Long-range Dependency Effects in Network Timekeeping

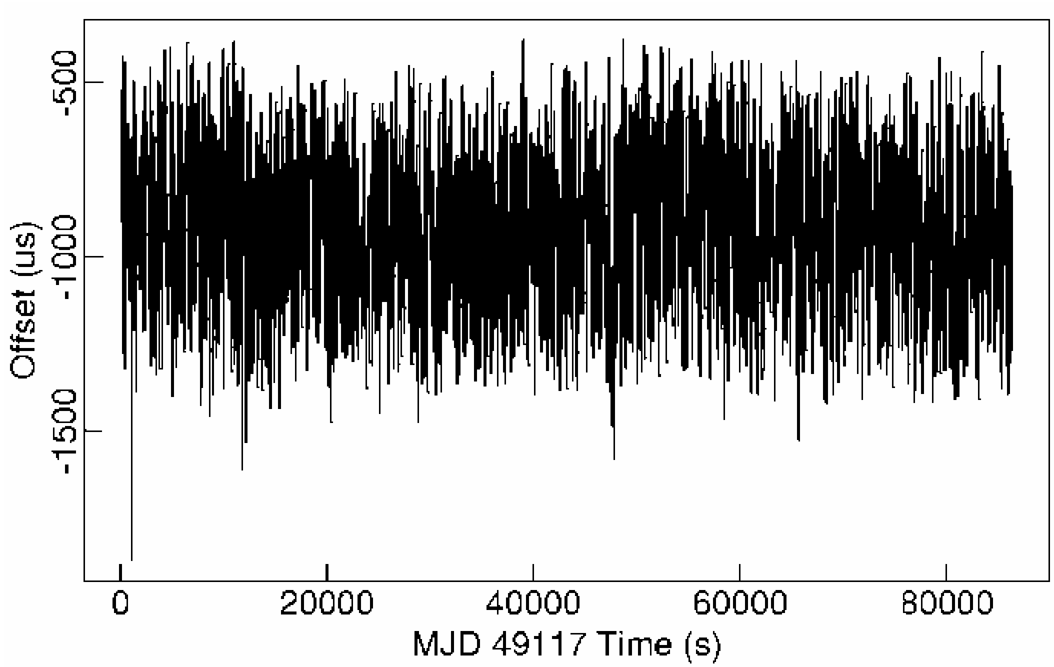
David L. Mills
University of Delaware
http://www.eecis.udel.edu/~mills
mailto:mills@udel.edu

Sir John Tenniel; *Alice's Adventures in Wonderland,* Lewis Carroll

Sources of error in network timekeeping

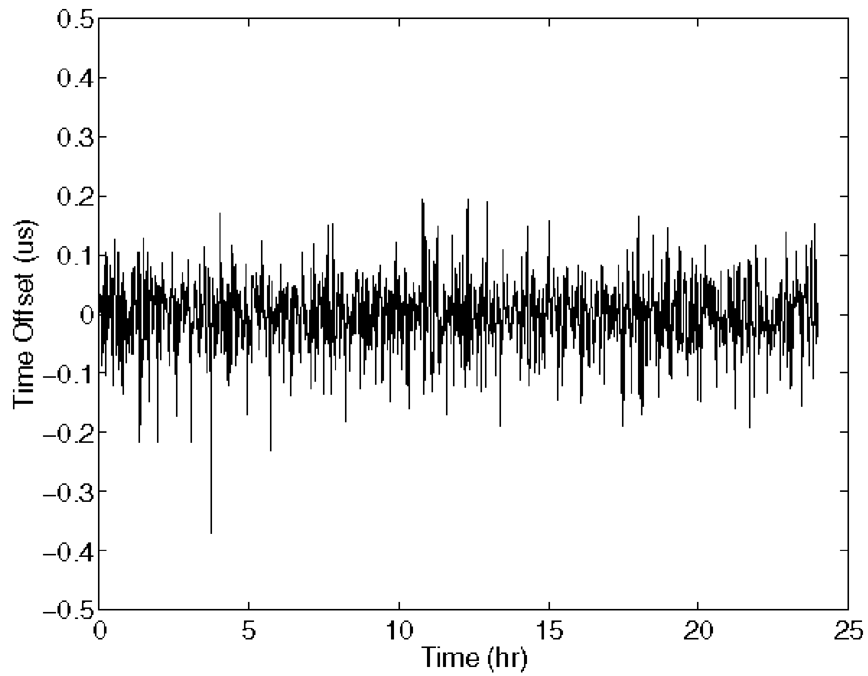
- Short-range distribution induced errors
  - Software latencies due to cache misses, context switches, page faults and process scheduling
  - Hardware latencies due to interrupts, network collisions, nonmaskable interrupts and timer/clock resolution
  - Asymmetric network propagation paths to and from the server
- Suspected long-range distribution induced errors
  - Network propagation path delay and jitter.
  - Jitter induced by wander in the system clock oscillator
- We need to prove/disprove whether long-range effects are in play.

Jitter witn a serial port hardware and driver



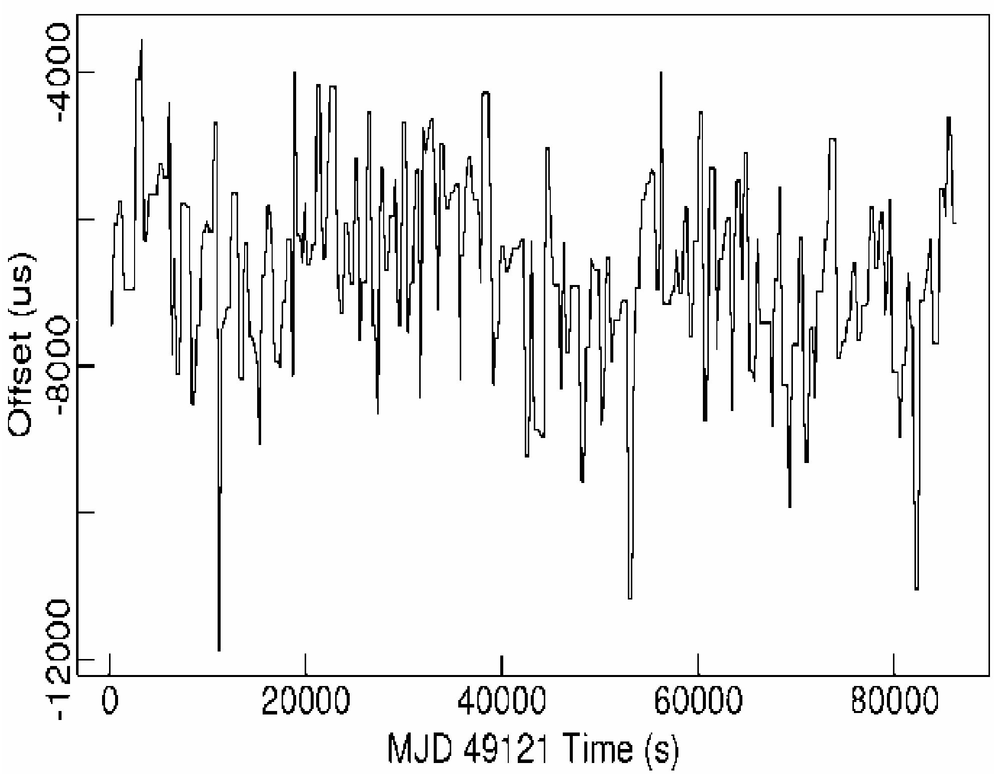
- Graph shows raw jitter of millisecond timecode and 9600-bps serial port. Samples are uniformly distributed over the character interval.
  - Additional latencies from 1.5 ms to 8.3 ms on SPARC IPC due to software driver and operating system; rare latency peaks over 20 ms
  - Using on-second format and median filter, residual jitter is less than 50 µs

Jitter with a PPS signal and Digital Alpha 433

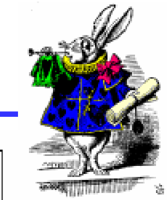


- Graph shows raw jitter of PPS timecode and parallel port due to interrupt latencies.
  - While not proven, the distribution looks very much like exponential.
  - Standard deviation 51.3 ns

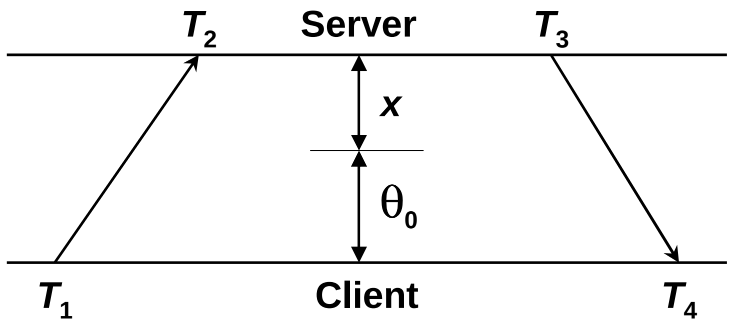
Jitter with a modem and ACTS service



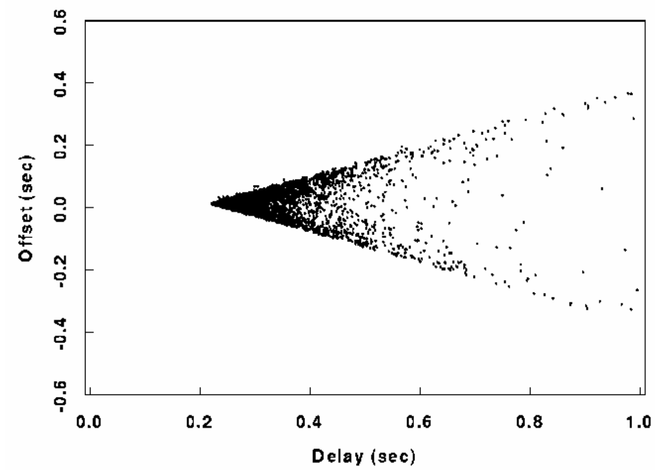
- Measurements use 2400-bps telephone modem and NIST Automated Computer Time Service (ACTS). Calls are placed at 16,384-s intervals.
  - Jitter is due primarily due to digital processing in the modem.
  - It is not clear what the distribution is, but it could include LRD.



Computing and filtering offset and delay samples

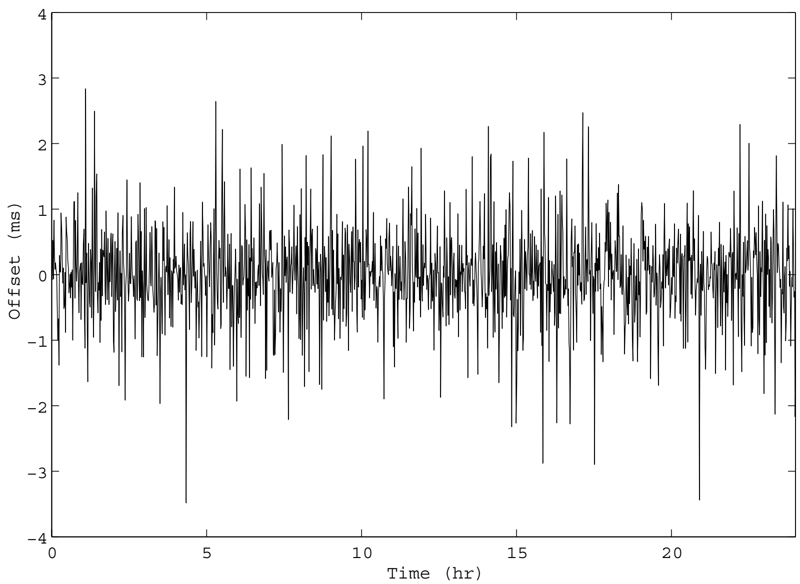


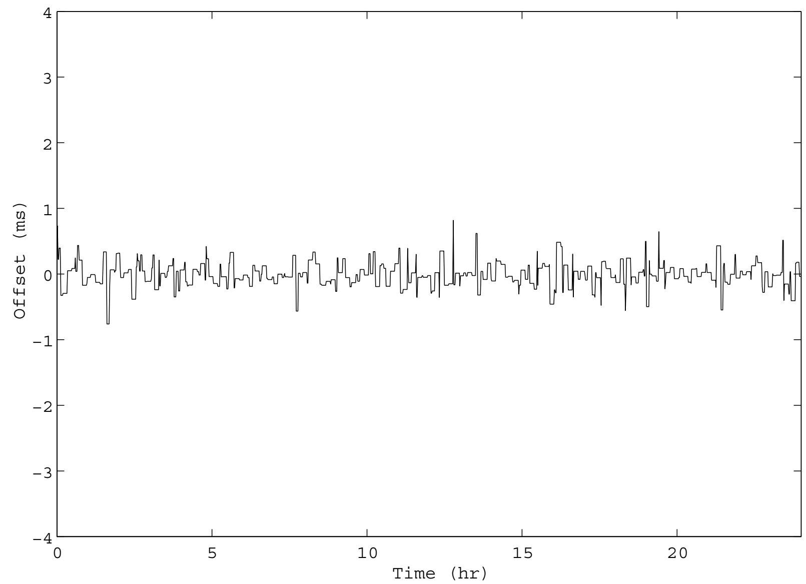
$$\theta = \tfrac{1}{2}[(T_2 - T_1) + (T_3 - T_4)]$$
$$\delta = (T_4 - T_1) - (T_3 - T_2)$$



- The most accurate offset θ_0 is measured at the lowest delay δ_0 (apex of the wedge scattergram).
- The correct time θ must lie within the wedge $\theta_0 \pm (\delta - \delta_0)/2$.
- The δ_0 is estimated as the minimum of the last eight delay measurements and (θ_0, δ_0) becomes the peer update.
- Each peer update can be used only once and must be more recent than the previous update.

Clock filter performance



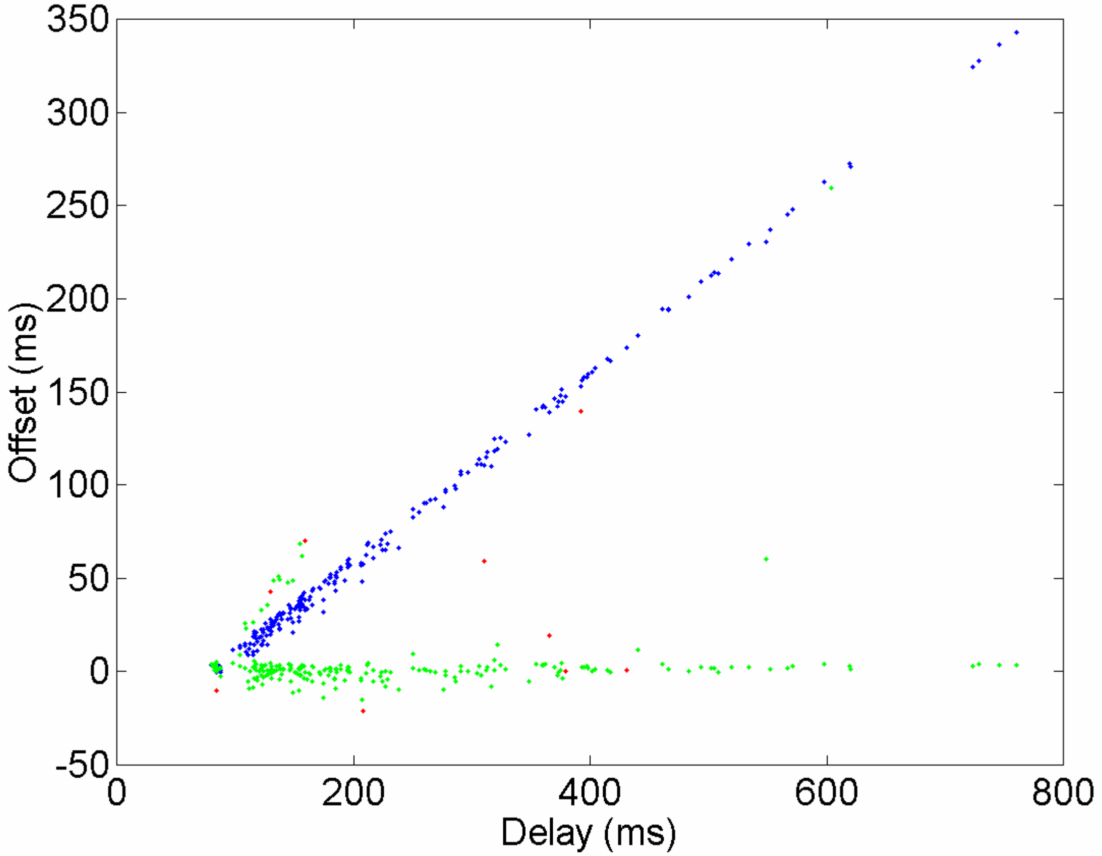


- Left figure shows raw time offsets measured for a typical path over a 24-hour period (mean error 724 μs, median error 192 μs)
- Right graph shows filtered time offsets over the same period (mean error 192 μs, median error 112 μs).
- The mean error has been reduced by 11.5 dB; the median error by 18.3 dB. This is impressive performance.

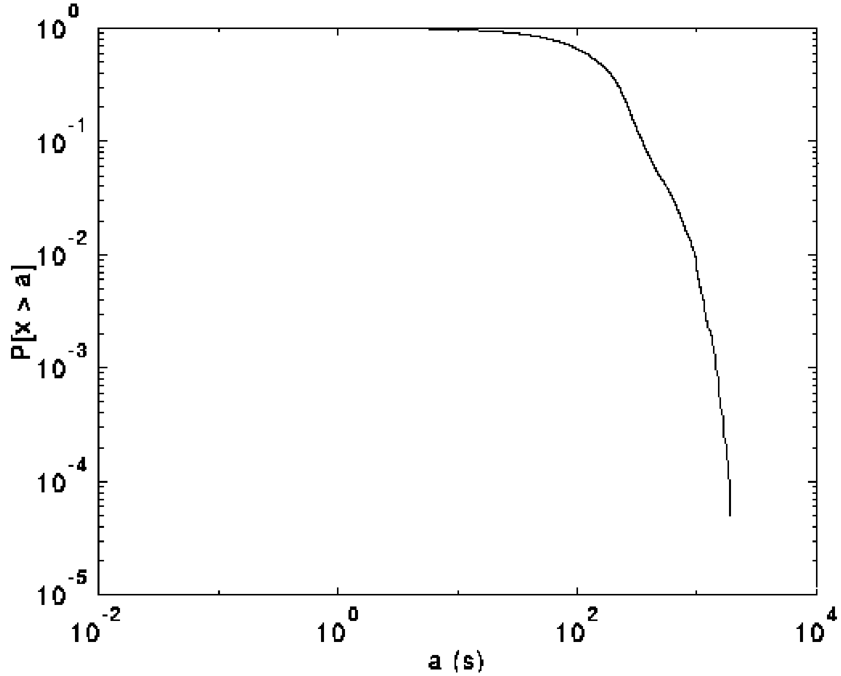
Asymmetric path delays

- We like to think that the delays on the outbound and inbound network paths are the same, or at least drawn from the same distribution.
- Such is not the case in several instances, one of which is shown in the wedge scattergram on the next slide.
  - The occasion arises with a slow PPP line while downloading a large file.
  - The download direction utilization is essentially 100 percent, while the other direction carries only ACKs and is only minimally utilized.
  - The delay distribution on the download direction depends on the packet length distribution, which is SRD.
  - The delay distribution on the other direction depends on the network jitter, which may or may not be LRD.

Huff&puff wedge scattergram



Raw roundtrip delay distribution function from survey



- Cumulative distribution function of absolute roundtrip delays
  - 38,722 Internet servers surveyed running NTP Version 2 and 3
  - Delays: median 118 ms, mean 186 ms, maximum 1.9 s(!)
  - Asymmetric delays can cause errors up to one-half the delay

Self-similar distributions

- Consider the (continuous) process $X = (X_t, -\infty < t < \infty)$
- If X_{at} and $a^H(X_t)$ have identical finite distributions for $a > 0$, then X is self-similar with parameter H.
- We need to apply this concept to a time series. Let $X = (X_t, t = 0, 1, \ldots)$ with given mean μ, variance σ^2 and autocorrelation function $r(k)$, $k \geq 0$.
- It's convienent to express this as $r(k) = k^{-\beta}L(k)$ as $k \to \infty$ and $0 < \beta < 1$.
- We assume $L(k)$ varies slowly near infinity and can be assumed a constant like 1.

Definition of self-similar distribution

- For $m = 1, 2, \ldots$ let $X^{(m)} = (X_k^{(m)}, k = 1, 2, \ldots)$, where m is a scale factor.
- Each $X_k^{(m)}$ represents a subinterval of m samples, and the subintervals are non-overlapping: $X_k^{(m)} = 1 / m\ (X^{(m)}_{(k-1)\,m}, + \ldots + X^{(m)}_{km-1})$, $k > 0$.
- For instance, $m = 2$ subintervals are (0,1), (2,3), …; $m = 3$ subintervals are (0, 1, 2), (3, 4, 5), …
- A process is (exactly) self-similar with parameter $H = 1 - \beta / 2$ if, for all $m = 1, 2, \ldots$, $\mathrm{var}[X^{(m)}] = \sigma^2 m - \beta$ and $r^{(m)}(k) = r(k) = 1 / 2\ [(k + 1)^{2H} - 2k^{2H} + (k - 1)^{2H}]$, $k > 0$, where $r^{(m)}$ represents the autocorrelation function of $X^{(m)}$.
- A process is (asymptotically) second-order self-similar if $r^{(m)}(k)$ -> $r(k)$ as $m \rightarrow \infty$.
- Plot $r(k) = k^{-\beta} = k^{1-2H}$ in log-log coordinates as a straight line with
  - $\beta = -1$ for $H = 0.5$, representing short-range dependent (SRD) distribution,
  - $-1 < \beta < 0$ for $0.5 < H < 1$, representing long-range dependent (LRD) distribution,
  - $\beta = 1$ for $H = 1$, representing a random-walk distribution.

Properties of self-similar distributions

- For self-similar distributions (0.5 < H < 1)
  - Hurst effect: the rescaled, adjusted range statistic is characterized by a power law; i.e., $E[R(m) / S(m)]$ is similar to m^H as $m \to \infty$.
  - Slowly decaying variance. the variances of the sample means are decaying more slowly than the reciprocal of the sample size.
  - Long-range dependence: the autocorrelations decay hyperbolically rather than exponentially, implying a non-summable autocorrelation function.
  - 1 / *f* noise: the spectral density *f*(.) obeys a power law near the origin.
- For memoryless or finite-memory distributions ($0 < H < 0.5$)
  - var[$X^{(m)}$] decays as to m^{-1}.
  - The sum of variances if finite.
  - The spectral density *f*(.) is finite near the origin.

Origins of self-similar processes

- Long-range dependent ($0.5 < H < 1$)
  - Fractional Gaussian Noise (F-GN)

$$r(k) = 1 / 2 \, [(k + 1)^{2H} - 2k^{2H} + (k - 1)^{2H}], \ k > 1$$

  - Fractional Brownian Motion (F-BM)
  - Fractional Autoregressive Integrative Moving Average (F-ARIMA
  - Random Walk (RW) (descrete Brownian Motion (BM))
- Short-range dependent
  - Memoryless and short-memory (Markov)
  - Just about any conventional distribution – uniform, exponential, Pareto
  - ARIMA

Simulation studies

- The object of these simulations is to confirm samples from a given distribution have short-range dependency (SRD) or long-range dependency (LRD).
  - X is a time series of N samples drawn from a distribution with given mean μ and variance σ.
  - $X^{(m)} = (X_k^{(m)}, k = 1, 2, \ldots)$, where $m = 1, 2, 4, \ldots$ is a scale factor increasing in powers of two.
  - X is divided in contiguous, non-overlapping intervals of size m indexed by k.
  - $a^{(m)} = (a_k^{(m)}, k = 1, 2, \ldots)$ is the time series corresponding to the average of the samples in each interval .
  - The variance-time graph plots variance $\sigma^2(a^{(m)})$ against m in log-log scales.

m = 1

k	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	…

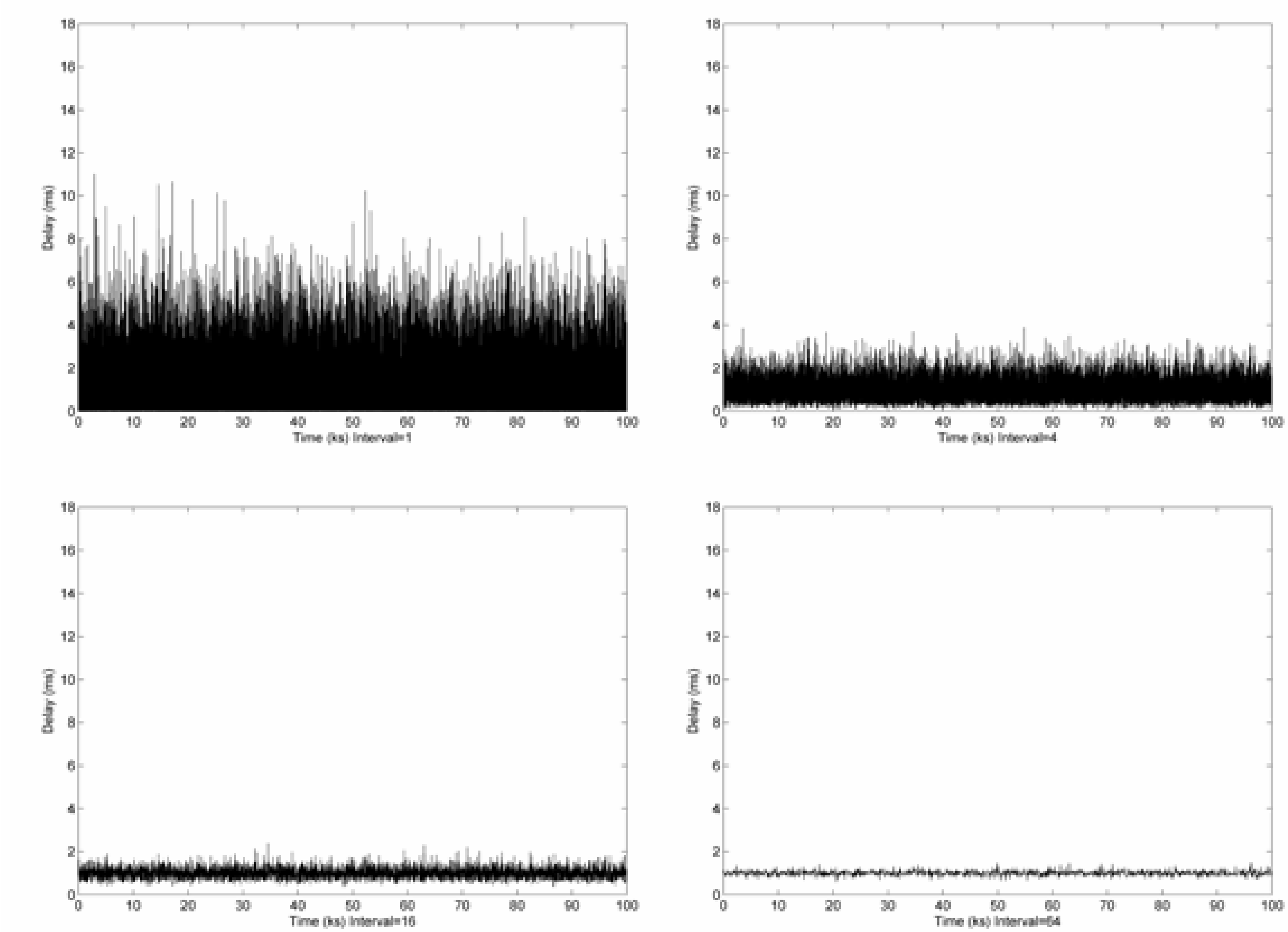
m = 2

k	$(X_1 + X_2) / 2$	$(X_3 + X_4) / 2$	$(X_5 + X_6) / 2$	$(X_7 + X_8) / 2$	…

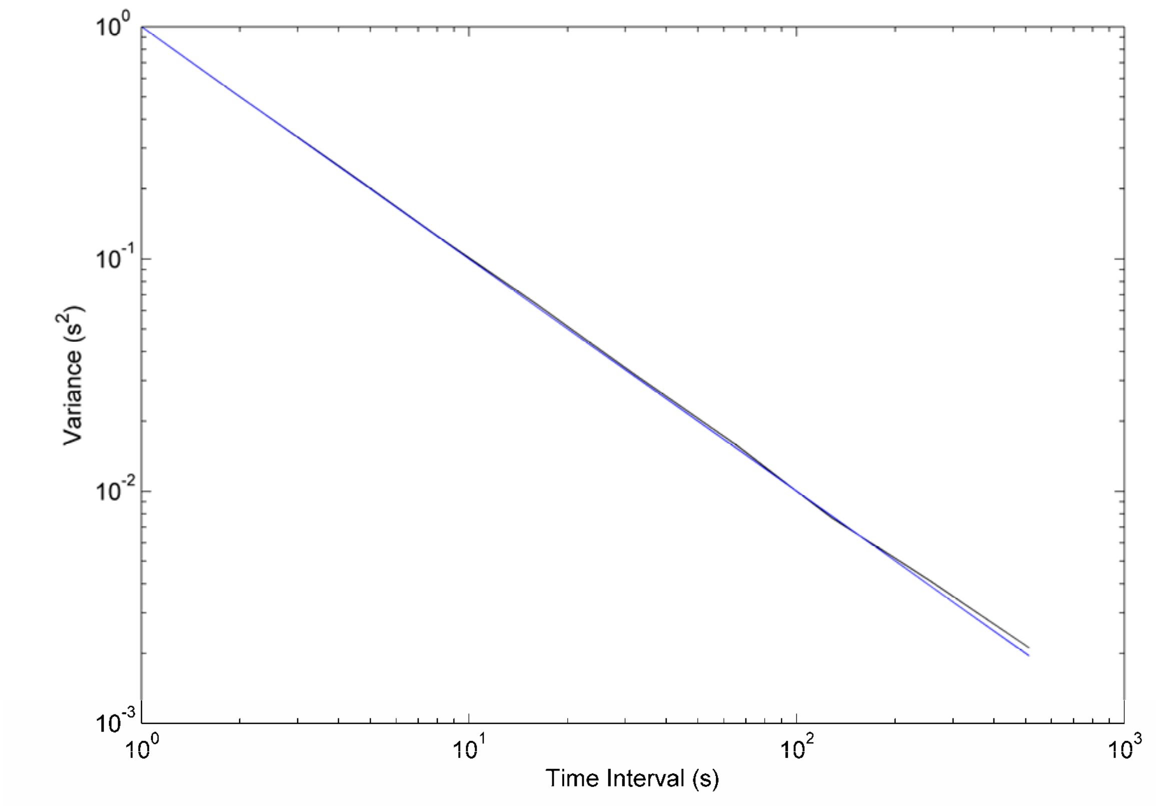
Exponential distribution

- The object of this experiment is to determine whether an exponential distribution has only SRD.
  - 100,000 samples generated from an exponential distribution with $\sigma = 1$.
  - The next slide shows the time series $X_k^{(m)}$ for values of $m = 1, 4, 16$ and 64. Note the weak self-similar characteristic.
  - The second slide shows the variance-time plot, which shows the Hurst parameter $H = 0.5$ and confirms the exponential distribution has only SRD.
  - This property is true also of other processes generated by uniform, Poisson, finite Markov and just about every other process without a heavy-tail autocorrelation function.

Exponential distribution *m* = 1, 4, 16, 64 s



Exponential distribution variance-time plot

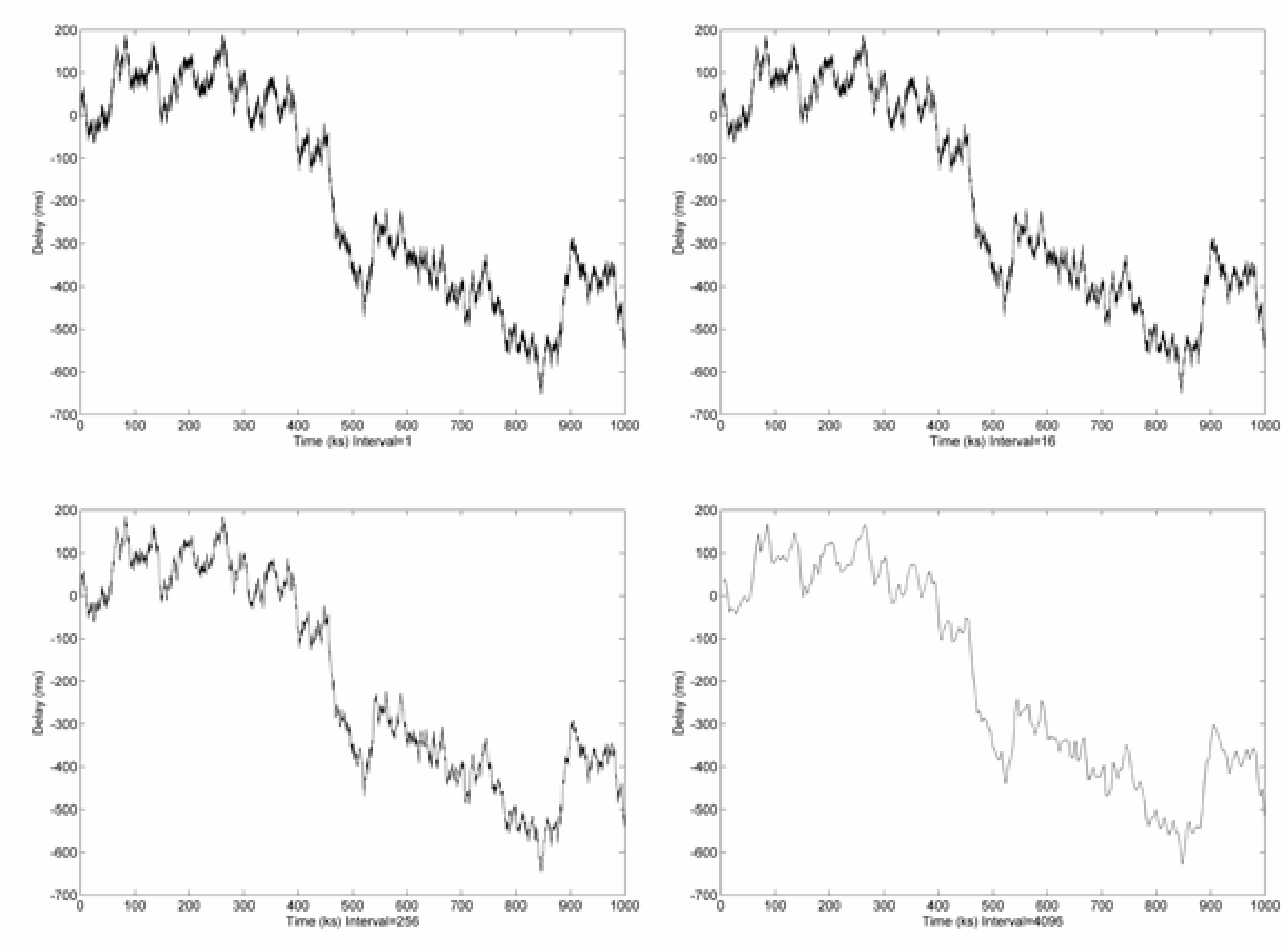


- Graph shows the variance from data averaged over specified intervals.
  - One curve shows the data, the other shows SRD with *H* = 0.5.
  - Both curves overlap almost everywhere, showing the distribution is SRD.

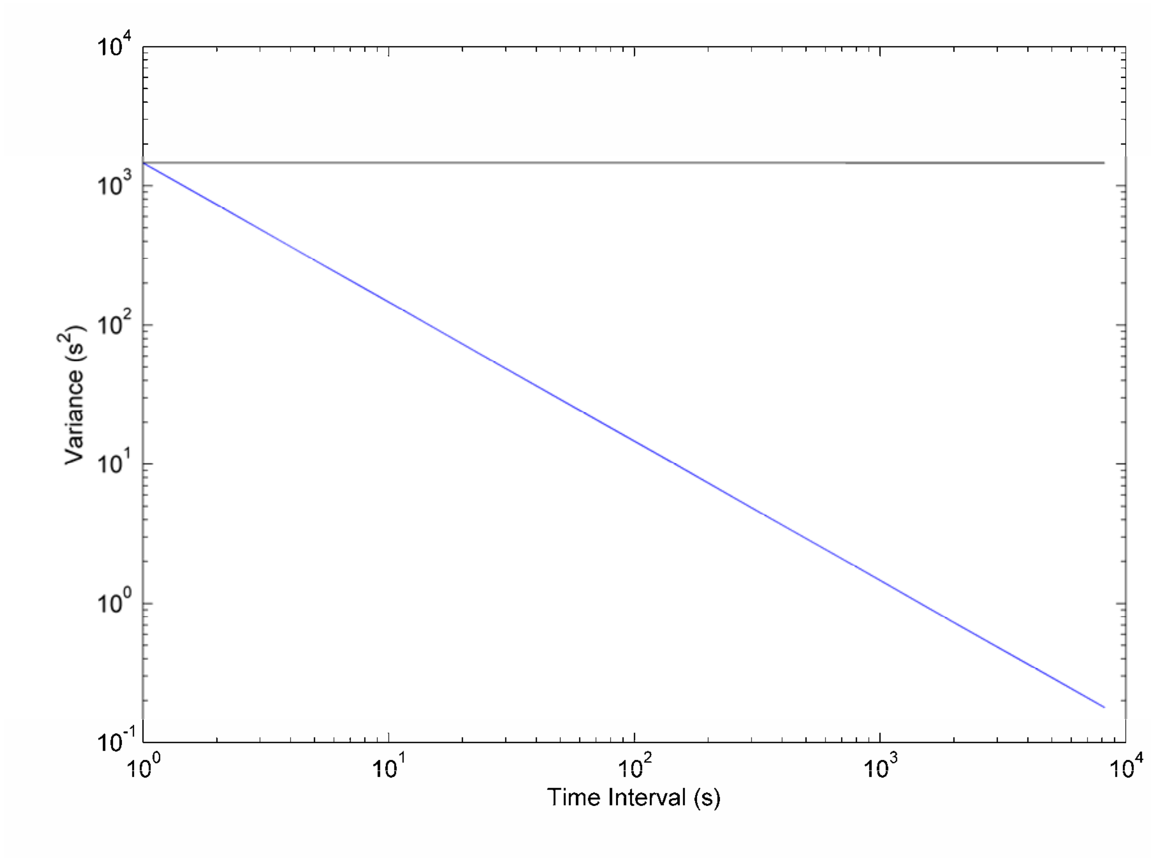
Random-walk distribution

- The object of this experiment is to determine whether a random-walk distribution is LRD.
  - 1,000,000 samples were generated from a random-walk distribution consisting of the integral of a Gaussian distribution with $\mu = 0$ and $\sigma = 0.1$.
  - The next slide shows the time series $X_k^{(m)}$ for $m = 1, 16, 256$ and 4096 seconds. Note the curves of the first three are almost identical, except for some high-frequency smoothing at $m = 4096$.
  - This is to be expected, since even at $m = 4096$ the intervals are small compared to the wiggle of the curve. This is characteristic of flicker (1 / f) noise and the fact the autocorrelation functions are non-summable.
  - Random-walk distributions ($H = 1$) are probably not good models for network delays, but they are good models for computer clock oscillator wander.

Random-walk distribution *m* = 1, 16, 256 and 4096 s



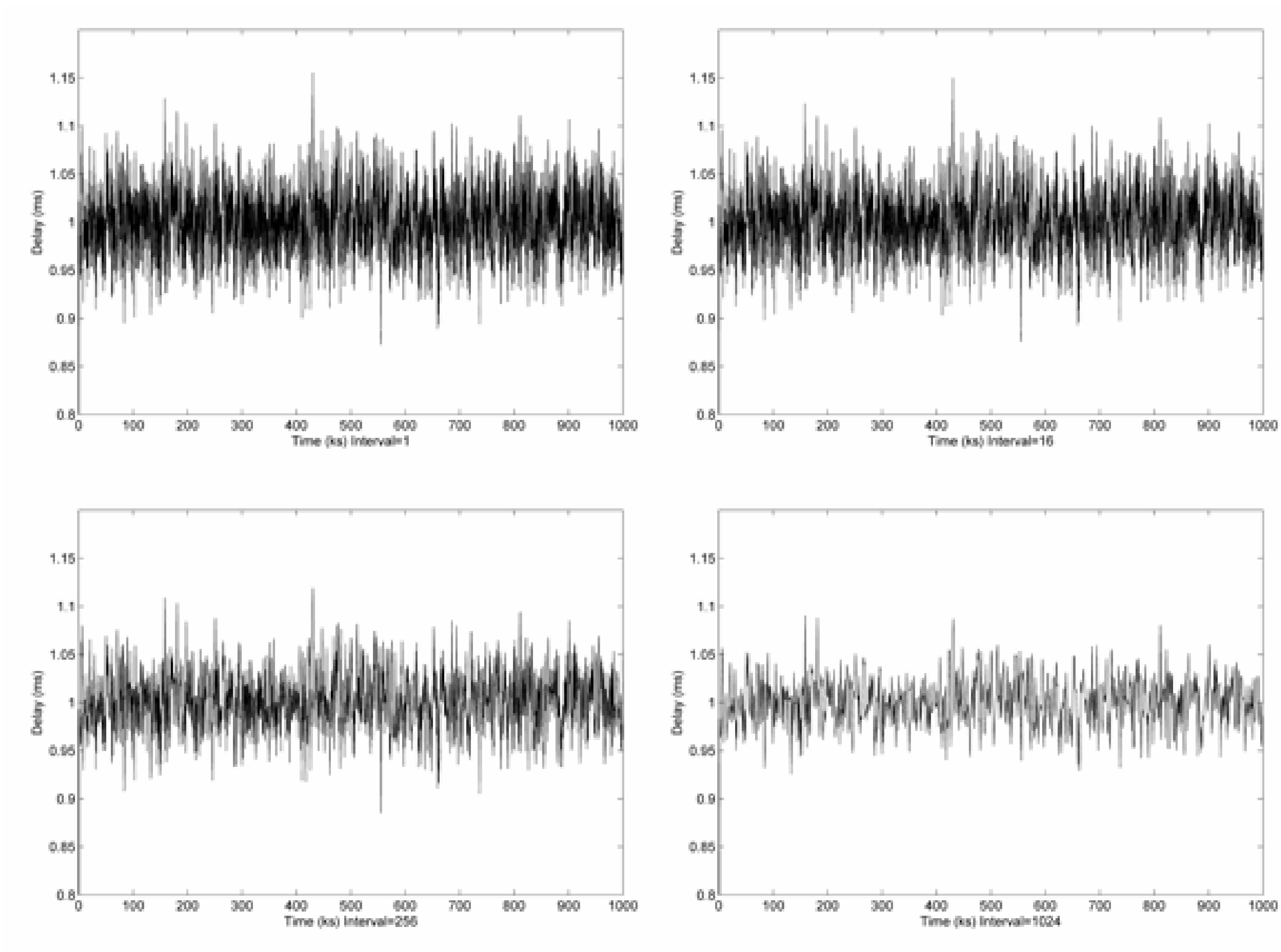
Random-walk distribution variance-time plot



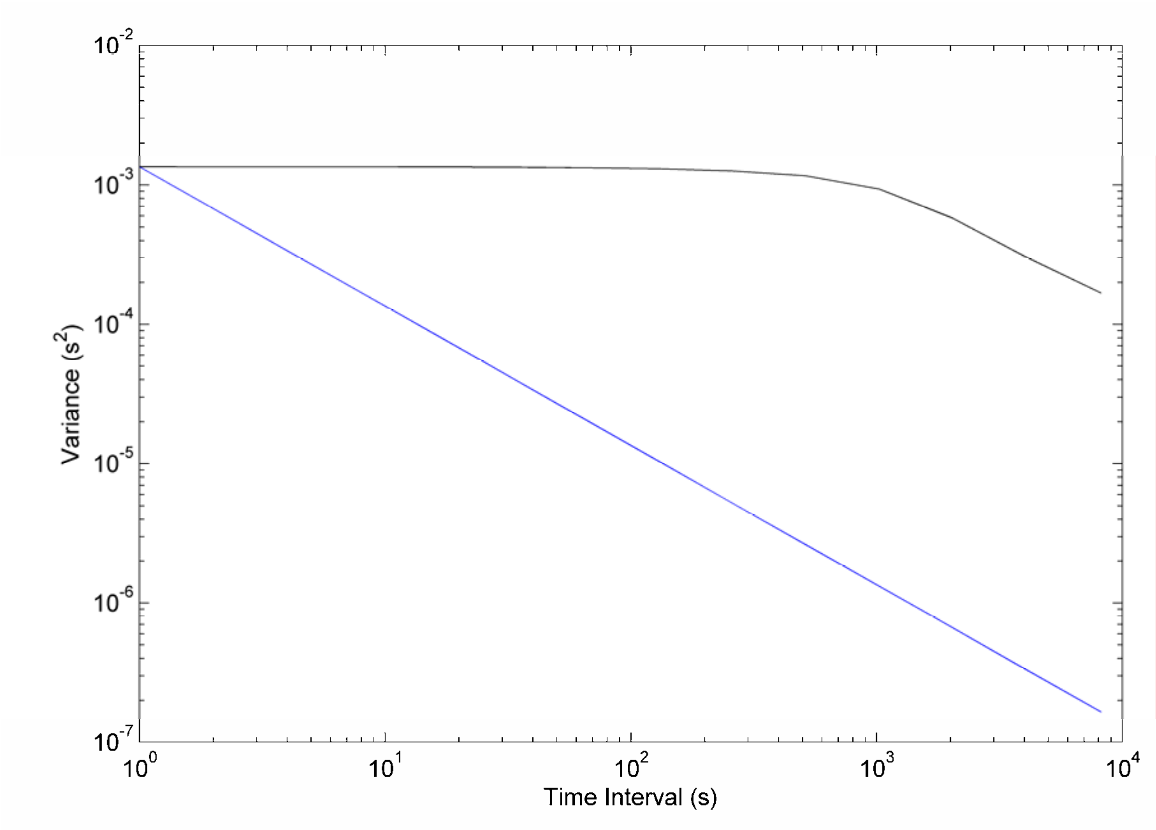
Filtered exponential distribution

- A strict random-walk distribution ($H = 1$) is probably not a good model for network delays. A better model would have H somewhere in the middle of $0.5 < H < 1$.
- Generating a strict self-similar time series for given H is computationally complex and expensive.
- So, try a filtered exponential distribution with given finite autocorrelation function $r(k) = k^{\beta}$ ($1 \le k \le n$, $0 \le \beta \le 1$). We choose $n = 1{,}000$ and $\beta = 1$.
- The next slide shows the time series $X_k^{(m)}$ for $m = 1, 16, 256$ and 1024 seconds. Note the curves of the first three are almost identical. There is some decay at 1024 s.
- The variance-time plot on the second page shows random-walk and characteristic at lags in the order of n and decays to SRD after tha.

Filtered exponential distribution *m* = 1, 16, 256 and 1024 s



Filtered exponential distribution variance-time plot

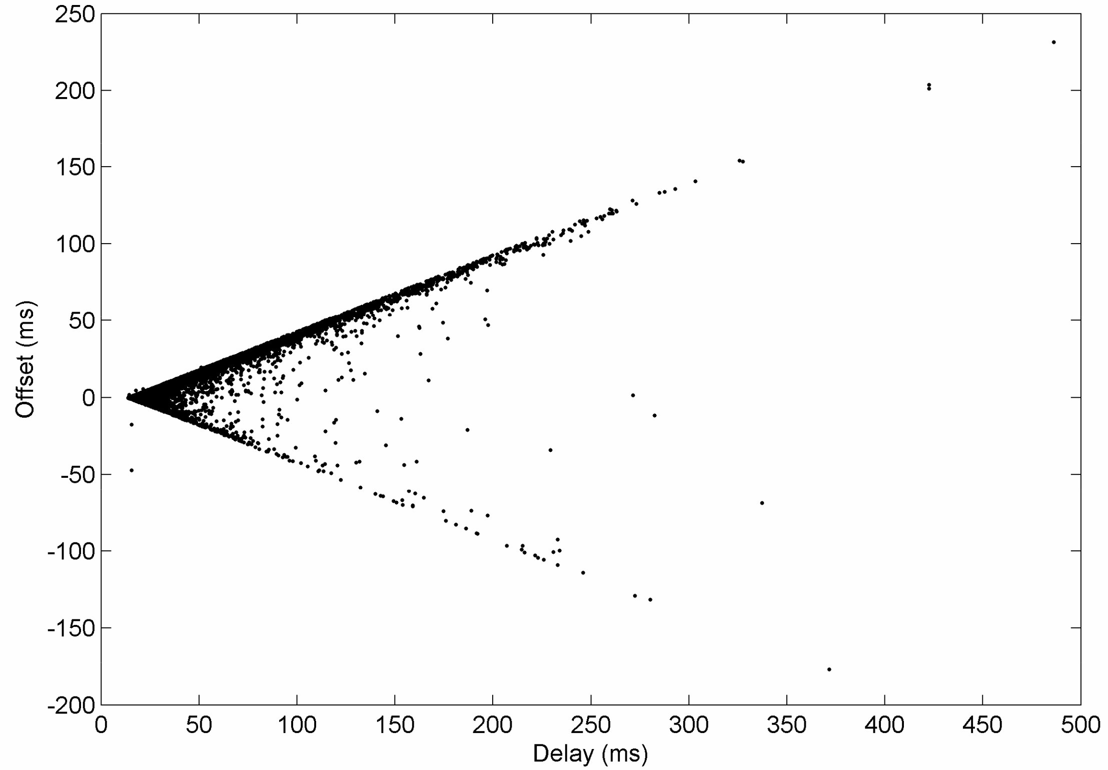


- Graph shows the variance from data averaged over specified intervals.
  - The upper curve from data shows filtered exponential.
  - The lower curve shows SRD with H = 0.5 for reference.

Experiment study – USNO data

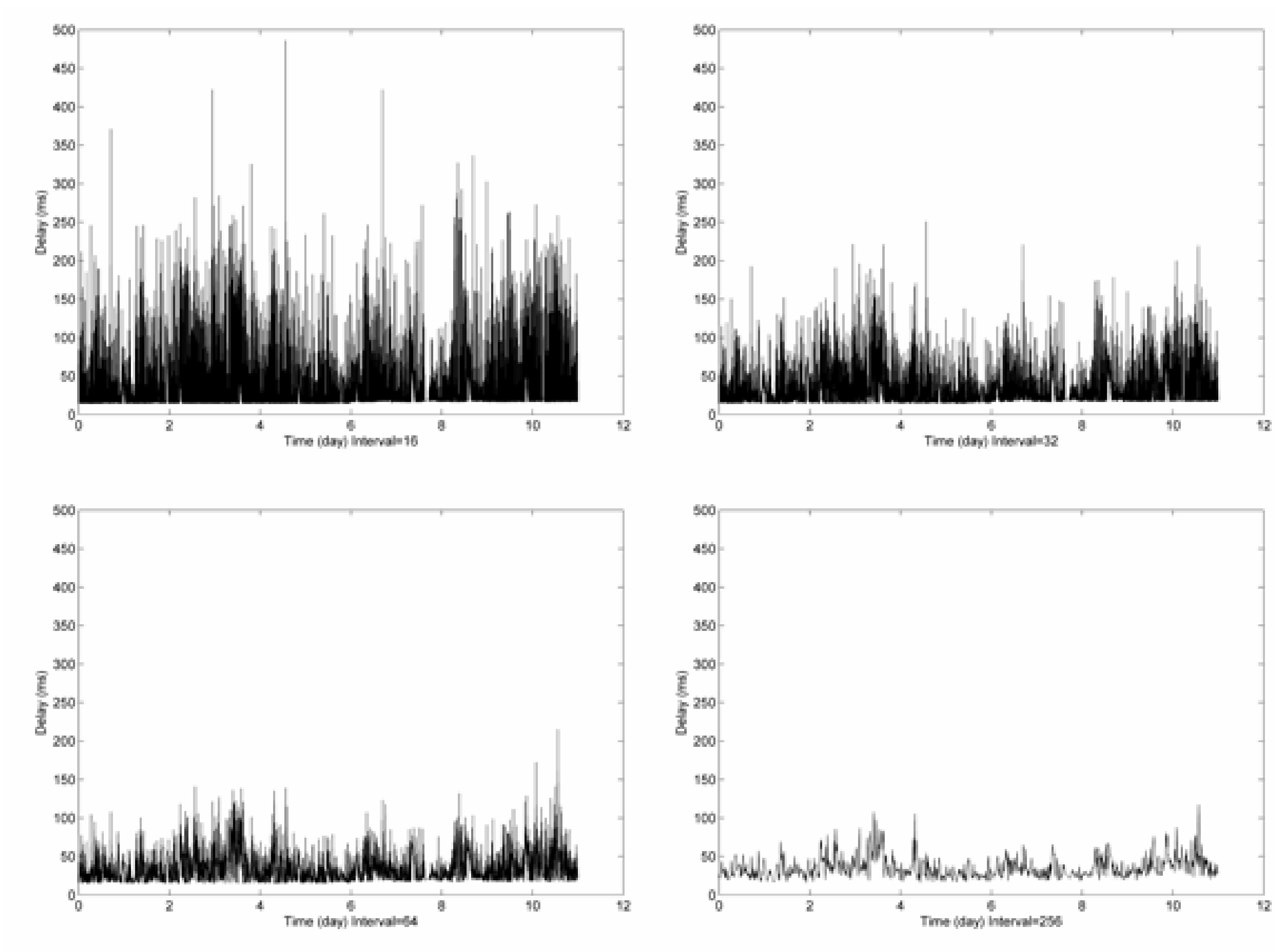
- The object of this experiment is to determine whether roundtrip delays measured over Internet paths by NTP show long-range dependency.
  - The Internet path was between primary time servers *pogo.udel.edu* at UDel and *tick.usno.navy.mil* in Washington, DC.
  - Measurements were made every 16 seconds over about 11 days.
  - The next slide shows the path delays are asymmetric. The roundtrip delay is the sum of the two one-way delays, which is the convolution of their distributions. In most cases we assume the two distributionsare the same.
  - The following slide shows the smoothed delay at averaging intervals m = 32, 64, 64 and 256 seconds. Note the weak self-similar characteristic.

USNO data wedge scattergram

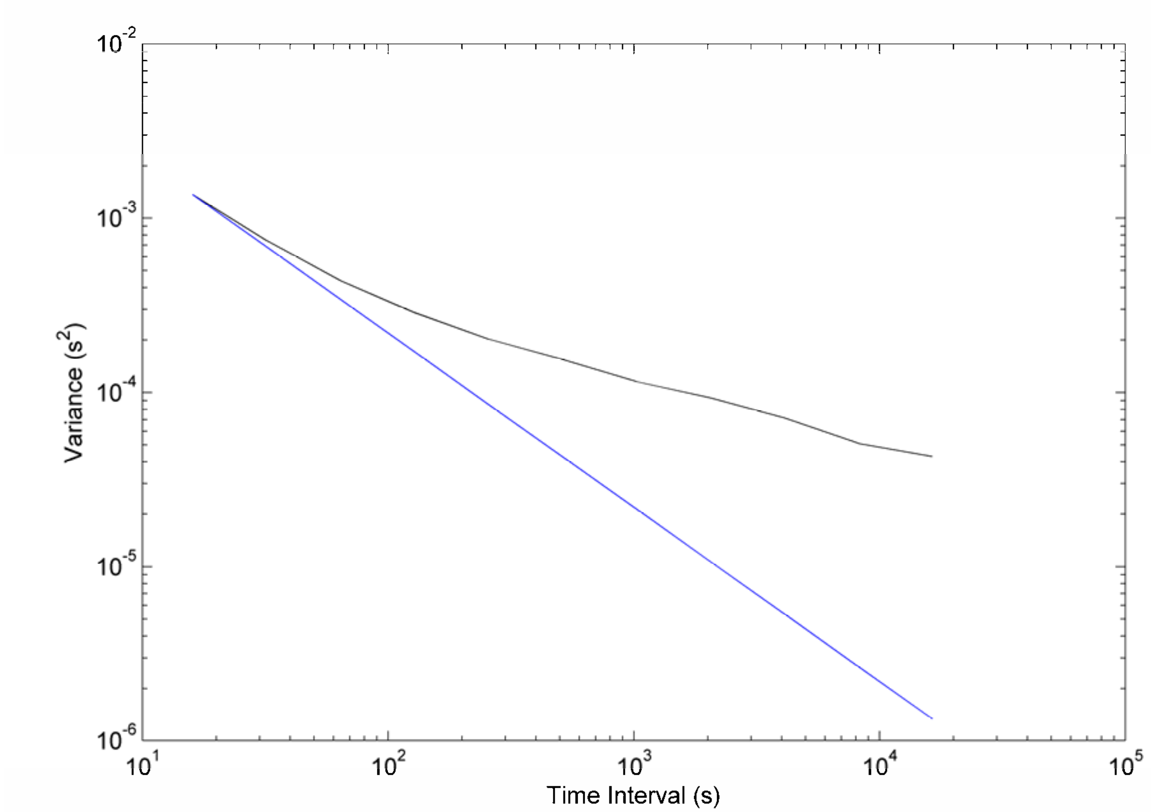


- Each dot represents a offset/delay sample.
  - The upper limb of the wedge represents packets inbound to USNO; the lower limb outbount.
  - Obviously, the traffic is asymmetric, so the delays should be as well.

USNO data delay *m* = 16, 32, 64 and 256 s



USNO data delay variance-time plot

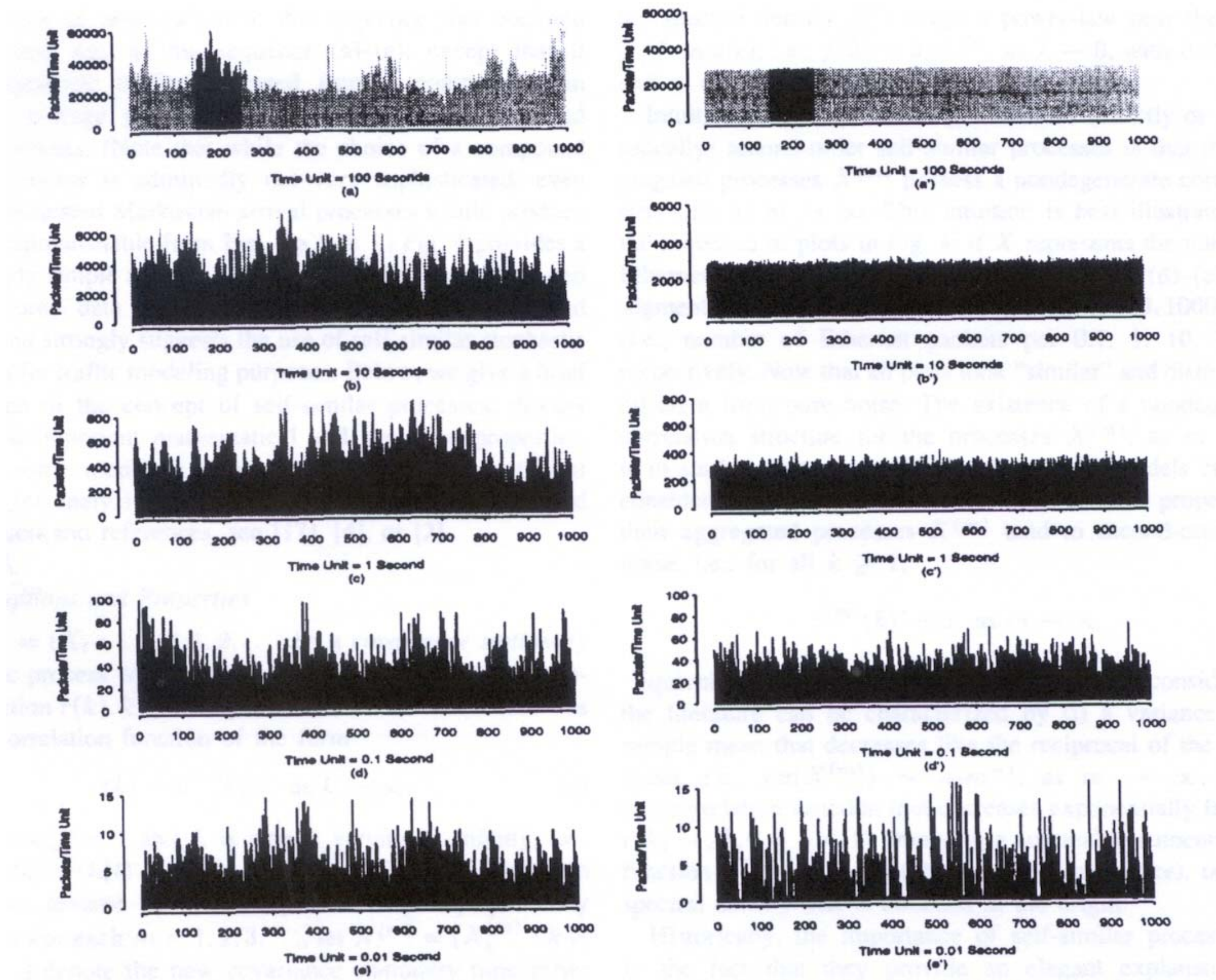


- Graph shows the variance from data averaged over specified intervals.
  - The upper curve from data shows LRD with 0.5 < H < 1.
  - The lower curve shows SRD with H = 0.5 for reference.

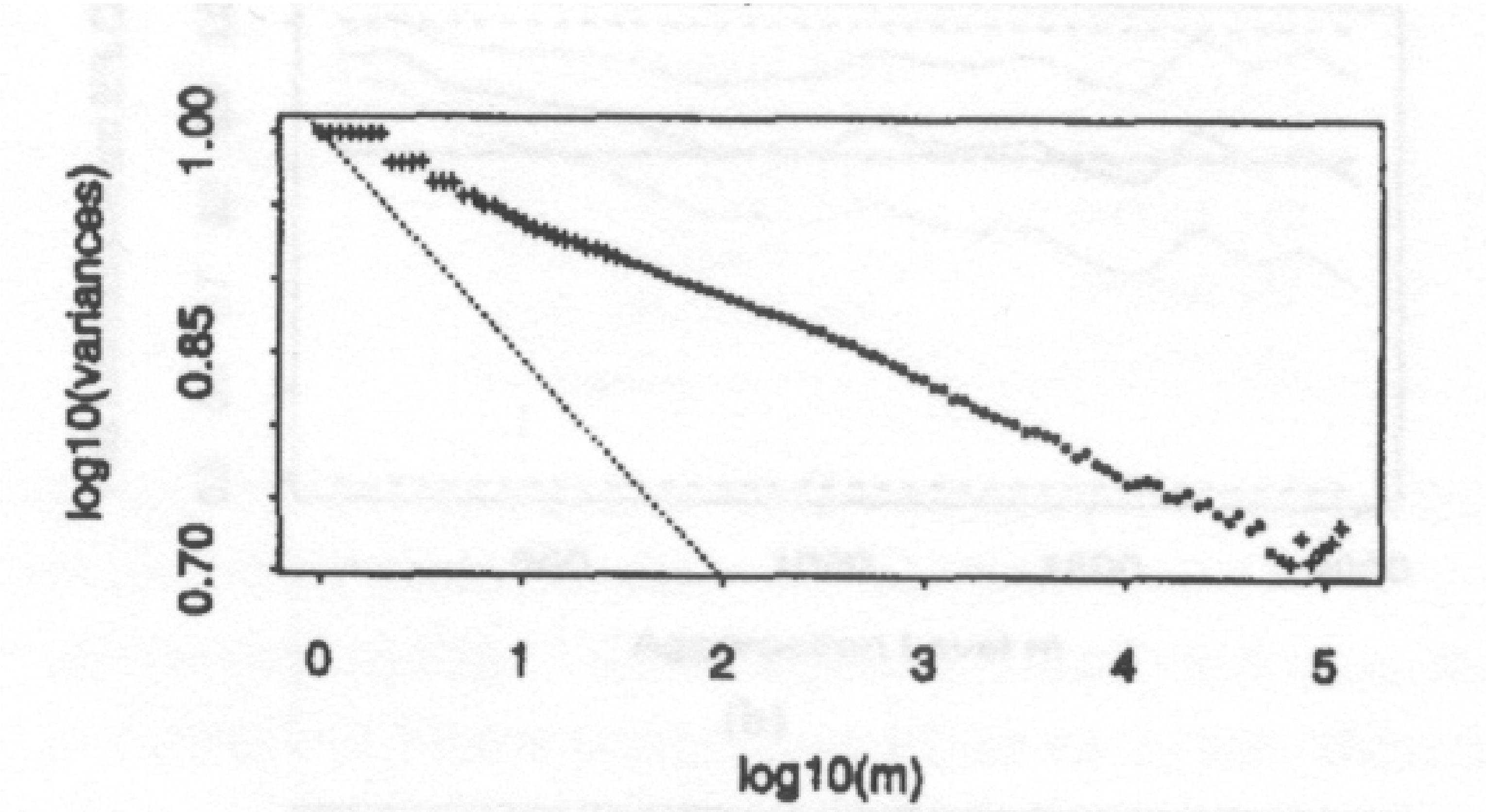
Data from Levine paper

- The following figures are from the paper:
  - Levine, W.E., M.S. Taqqu, W. Willinger and D.V. Wilson. On the self-similar nature of Ethernet traffic (extended version). IEEE/ACM Trans. Networking 2, 1 (February 1984), 1-15.
- They show the same thing, that network delay distributions have LRD in some degree or other.
- The next slide shows an example of a self-similar distribution at five different values of *m* for network traffic (left) and samples drawn from an exponential distribution (right).
- The fact those on the left look substantially “like each other” suggests the distribution has more LRD than SRD.
- The fact those on the right look very different suggests the underlying distribution has more SRD and LRD.

Examples of self-similar traffic on a LAN

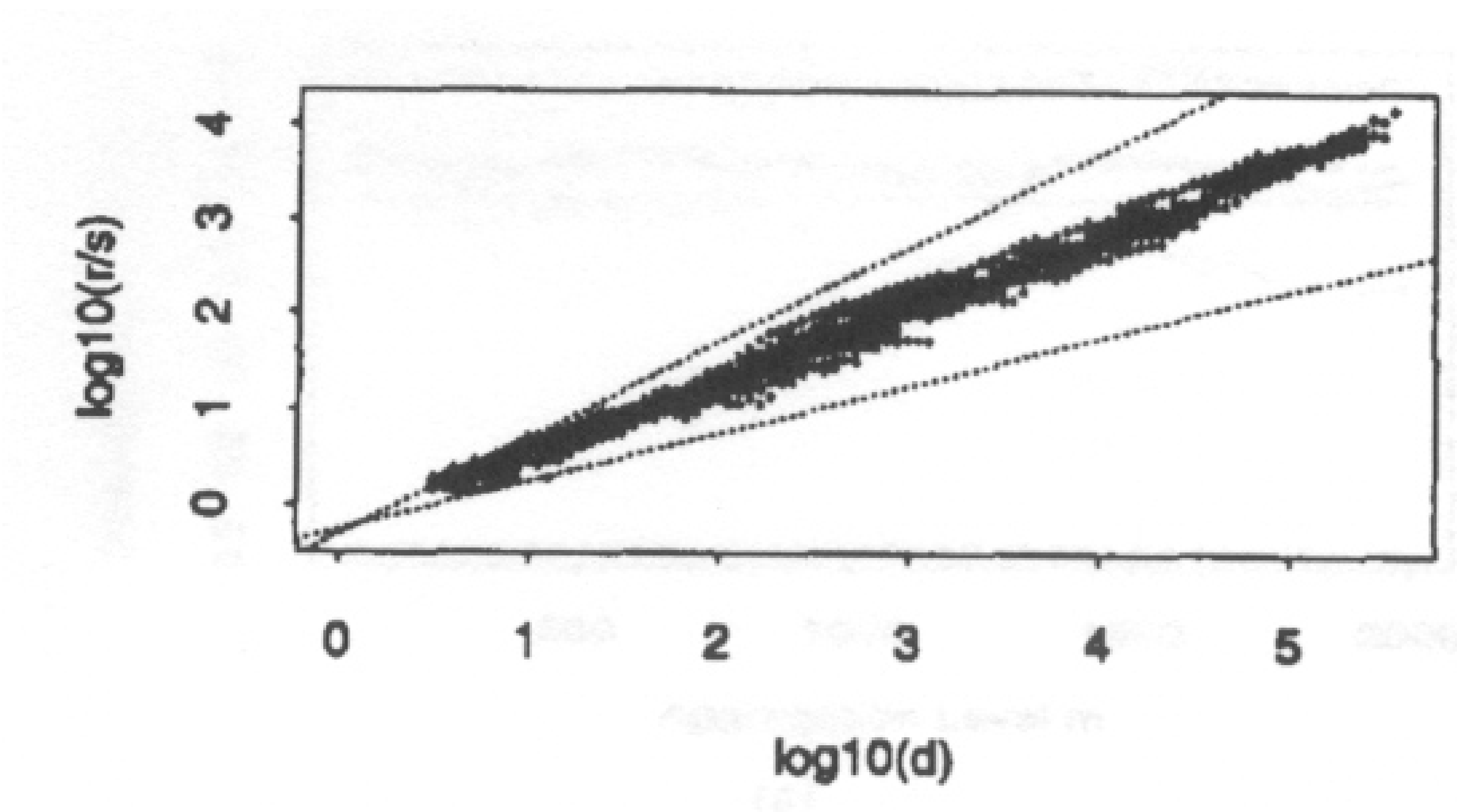


Variance-time plot



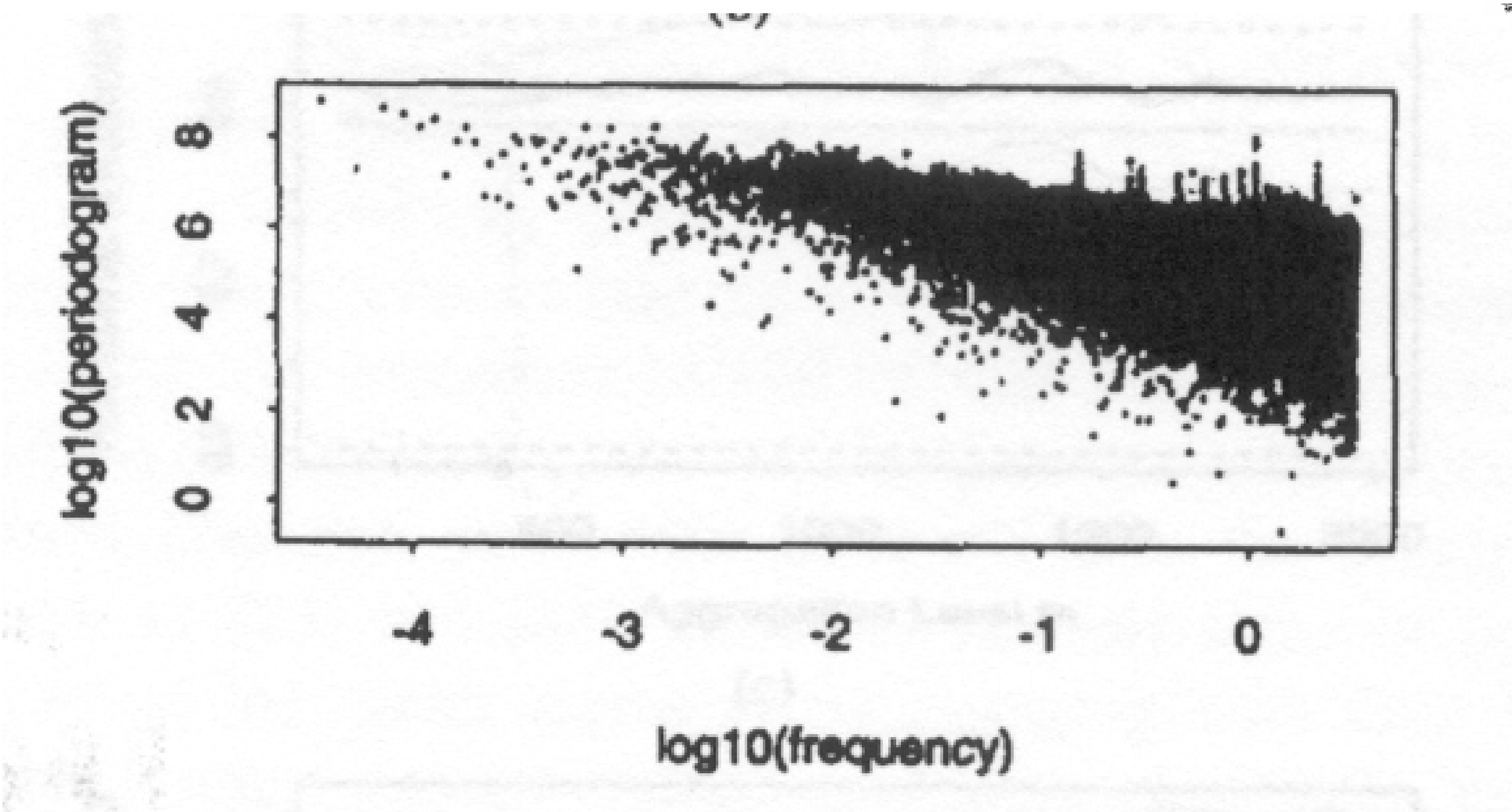
- This is a variance-time plot from the network traffic. The lower line is for H = 0.5. Apparently, the network traffic has LRD 0.5 < H < 1.

R/S plot



- This is a S/R (poc) plot from the network traffic. This further confirms the network traffic has LRD 0.5 < H < 1.

Periodogram (discrete Fourier transform) plot



- This is a periodogram (Fourier transform) from the network traffic. this further confirms the network traffic has LRD 0.5 < H < 1.