CISC 320 010 Introduction to Algorithms (Fall 2005)

Homework 1
Handed out: September 8, 2005
Due date: September 20, 2005

Your handwriting must be legible, and your answers should be rigorous, concise and in proper order. Please note that the work handed in must be your own.

1. CLRS 3-1 (18 points)

2. (30 points) Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, or Θ of B. Assume that are c > 1 and ε >0 constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

	A	B	O	o	Ω	Θ
1	$\sqrt{n}$	$n^{\sin n}$				
2	$n^{\lg(c)}$	$c^{\lg(n)}$				
3	2^n	$n^{n/2}$				
4	$\lg(n)$	n^{ε}				
5	$\lg(n!)$	$\lg(n^n)$				

3. CLRS 4.3-2 (17 points) The recurrence $T(n) = 7T(n/2) + n^2$ describes the running time of an algorithm *A*. A competing algorithm *A'* has a running time of $T'(n) = aT'(n/2) + n^2$. What is the largest integer value for *a* such that *A'* is asymptotically faster than *A*?

4. (35 points) Algorithm *A* is a divide and conquer type with recurrence relation $T(n) = 4T(n/2) + n^2 lg(n)$. Answer the following questions about the recursive tree of *A* on input size $n = 2^m$, where *m* is an positive integer.
   a. The root starts at level 0, then what is the input size to a node at level *i*?
   b. How many nodes at level *i*?
   c. How much total work is actually done at level *i*?
   d. The root is at level 0. What level are the leaves at?
   e. Write a series that represent the running time for the algorithm for inputs of size $n = 2^m$, and give explicitly the starting and ending value for the index in the series.
   f. Solve the series.
   g. Give the tightest *big-O* bound for *T(n)*.