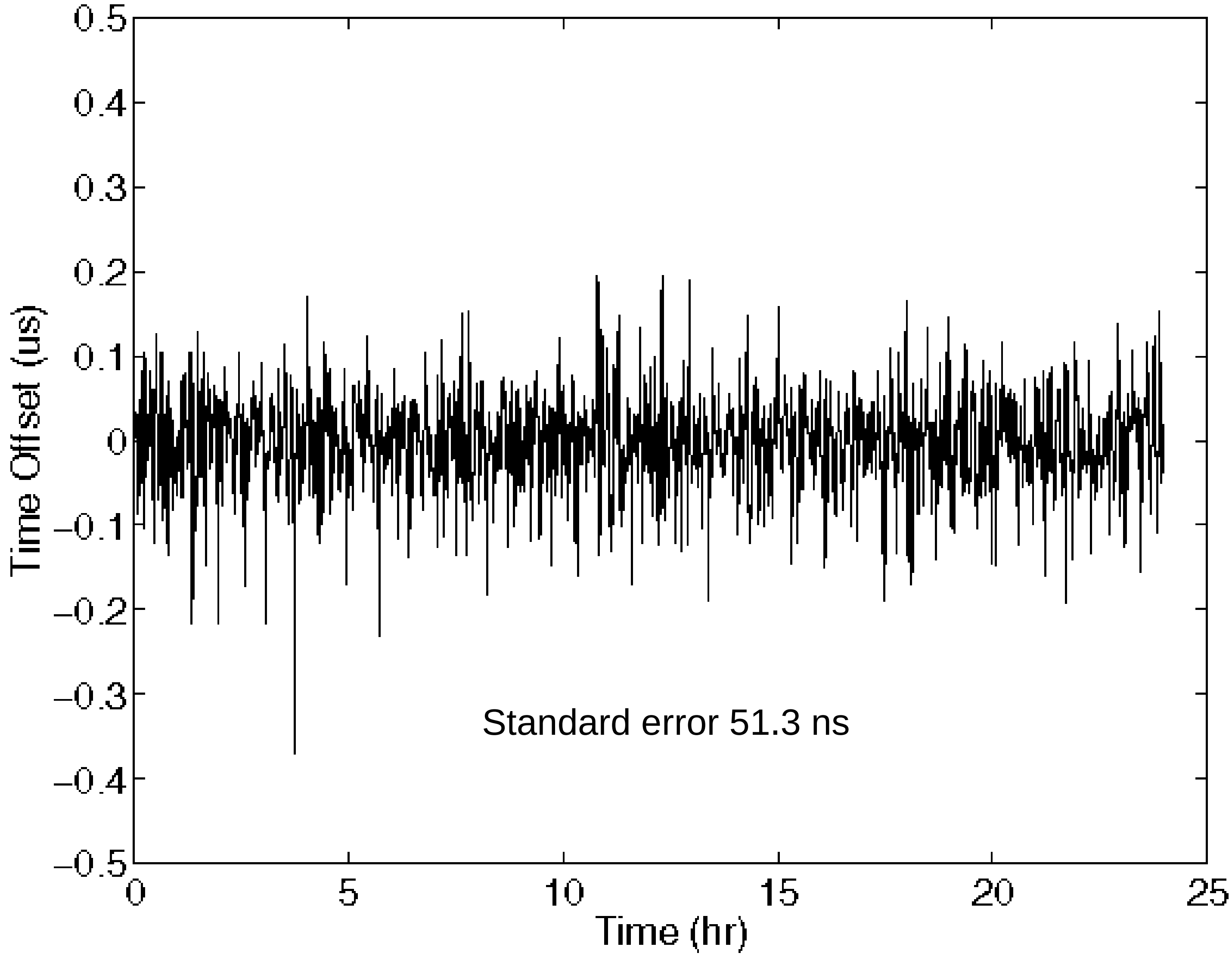
Clock filter performance



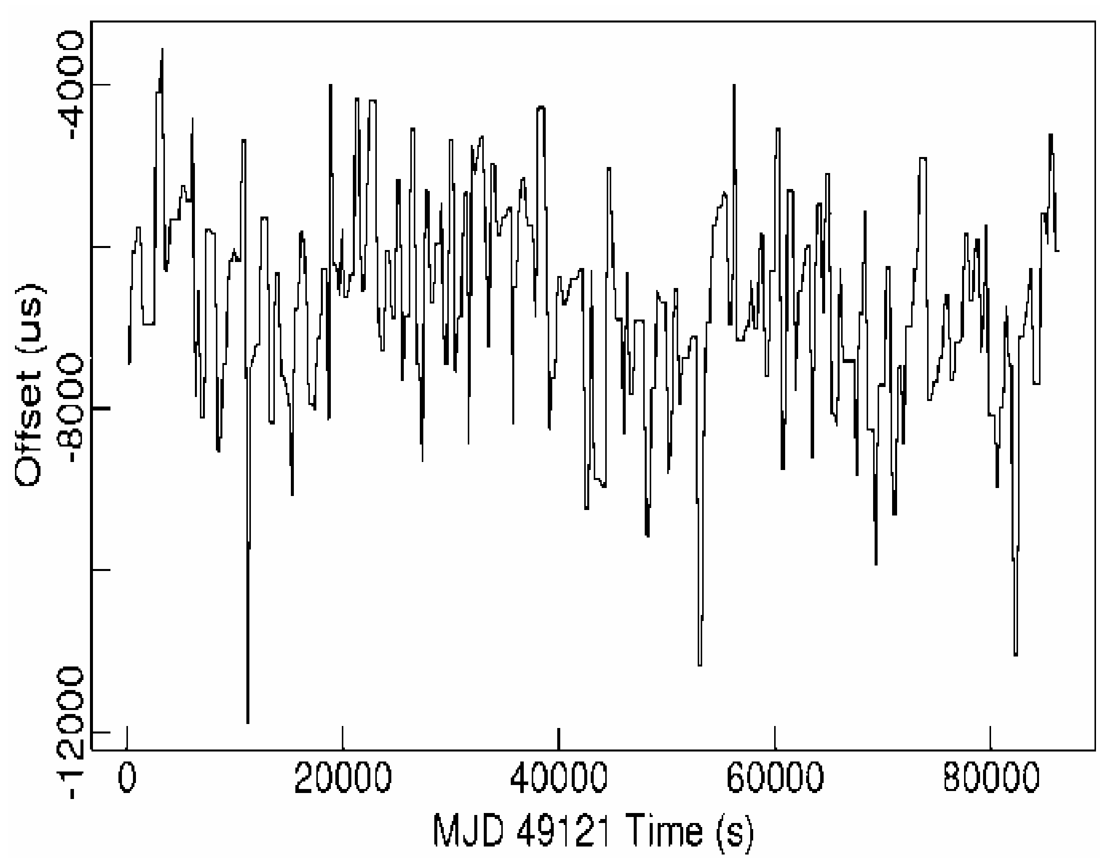


- Left figure shows raw time offsets measured for a typical path over a 24-hour period (mean error 724 μs, median error 192 μs)
- Right graph shows filtered time offsets over the same period (mean error 192 μs, median error 112 μs).
- The mean error has been reduced by 11.5 dB; the median error by 18.3 dB. This is impressive performance.

Measured PPS time error for Alpha 433

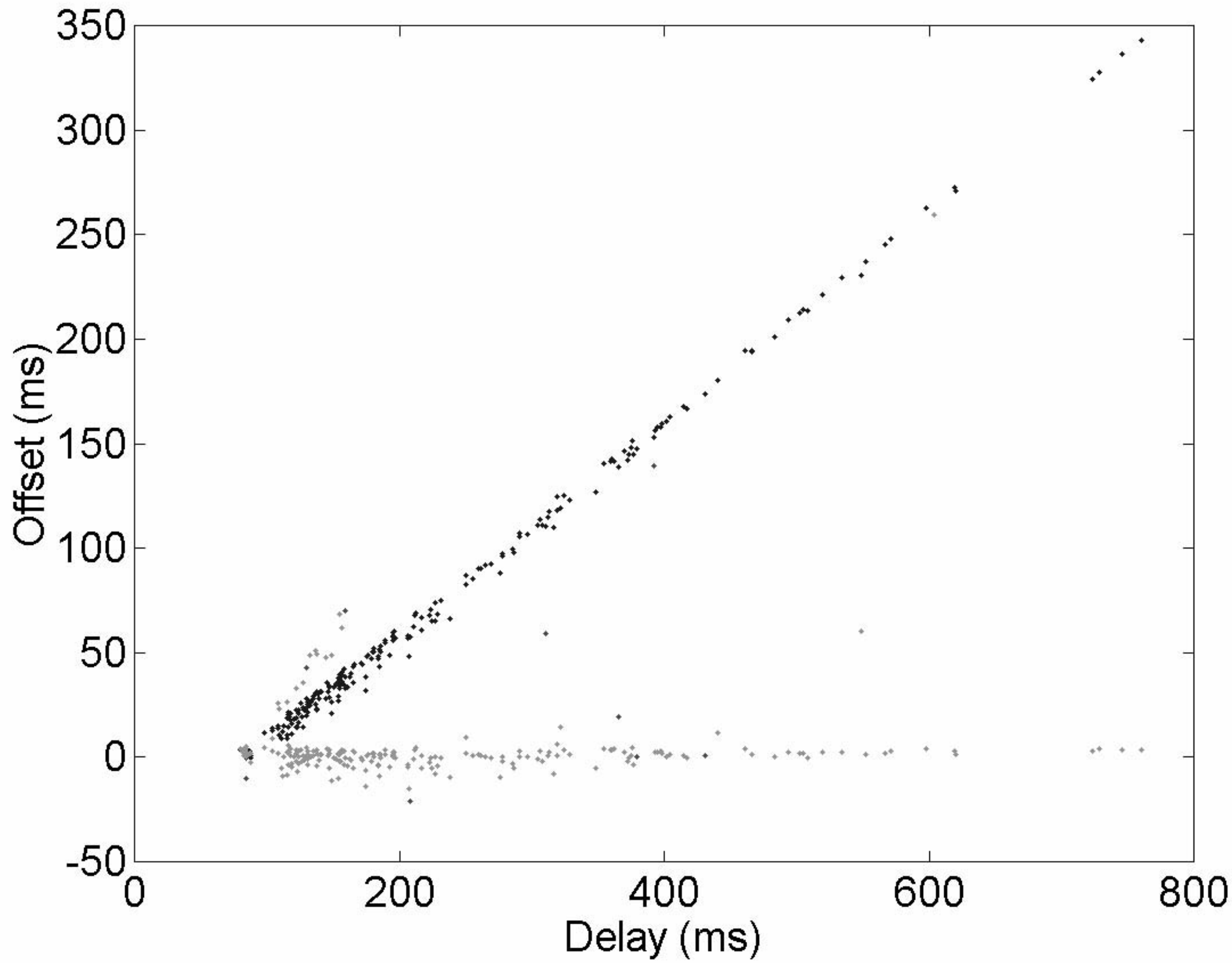


Performance with a modem and ACTS service

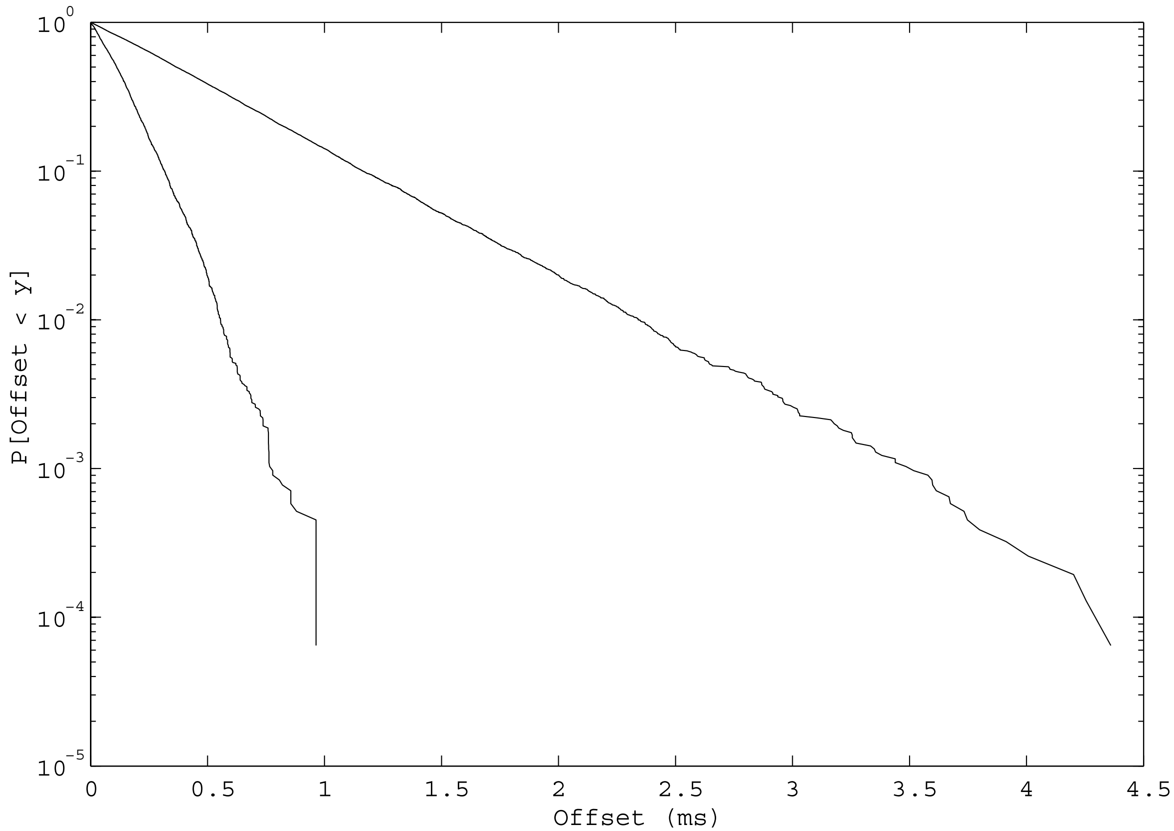


- Measurements use 2300-bps telephone modem and NIST Automated Computer Time Service (ACTS)
- Calls are placed via PSTN at 16,384-s intervals

Huff&puff wedge scattergram

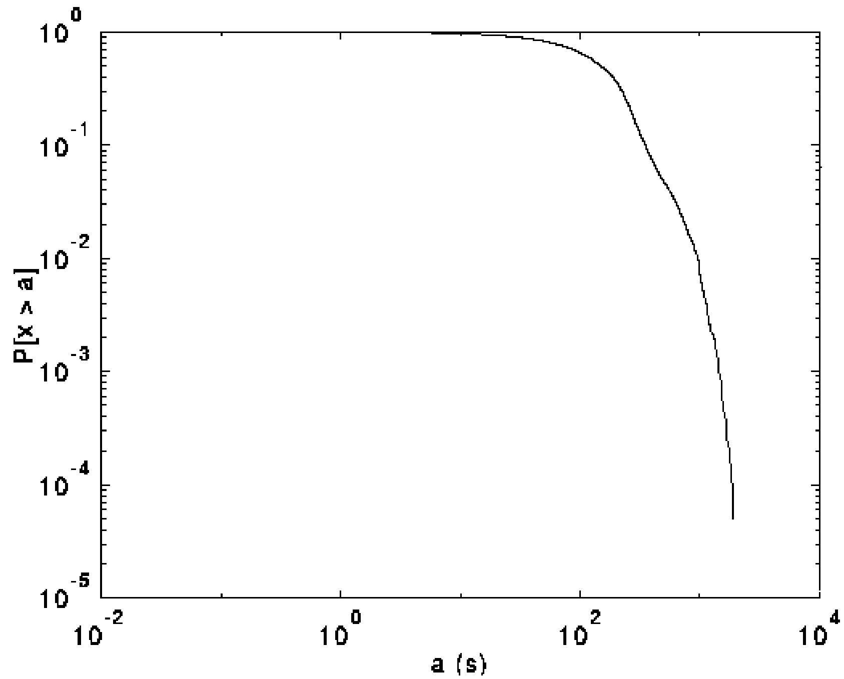


Network offset time jitter



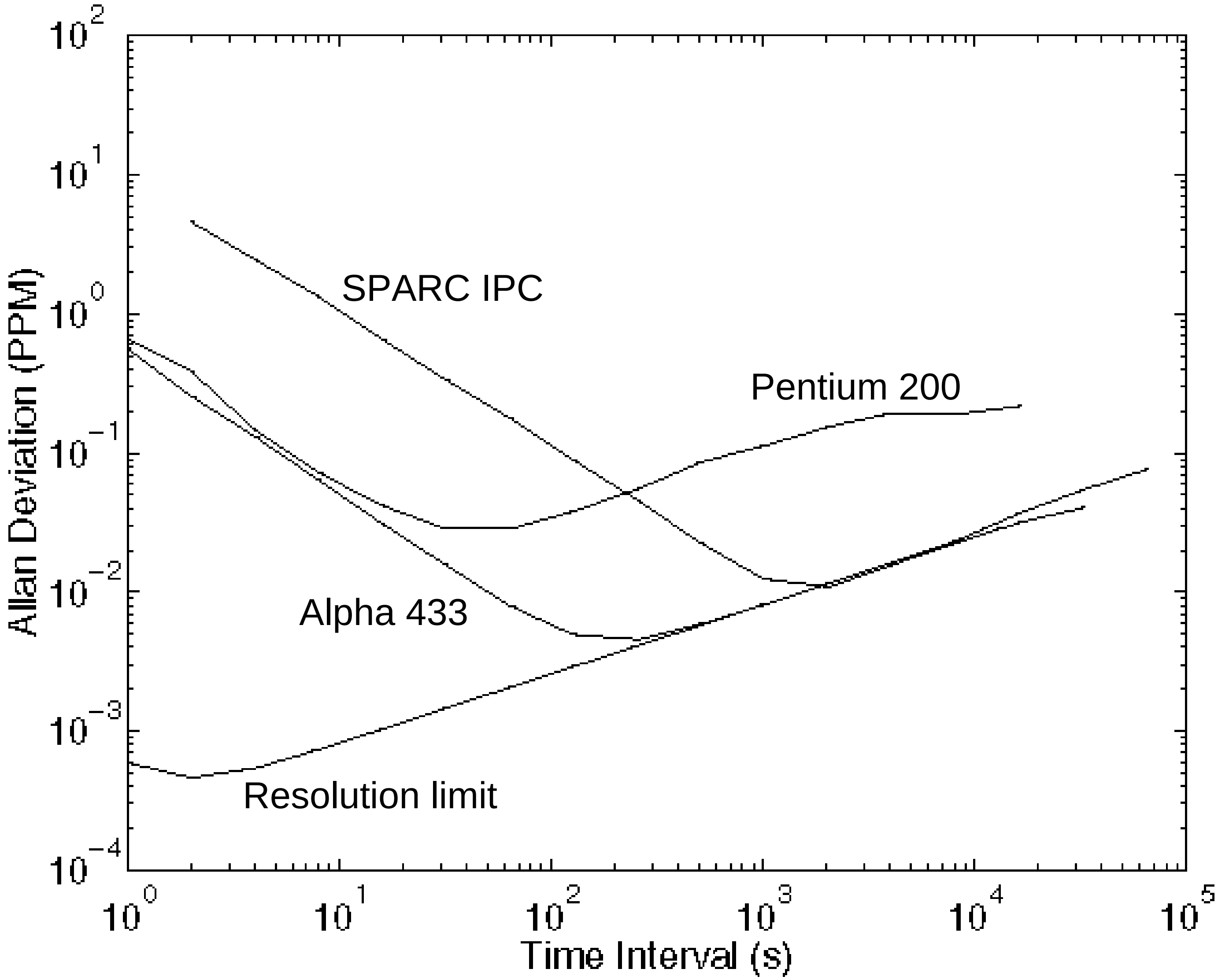
- The traces show the cumulative probability distributions for
  - Upper trace: raw time offsets measured over a 12-day period
  - Lower trace: filtered time offsets after the clock filter

Roundtrip delays

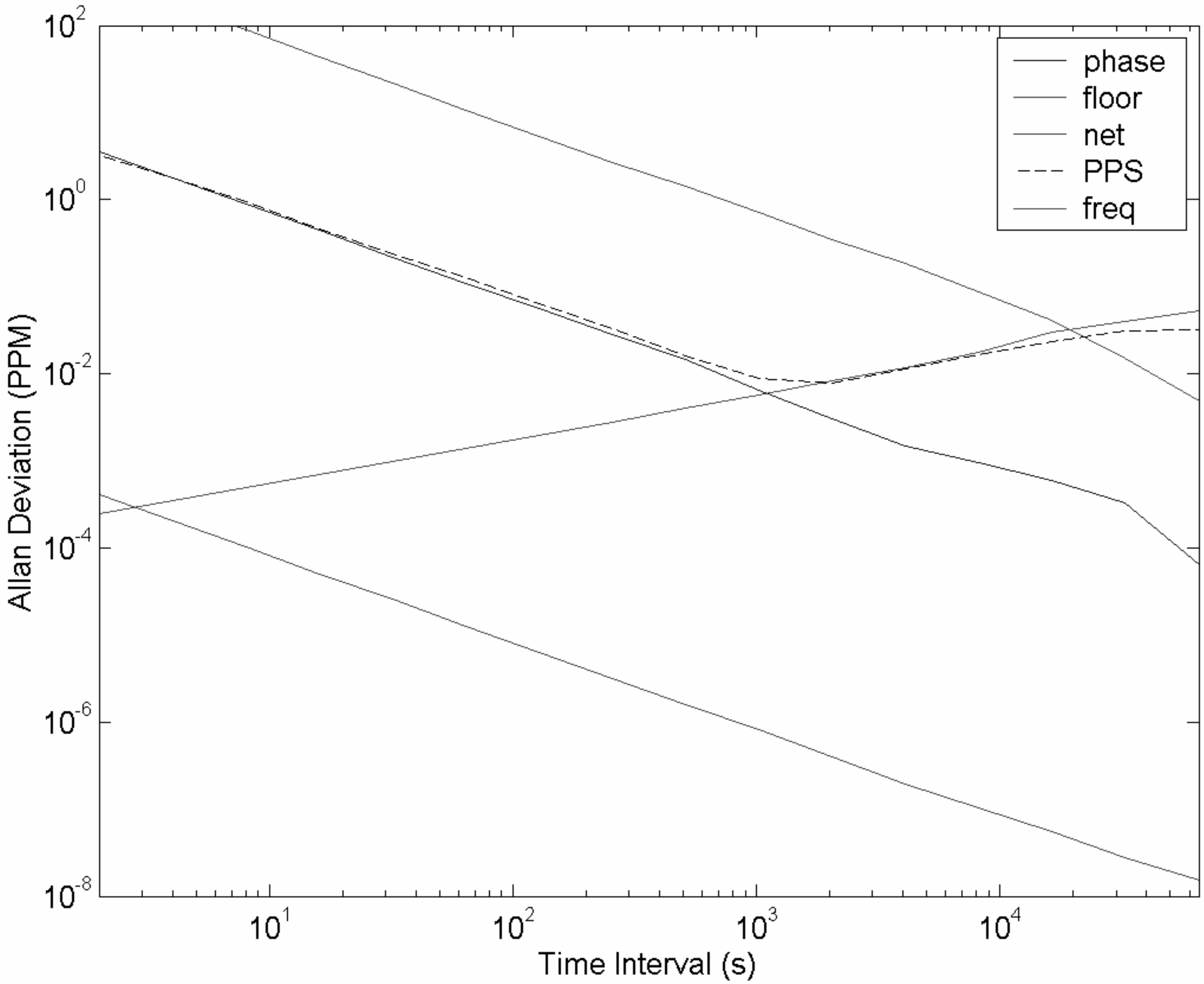


- Cumulative distribution function of absolute roundtrip delays
  - 38,722 Internet servers surveyed running NTP Version 2 and 3
  - Delays: median 118 ms, mean 186 ms, maximum 1.9 s(!)
  - Asymmetric delays can cause errors up to one-half the delay

Allan deviation characteristics compared



Allan deviation calibration



Self-similar distributions

- Consider the (continuous) process $X = (X_t, -\inf < t < \inf)$
- If X_{at} and $a^H(X_t)$ have identical finite distributions for $a > 0$, then X is self-similar with parameter H.
- We need to apply this concept to a time series. Let $X = (X_t, t = 0, 1, \ldots)$ with given mean μ, variance σ^2 and autocorrelation function $r(k)$, $k >= 0$.
- It's convienent to express this as $r(k) = k^{-\beta}L(k)$ as k -> inf and $0 < \beta < 1$.
- We assume $L(k)$ varies slowly near infinity and can be assumed constant.

Self-similar definition

- For $m = 1, 2, \ldots$ let $X^{(m)} = (X_k^{(m)}, k = 1, 2, \ldots)$, where m is a scale factor.
- Each $X_k^{(m)}$ represents a subinterval of m samples, and the subintervals are non-overlapping: $X_k^{(m)} = 1 / m\ (X^{(m)}_{(k-1)m}, + \ldots + X^{(m)}_{km-1})$, $k > 0$.
- For instance, $m = 2$ subintervals are (0,1), (2,3), …; $m = 3$ subintervals are (0, 1, 2), (3, 4, 5), …
- A process is (exactly) self-similar with parameter $H = 1 - \beta / 2$ if, for all $m = 1, 2, \ldots$, $\mathrm{var}[X^{(m)}] = \sigma^2 m - \beta$ and

$$r^{(m)}(k) = r(k) = 1 / 2\ ([k + 1]^{2H} - 2k^{2H} + [k - 1]^{2H}),\ k > 0,$$

  where $r^{(m)}$ represents the autocorrelation function of $X^{(m)}$.
- A process is (asymptotically) second-order self-similar if $r^{(m)}(k)$ -> $r(k)$ as m -> inf

Properties of self-similar distributions

- For self-similar distributions (0.5 < H < 1)
  - Hurst effect: the rescaled, adjusted range statistic is characterized by a power law; i.e., $E[R(m) / S(m)]$ is similar to m^H as m -> inf.
  - Slowly decaying variance. the variances of the sample means are decaying more slowly than the reciprocal of the sample size.
  - Long-range dependence: the autocorrelations decay hyperbolically rather than exponentially, implying a non-summable autocorrelation function.
  - 1 / *f* noise: the spectral density *f*(.) obeys a power law near the origin.
- For memoryless or finite-memory distributions ($0 < H < 0.5$)
  - var[$X^{(m)}$] decays as to m^{-1}.
  - The sum of variances if finite.
  - The spectral density *f*(.) is finite near the origin.

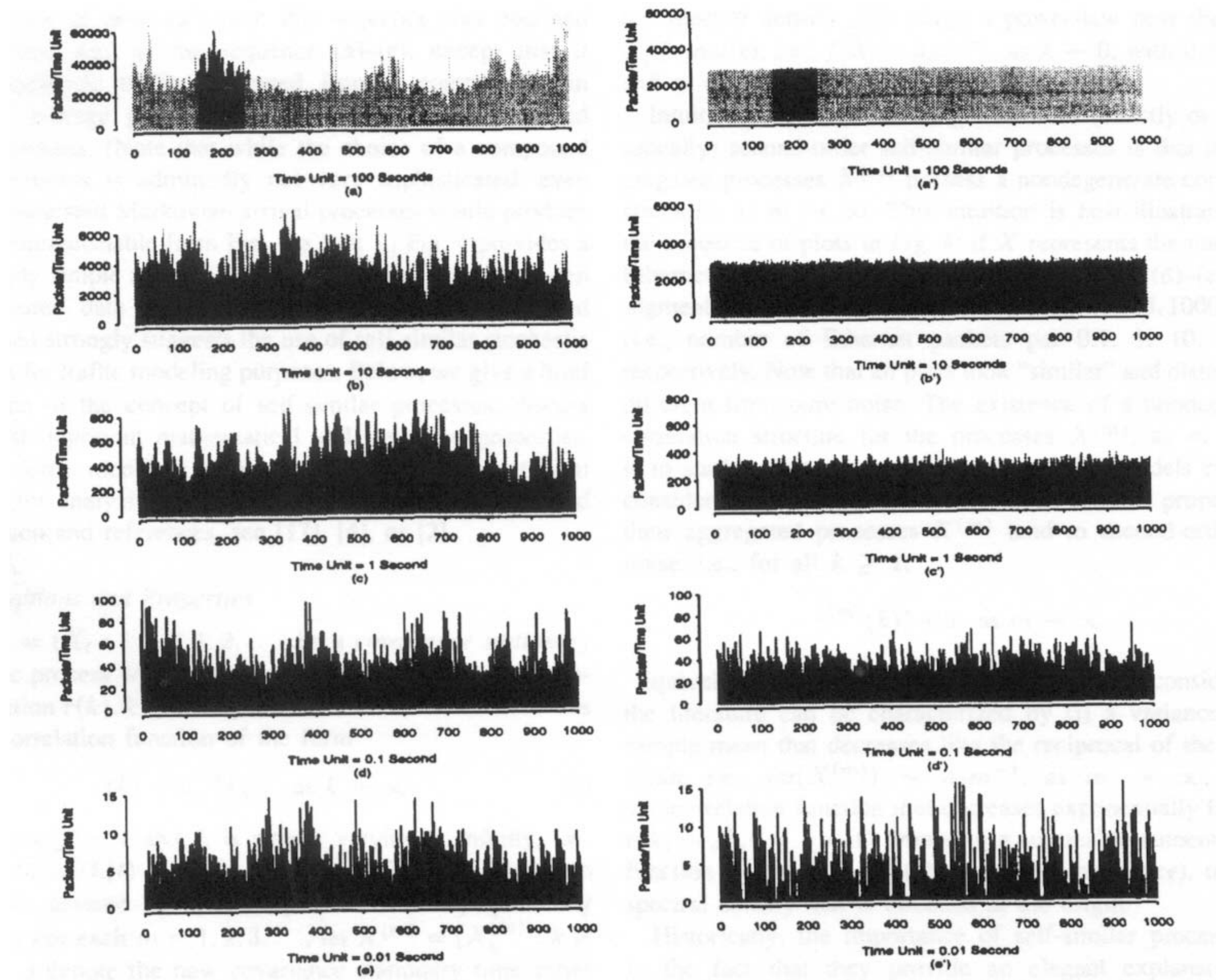
Origins of self-similar processes

- Long-range dependent ($0.5 < H < 1$)
  - Fractional Gaussian Noise (F-GN)

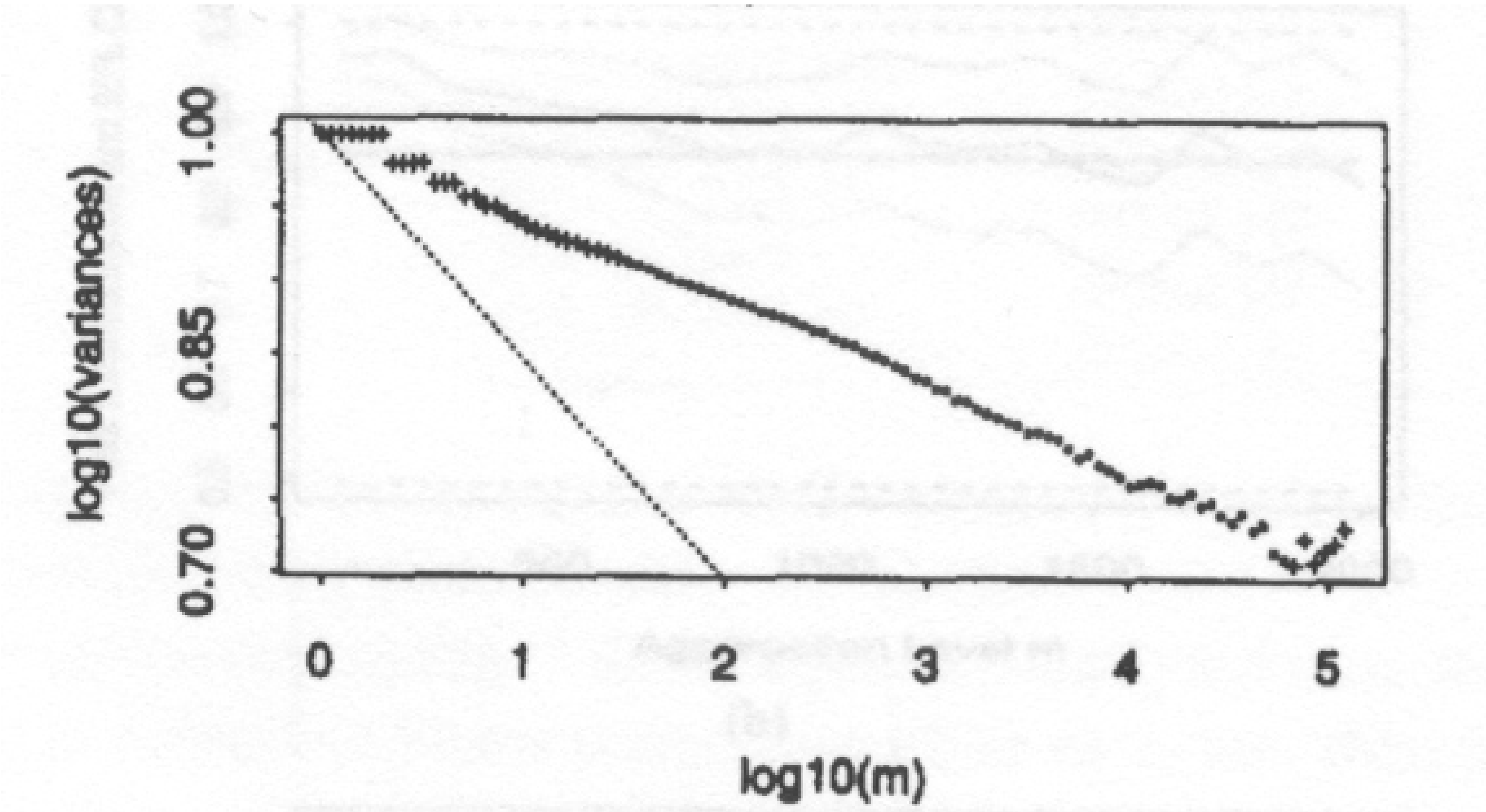
    $$r(k) = 1 / 2 \left([k + 1]^{2H} - 2k^{2H} + [k - 1]^{2H}\right),\ k > 1$$

  - Fractional Brownian Motion (F-BM)
  - Fractional Autoregressive Integrative Moving Average (F-ARIMA
  - Random Walk (RW) (descrete Brownian Motion (BM)
- Short-range dependent
  - Memoryless and short-memory (Markov)
  - Just about any conventional distribution – uniform, exponential, Pareto
  - ARIMA

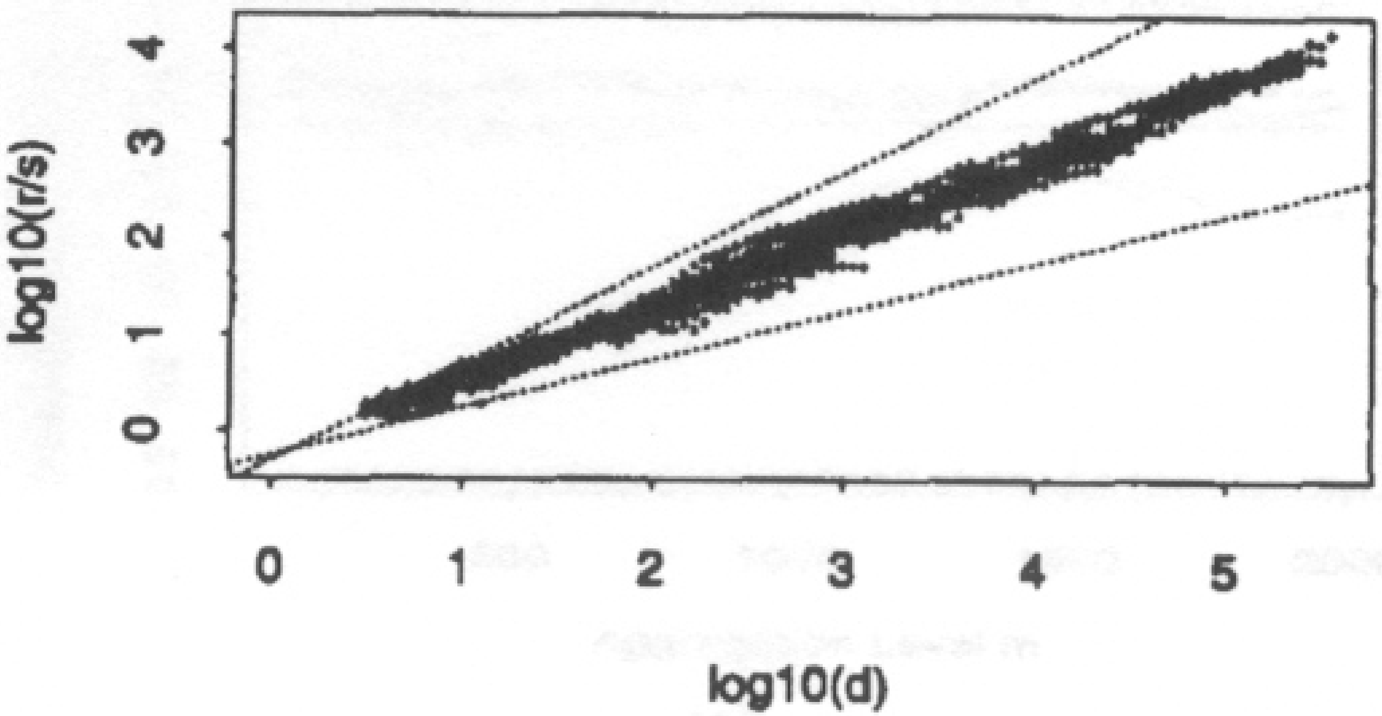
Examples of self-similar traffic on a LAN



Variance-time plot



R/S plot



Periodogram plot