Earley Algorithm
Chapter 13.4

Lecture #9

October 2009

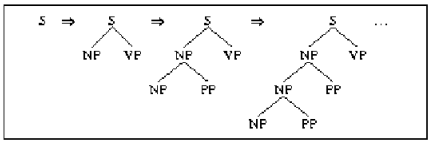
Review

- Top-Down vs. Bottom-Up Parsers
  - Both generate too many useless trees
  - Combine the two to avoid over-generation: Top-Down Parsing with Bottom-Up look-ahead
- Left-corner table provides more efficient look-ahead
  - Pre-compute all POS that can serve as the leftmost POS in the derivations of each non-terminal category
- More problems remain..

Left Recursion

- Depth-first search will never terminate if grammar is left recursive (e.g. NP --> NP PP)

$$(A \xrightarrow{*} \alpha AB, \alpha \xrightarrow{*} \varepsilon)$$



Left-Recursion

- What happens in the following situation
  - S -> NP VP
  - S -> Aux NP VP
  - NP -> NP PP
  - NP -> Det Nominal
  - …
  - With the sentence starting with
    - Did the flight…

Rule Ordering

- S -> Aux NP VP
- S -> NP VP
- NP -> Det Nominal
- NP -> NP PP

- The key for the NP is that you want the recursive option after any base case. Duhh.

Rule Ordering

- S -> Aux NP VP
- S -> NP VP
- NP -> Det Nominal
- NP -> NP PP

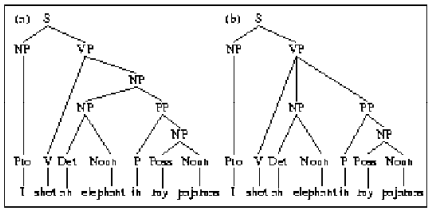
- What happens with
  - Book that flight

- Solutions:
  - Rewrite the grammar (automatically?) to a weakly equivalent one which is not left-recursive
    e.g. The man {on the hill with the telescope…}
    NP → NP PP
    NP → Nom PP
    NP → Nom
    …becomes…
    NP → Nom NP'
    NP' → PP NP'
    NP' → e
    - This may make rules unnatural

  - Harder to eliminate non-immediate left recursion
    - NP --> Nom PP
    - Nom --> NP
  - Fix depth of search explicitly
  - Rule ordering: non-recursive rules first
    NP --> Det Nom
    NP --> NP PP

Structural ambiguity:

- Multiple legal structures
  - Attachment (e.g. I saw a man on a hill with a telescope)
  - Coordination (e.g. younger cats and dogs)
  - NP bracketing (e.g. Spanish language teachers)

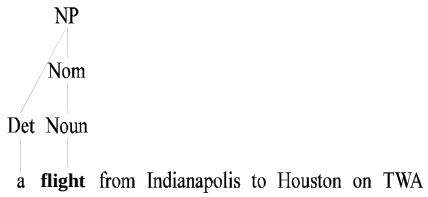


- Solution?
  - Return all possible parses and disambiguate using "other methods"

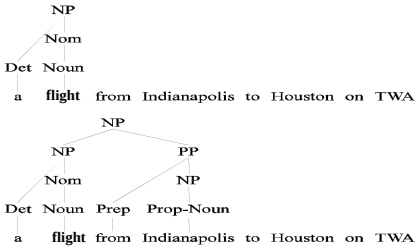
Avoiding Repeated Work

- Parsing is hard, and slow. It's wasteful to redo stuff over and over and over.
- Consider an attempt to top-down parse the following as an NP

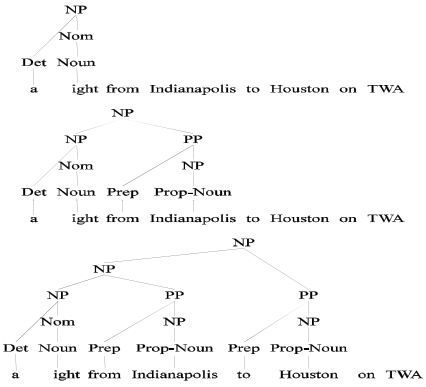
A flight from India to Houston on TWA



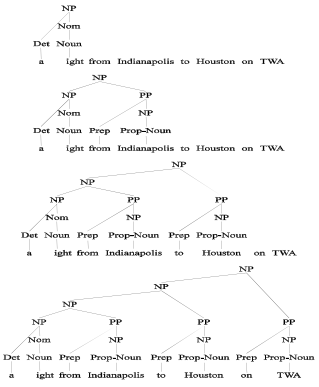
13



14



15



16

Dynamic Programming

- We need a method that fills a table with partial results that
  - Does not do (avoidable) repeated work
  - Does not fall prey to left-recursion
  - Solves an exponential problem in polynomial time (sort of)

17

Dynamic Programming

- Create table of solutions to sub-problems (e.g. subtrees) as parse proceeds
- Look up subtrees for each constituent rather than re-parsing
- Since all parses implicitly stored, all available for later disambiguation
- Examples: Cocke-Younger-Kasami (CYK) (1960), Graham-Harrison-Ruzzo (GHR) (1980) and Earley (1970) algorithms

18

Earley's Algorithm

- Uses dynamic programming to do parallel top-down search in (worst case) $O(N^3)$ time
- First, L2R pass fills out a chart with N+1 states (N: the number of words in the input)
  - Think of chart entries as sitting between words in the input string keeping track of states of the parse at these positions
  - For each word position, chart contains set of states representing all partial parse trees generated to date. E.g. chart[0] contains all partial parse trees generated at the beginning of the sentence

19

Earley Parsing

- Fills a table in a single sweep over the input words
  - Table is length N+1; N is number of words
  - Table entries represent
    - Completed constituents and their locations
    - In-progress constituents
    - Predicted constituents

20

States

- The table-entries are called states and are represented with dotted-rules.

S -> ˙ VP	A VP is predicted
NP -> Det ˙ Nominal	An NP is in progress
VP -> V NP ˙	A VP has been found

21

States/Locations

- It would be nice to know where these things are in the input so…[x,y] tells us where the state begins (x) and where the dot lies (y) wrt the input

S -> ˙ VP [0,0]		A VP is predicted at the start of the sentence
NP -> Det ˙ Nominal	[1,2]	An NP is in progress; the Det goes from 1 to 2
VP -> V NP ˙	[0,3]	A VP has been found starting at 0 and ending at 3

22

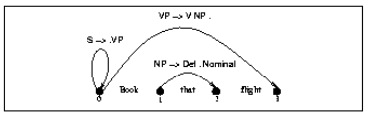
$_0$ Book $_1$ that $_2$ flight $_3$

S --> • VP, [0,0]
- First 0 means S constituent begins at the start of the input
- Second 0 means the dot here too
- So, this is a top-down prediction

NP --> Det • Nom, [1,2]
- the NP begins at position 1
- the dot is at position 2
- so, Det has been successfully parsed
- Nom predicted next

23

VP --> V NP •, [0,3]
- Successful VP parse of entire input



24

Successful Parse

- Final answer found by looking at last entry in chart
- If entry resembles S --> α • [0,N] then input parsed successfully
- But note that chart will also contain a record of all possible parses of input string, given the grammar -- not just the successful one(s)

25

Earley

- As with most dynamic programming approaches, the answer is found by looking in the table in the right place.
- In this case, there should be an S state in the final column that spans from 0 to n+1 and is complete.
- If that's the case you're done.
  - S – α · [0,n+1]

26

Earley

- So sweep through the table from 0 to n+1…
  - New predicted states are created by
  - New incomplete states are created by advancing existing states as new constituents are discovered
  - New complete states are created in the same way.

27

Earley

- More specifically…
  1. Predict all the states you can upfront
  2. Read a word
     1. Extend states based on matches
     2. Add new predictions
     3. Go to 2
  3. Look at N+1 to see if you have a winner

28

Parsing Procedure for the Earley Algorithm

- Move through each set of states in order, applying one of three operators to each state:
  - predictor: add predictions to the chart
  - scanner: read input and add corresponding state to chart
  - completer: move dot to right when new constituent found
- Results (new states) added to current or next set of states in chart
- No backtracking and no states removed: keep complete history of parse

29

Predictor

- Intuition: new states represent top-down expectations
- Applied when non part-of-speech non-terminals are to the right of a dot

  S --> • VP [0,0]
- Adds new states to ***current*** chart
  - One new state for each expansion of the non-terminal in the grammar

    VP --> • V [0,0]

    VP --> • V NP [0,0]

30

Scanner

- New states for predicted part of speech.
- Applicable when part of speech is to the right of a dot
  VP --> • V NP [0,0] 'Book…'
- Looks at current word in input
- If match, adds state(s) to ***next*** chart
  VP --> V • NP [0,1]

Completer

- Intuition: parser has discovered a constituent, so must find and advance all states that were waiting for this
- Applied when dot has reached right end of rule
  NP --> Det Nom • [1,3]
- Find all states w/dot at 1 and expecting an NP
  VP --> V • NP [0,1]
- Adds new (completed) state(s) to ***current*** chart
  VP --> V NP • [0,3]


```
function EARLEY-PARSE(words, grammar) returns chart

  ENQUEUE(γ → • S, [0,0]), chart[0])
  for i ← from 0 to LENGTH(words) do
    for each state in chart[i] do
      if INCOMPLETE?(state) and
            NEXT-CAT(state) is not a part of speech then
          PREDICTOR(state)
      elseif INCOMPLETE?(state) and
            NEXT-CAT(state) is a part of speech then
          SCANNER(state)
      else
          COMPLETER(state)
    end
  end
  return(chart)

procedure PREDICTOR((A → α • B β, [i,j]))
  for each (B → γ) in GRAMMAR-RULES-FOR(B, grammar) do
      ENQUEUE((B → • γ, [j,j]), chart[j])
  end
procedure SCANNER((A → α • B β, [i,j]))
  if B ⊂ PARTS-OF-SPEECH(word[j]) then
      ENQUEUE((B → word[j], [j,j+1]), chart[j+1])
procedure COMPLETER((B → γ •, [j,k]))
  for each (A → α • B β, [i,j]) in chart[j] do
      ENQUEUE((A → α B • β, [i,k]), chart[k])
  end
procedure ENQUEUE(state, chart-entry)
  if state is not already in chart-entry then
      PUSH(state, chart-entry)
  end
```


Book that flight (Chart [0])

- Seed chart with top-down predictions for S from grammar

γ → • **S**	[0,0]	Dummy start state
S → • NP VP	[0,0]	Predictor
S → • Aux NP VP	[0,0]	Predictor
S → • VP	[0,0]	Predictor
NP → • Det Nom	[0,0]	Predictor
NP → • PropN	[0,0]	Predictor
VP → • V	[0,0]	Predictor
VP → • V NP	[0,0]	Predictor

CFG for Fragment of English

S → NP VP	Det → that | this | a
S → Aux NP VP	N → book | flight | meal | money
S → VP	V → book | include | prefer
NP → Det Nom	Aux → does
Nom → N	
Nom → N Nom	Prep →from | to | on
NP →PropN	PropN → Houston | TWA
VP → V	Nom → Nom PP
VP → V NP	PP → Prep NP

- When dummy start state is processed, it's passed to Predictor, which produces states representing every possible expansion of S, and adds these and every expansion of the left corners of these trees to bottom of Chart[0]
- When VP --> • V, [0,0] is reached, Scanner called, which consults first word of input, Book, and adds first state to Chart[1], V --> Book •, [0,1]
- Note: When VP --> • V NP, [0,0] is reached in Chart[0], Scanner expands the rule yielding VP --> V . NP, [0,1] but does not put V --> Book •, [0,1] in again.

Example

Chart[0]

γ → • *S*	[0,0]	Dummy start state
S → • *NP VP*	[0,0]	Predictor
NP → • *Det NOMINAL*	[0,0]	Predictor
NP → • *Proper-Noun*	[0,0]	Predictor
S → • *Aux NP VP*	[0,0]	Predictor
S → • *VP*	[0,0]	Predictor
VP → • *Verb*	[0,0]	Predictor
VP → • *Verb NP*	[0,0]	Predictor

Chart[1]

Verb → *book* •	[0,1]	Scanner
VP → *Verb* •	[0,1]	Completer
S → *VP* •	[0,1]	Completer
VP → *Verb* • *NP*	[0,1]	Completer
NP → • *Det NOMINAL*	[1,1]	Predictor
NP → • *Proper-Noun*	[1,1]	Predictor

Chart[1]

V→ book •	[0,1]	Scanner
VP → V •	[0,1]	Completer
VP → V • NP	[0,1]	Completer
S → VP •	[0,1]	Completer
NP → • Det Nom	[1,1]	Predictor
NP → • PropN	[1,1]	Predictor

V--> book • passed to Completer, which finds 2 states in Chart[0,0] whose left corner is V and adds them to Chart[0,1], moving dots to right

- When VP → V • is itself processed by the Completer, S → VP • is added to Chart[1] since VP is a left corner of S
- Last 2 rules in Chart[1] are added by Predictor when VP → V • NP is processed
- And so on….

Example

Chart[1]

Verb → *book* •	[0,1]	Scanner
VP → *Verb* •	[0,1]	Completer
S → *VP* •	[0,1]	Completer
VP → *Verb* • *NP*	[0,1]	Completer
NP → • *Det NOMINAL*	[1,1]	Predictor
NP → • *Proper-Noun*	[1,1]	Predictor

Chart[2]

Det → *that* •	[1,2]	Scanner
NP → *Det* • *NOMINAL*	[1,2]	Completer
NOMINAL → • *Noun*	[2,2]	Predictor
NOMINAL → • *Noun NOMINAL*	[2,2]	Predictor

Example

Chart[2]

Det → *that* •	[1,2]	Scanner
NP → *Det* • *NOMINAL*	[1,2]	Completer
NOMINAL → • *Noun*	[2,2]	Predictor
NOMINAL → • *Noun NOMINAL*	[2,2]	Predictor

Chart[3]

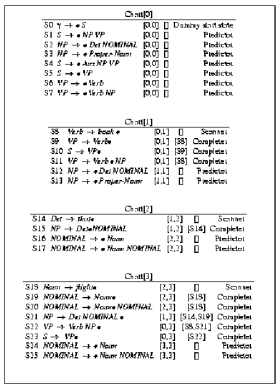
Noun → *ight* •	[2,3]	Scanner
NOMINAL → *Noun* •	[2,3]	Completer
NOMINAL → *Noun* • *NOMINAL*	[2,3]	Completer
NP → *Det NOMINAL* •	[1,3]	Completer
VP → *Verb NP* •	[0,3]	Completer
S → *VP* •	[0,3]	Completer
NOMINAL → • *Noun*	[3,3]	Predictor
NOMINAL → • *Noun NOMINAL*	[3,3]	Predictor

What is it?

- What kind of parser did we just describe (trick question).
  - Earley parser… yes
  - Not a parser – a recognizer
    - The presence of an S state with the right attributes in the right place indicates a successful recognition.
    - But no parse tree… no parser

How do we retrieve the parses at the end?

- Augment the Completer to add ptr to prior states it advances as a field in the current state
  - I.e. what state did we advance here?
  - Read the ptrs back from the final state
- Do we NEED the pointers?



Useful Properties

- Error handling
- Alternative control strategies

Error Handling

- What happens when we look at the contents of the last table column and don't find a S --> α• rule?
  - Is it a total loss? No...
  - Chart contains every constituent and combination of constituents possible for the input given the grammar
- Also useful for partial parsing or shallow parsing used in information extraction

Alternative Control Strategies

- Change Earley top-down strategy to bottom-up or ...
- Change to best-first strategy based on the probabilities of constituents
  - Compute and store probabilities of constituents in the chart as you parse
  - Then instead of expanding states in fixed order, allow probabilities to control order of expansion

Summing Up

- Ambiguity, left-recursion, and repeated re-parsing of subtrees present major problems for parsers
- Solutions:
  - Combine top-down predictions with bottom-up look-ahead
  - Use dynamic programming
  - Example: the Earley algorithm
- Next time: Read Ch 15