CISC 320 Introduction to Algorithms Fall 2005

Course Overview

Administrative

- Course syllabus
- Course webpage: http://www.cis.udel.edu/~lliao/cis320f05
- Office hours: Tuesdays and Thursdays 10:00AM-11:30AM
- TA: Mani Thomas (mani@Udel.Edu)
  - Tuesdays 3:30PM-5:30PM, 115B Pearson Hall
- Your name and email
- A random ID will be assigned to you for the purpose of posting grades.

A letter from industry

> Subject: Industry perspective on CS
> Date: 12 Jul 2003 13:48:26 -0700
>
> I had a conversation 2 weeks ago with a colleague at Intel.
> His group was looking for a few CS people last year. He said that
> almost everyone he interviewed was "completely useless: they know
> nothing about algorithms, analysis, or any kind of mathematical
> reasoning; the only thing they know is Java." This seems to match
> what I hear a lot of people say around here about recent CS grads.
> It's like they go to college for 4 years and come out knowing one
> thing.
>
> Has CIS switched from C++ to Java? That would be a mistake if any of
> them want to work for Intel -- I think the assumption here is if your
> primary language is Java, then you're worthless. (BTW, most of our
> work is a mix of C++ and C, with the occasional Perl script here and
> there.)
>
> Kevin
- --

What is an algorithm?

- Step-by-step, computer executable, solution to a "computable" problem.
- Yes, non-computable problems do exist.
  - E.g., Turing's Halting Problem
  - Computability is a subject of study of CISC 401
- Computable problems are many: -)
  - The Human Genome Project: sequence assembly, gene identification …
  - The Internet: Information search – Google it.
  - E-Commerce: data security (RSA Public-Key cryptography)
    - RSA won the Turing Award in 2002. R is Rivest, one of the authors of our textbook.
  - Airline Scheduling:
  - …

Logistics (by problem types) adopted

- Part 1: Sorting algorithms and Selection algorithms
- Part 2: some advanced Data Structures: Hash tables, Red-Black Trees, and Disjoint Sets.
- Part 3: Graph algorithms
- Part 4: NP-completeness; Approximation algorithms; Parallel algorithms

Workload

- Five homework assignments (8% each)
  - Four "paper-and-pencil" and one programming
- Two exams (30% each)
  - One is on Oct. 18, and the other is given during the final
  - Mainly facts problems, e.g., describing an algorithm learned in the class

Bottom line

- Contents
  - Algorithms
    - Able to recite the main ideas
      - Convince others and yourself it works
      - Complexity
      - Other characteristics, e.g., on-line v.s. off-line.
    - Able to write (pseudo) code for it
- Methods
  - Several design techniques
  - Proof techniques (induction, decision trees)
  - Ways of abstract thinking

Designing Algorithms

- Designing paradigms
  - Brute Force
  - Divide and Conquer
  - Greedy
  - Dynamic Programming
  - Randomized
  - Parallel

Analyzing Algorithms

- Correctness
  - Can be proved (recursion and induction)
  - Approximation
- Complexity (efficiency)
  - Time and Space
  - Worst-case, Average-case, and Best case
  - Asymptotical
  - Optimality
- Simplicity and Clarity

How to solve it? (Polya's book)

- What is the problem?
- Can we solve it anyway (Brute force)?
  - Is our solution correct?
    - All inputs
    - Special inputs (boundaries and/or limits)
- Can we solve it more efficiently?
  - Have we used all info available?
    - Need specific data structure to store the input and/or intermediaries?
  - Can we divide and conquer, be greedy, do dynamic programming, etc.?
  - Can we improve average-case, if not worst-case, performance? Amortizing? Do we need to randomize to enforce a favorable average-case performance?
  - …

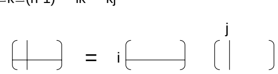
Optimality

- An algorithm is Optimal: if its time complexity = the complexity of the problem.

Complexity of problems = necessary and sufficient work to solve the problem.

- Lower bound: the least work (or steps) needed; but no guarantee to solve the problem.
- Upper bound: the sufficient work (or steps) needed; from known algorithms that solve the problem.
- Complexity of the problem is given by the lower and upper bounds that meet each other.
- Complexity of the problem is unknown when there is a gap between the tightest known lower and upper bounds.
  - E.g., multiplication of two n-by-n matrices: Tightest lower bound is $O(n^2)$ and tightest upper bound is $O(n^{2.376})$.

- Example 1: finding the largest entry in an array.
  - we do not know the complexity because the problem is not well-defined. E.g., if the array is already sorted?
- Example 2: matrix multiplication
  - $C_{ij} = \Sigma_{0 \le k \le (n-1)} A_{ik} B_{kj}$



- Brute force (n^3)
- Strassen's algorithm ($n^{2.81}$)
- Complexity of matrix multiplication is not yet known.

Example 3: Sequential search of an *unordered* array E

```
int seqSearch (int[] E, int n, int k)
1   int ans, index;
2   ans = -1;
3   for (i = 0; i < n; i++) {
4         if(k == E[i])
5             ans = i;
6             break;
7   return ans;
```

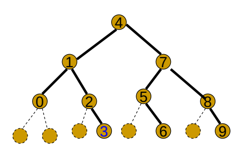
```
Number of comparisons (execution of line 4)
Worst-case: n
Best-case: 1
Average-case: ???
```


- Example 4: Searching an *ordered* array
  - Alg1
    - Early stop => improve the average-case, no impact on the worst-case
  - Alg2
    - Search every j entry => n/j + j comparisons
  - Alg3
    - Optimizing on j => $T_{wc}(n) = 2\sqrt{n}$ comparisons
  - Alg4: binary search => $T_{wc}(n) = \lg n$
    - This is an optimal algorithm. Why?
    - The binary search is not an *on-line* algorithm

Complexity of searching an ordered array of integers via comparisons is lg(n+1), where n is the array size.

- Proof
  - Decision tree (a binary tree) is to visualize operation flow of an algorithm
  - Let p = max # of comparisons = # of nodes on the Longest path of the decision tree
  - Let N = max # of nodes in decision tree. $N \le 1 + 2 + 4 + \ldots + 2^{p-1} = 2^p - 1$ (given the height, a balanced binary tree can hold more nodes than unbalanced).
  - $p \ge \lg(N + 1)$
  - $N \ge n$, *where n is the array size.* *(because every entry in array must appear at least once on the decision tree for the algorithm to work correctly)*
    - Note: this does not say every node will actually be visited during a run of the algorithm.

index:	0	1	2	3	4	5	6	7	8	9
E1:	21	33	45	51	60	67	72	78	79	85



- Spectrum of computational complexity