# Partition Optimization in LDPC Coded OFDM Systems with PTS PAPR Reduction

Li Li, Daiming Qu and Tao Jiang

Abstract-A joint decoding scheme was proposed in [1] to recover low-density parity-check (LDPC) codeword and partial transmit sequence (PTS) phase factors, for OFDM systems with low peak-to-average power ratio (PAPR). However, the errorcorrecting performance of the joint decoding scheme heavily relies on how the OFDM subcarriers are partitioned into groups in the PTS scheme. With a pseudo-random partition, the joint decoding scheme provides satisfactory error-correcting performance only when the number of PTS groups is very small [1]. In this paper, we formulate an optimization problem to improve the joint decoding performance by optimizing the partition. Furthermore, two greedy-based algorithms are proposed to solve the problem. Simulation results show that the joint decoding scheme with the proposed partition algorithms provides satisfactory error-correcting performance for a larger number of PTS groups, than it does with the pseudo-random partition. With the improved performance, better PAPR performance can be supported.

*Index Terms*—Peak-to-average power ratio (PAPR), orthogonal frequency division multiplexing (OFDM), low-density parity-check (LDPC), greedy algorithm, partial transmit sequence (PTS).

#### I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) has been widely adopted in various wireless communication standards, due to its capability to efficiently cope with frequency selective channels. However, one major drawback of OFDM systems is high peak-to-average power ratio (PAPR). Among a variety of PAPR reduction techniques [2], the partial transmit sequence (PTS) scheme has attracted a lot of attention, since it introduces no distortion in the transmitted signal and achieves significant PAPR reduction [3], [4]. However, the PTS phase factor information is required at the receiver as side information, which decreases the transmission efficiency or complicates the system design.

In [1], a joint decoding scheme was proposed to recover low-density parity-check (LDPC) [9]–[11] codeword and PTS phase factors, which avoids the transmission of PTS side information. Particularly, the PTS processing is viewed as a stage of coding, and the parity-check matrix and Tanner graph of the concatenated LDPC-PTS code are derived in [1]. With

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Li Li, Daiming Qu and Tao Jiang are with Department of Electronics and Information Engineering, Huazhong University of Science and Technology, Wuhan, 430074, China. (Corresponding author: Daiming Qu) the derived parity-check matrix and Tanner graph, the LDPC codeword and PTS phase factors are jointly decoded using a standard LDPC decoder. Compared with the other schemes to recover the phase factors, such as [5]–[8], the joint decoding scheme simplifies the system design, since it does not require the detection of phase factors before decoding.

In this paper, it is pointed out that the error-correcting performance of the joint decoding scheme heavily relies on how the OFDM subcarriers are partitioned into groups in the PTS scheme. With a pseudo-random partition, the joint decoding scheme provides satisfactory error-correcting performance only when the number of PTS groups is very small [1]. Then, we formulate an optimization problem to improve the joint decoding performance by optimizing the partition, and propose two greedy-based algorithms to solve the problem. Simulation results show that, compared with the pseudo-random partition, the proposed partition method offers much better joint decoding performance in terms of error correcting, convergence speed and complexity.

The remainder of this paper is organized as follows. In Section II, we briefly recall the joint decoding scheme in [1] with a more general presentation. The optimization problem and partition algorithms are presented in Section III. Then, the simulation results are presented in Section IV, followed by conclusions and future works in Section V.

## II. SYSTEM MODEL

In the PTS scheme, the OFDM subcarriers are partitioned into several groups, then the groups are phase rotated separately with proper phase factors [3], [4]. As explained in [1], rotating a phase-shift keying (PSK) or quadrature-amplitude modulation (QAM) symbol with Gray mapping is equivalent to flipping several bits of the symbol, when the rotating phase factors are chosen from  $\{1; -1\}$ . For example, there are two bits stand for the sign of the I-phase and Q-phase component, respectively, among the four bits of a Gray mapping 16QAM symbol. In the case of multiplying the Gray mapping 16QAM symbol by phase factor of -1, only the two sign bits are flipped and the other two bits are never affected. Therefore, for clarity purpose, we briefly recall the joint decoding scheme [1] with a more general presentation, in which the phase rotating is replaced with bit flipping.

## A. PAPR Reduction by Flipping Bit Groups in LDPC Coded OFDM Systems

As shown in Fig. 1, the LDPC codeword of length N, denoted as A, is partitioned into U coded bit groups

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Fig. 1. The system diagram for PAPR reduction by flipping bit groups.

 $\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_U$ . For clarity, we reorder the coded bits such that  $\mathbf{A} = [\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_U]$ . Moreover, let S denote the set of bit indexes of  $\mathbf{A}$ , i.e.,  $S = \{1, 2, \cdots, N\}$ , and  $S_u$  denote the set of bit indexes of  $\mathbf{A}_u$  ( $u = 1, 2, \cdots, U$ ). Apparently,  $S_u \subset S$ , and  $S_u \bigcap S_{u'} = \emptyset$  if  $u \neq u'$ . Let  $N_u$  ( $0 < N_u < N, \sum_{u=1}^U N_u = N$ ) denote the number of elements in  $S_u$  and  $i_u^k$  denote the k-th element of  $S_u$ , then  $S_u = \{i_u^1, i_u^2, \cdots, i_u^{N_u}\}$ , for  $u \in 1, 2, \cdots, U$ . Note that,  $N_u$  is not necessarily the same for different u. Accordingly, the parity-check matrix of the LDPC code, denoted by  $\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_U$ , respectively. Let  $\mathbf{h}(i)$  ( $1 \leq i \leq N$ ) denote the *i*th column of  $\mathbf{H}$ , then  $\mathbf{H}_u = [\mathbf{h}(i_u^1), \mathbf{h}(i_u^2), \cdots, \mathbf{h}(i_u^{N_u})]$ , for  $u \in 1, 2, ..., U$ .

To reduce PAPR, the phase optimization module generate U-1 phase bits, denoted as  $b_u$  ( $b_u \in \{0,1\}, 1 \le u \le U-1$ ), to control the flipping of the corresponding bit group  $\mathbf{A}_u$ : the bit group  $\mathbf{A}_u$  is flipped if  $b_u$  equals to 1, otherwise  $\mathbf{A}_u$  remains unchanged. Let  $\mathbf{A}_u$  ( $u = 1, 2, \dots, U$ ) denote the output bits of the flipping operation at the transmitter, then

$$\widetilde{\mathbf{A}}_{u} = \begin{cases} \mathbf{A}_{u} \oplus [\underbrace{b_{u}, \dots, b_{u}}_{N_{u}}], & u = 1, 2, \cdots, U - 1\\ \mathbf{A}_{U}, & u = U \end{cases}, \quad (1)$$

where  $\oplus$  represents modulo 2 addition. Let **A** denote the codeword after the flipping operation, i.e.,  $\widetilde{\mathbf{A}} = [\widetilde{\mathbf{A}}_1, \cdots, \widetilde{\mathbf{A}}_{U-1}, \mathbf{A}_U]$ . Note that, the coded bits in  $\mathbf{A}_U$  are never flipped due to two facts: 1) it is not necessary to involve all subcarrier groups in phase rotation of the PTS processing [3], [4]; 2) some bits are never affected even their corresponding symbols are multiplied by -1 [1]. Finally, the transmitted signal is formed after PSK/QAM mapping and OFDM multiplexing.

Apparently, the original codeword  $\mathbf{A}$  can be transformed into  $2^{U-1}$  different candidates (including  $\mathbf{A}$  itself with all-zero phase bits), with the flipping operation. For the best case, the candidate corresponding to the time signal with the minimum PAPR is selected for transmission. Thus, the PAPR can be significantly reduced by flipping the bit groups, especially when the group number U is large.

## B. Joint Decoding of LDPC Codewords and Phase bits

As proposed in [1], the PTS processing is viewed as a stage of coding, and the parity-check matrix and Tanner graph of the concatenated LDPC-PTS code are derived. With the derived parity-check matrix and Tanner graph, the LDPC codeword and phase bits can be jointly decoded using a standard LDPC decoder. Here, the concatenated LDPC-PTS codeword, which consists of  $\widetilde{A}$  and phase bits, is denoted by,

$$\mathbf{A}_{\mathrm{EX}} = [(\mathbf{A}_{\mathrm{EX}})_1, \cdots, (\mathbf{A}_{\mathrm{EX}})_{U-1}, \mathbf{A}_U]$$
(2)

where  $(\mathbf{A}_{\mathrm{EX}})_u = [\mathbf{A}_u, b_u]$  for  $u = 1, 2, \cdots, U-1$ . Obviously, the transmitted codeword  $\mathbf{\tilde{A}}$  is a punctured version of  $\mathbf{A}_{\mathrm{EX}}$ . The extended parity-check matrix that corresponds to  $\mathbf{A}_{\mathrm{EX}}$ is denoted as  $\mathbf{H}_{\mathrm{EX}} = [(\mathbf{H}_{\mathrm{EX}})_1, \cdots, (\mathbf{H}_{\mathrm{EX}})_{U-1}, \mathbf{H}_U]$ , where  $(\mathbf{H}_{\mathrm{EX}})_u$  is the extension of  $\mathbf{H}_u$  by appending column  $\mathbf{g}_u$ , which corresponds to the phase bit  $b_u$ , i.e.,

$$(\mathbf{H}_{\mathrm{EX}})_u = [\mathbf{h}(i_u^1), \cdots, \mathbf{h}(i_u^{N_u}), \mathbf{g}_u], \ u = 1, \cdots, U - 1.$$
 (3)

According to Theorem 1 in [1], the appended column  $\mathbf{g}_u$  is generated with the following rule in GF(2):

$$\mathbf{g}_u = \sum_{i_u^k \in \mathcal{S}_u} \mathbf{h}(i_u^k), \text{ for } u = 1, 2, \cdots, U - 1.$$
(4)

With the appended columns, each row of  $(\mathbf{H}_{EX})_u$   $(1 \le u \le U-1)$  has even hamming weight, i.e, the number of nonzero elements in each row of  $(\mathbf{H}_{EX})_u$  is even. Note that, there is no need to extend  $\mathbf{H}_U$ , since the bit group  $\mathbf{A}_U$  is never flipped. In addition, the Tanner graph corresponding to  $\mathbf{H}_{EX}$  is called as extended Tanner graph, which is the extension of the original Tanner graph by adding U - 1 phase nodes and the edges connected to the phase nodes, where the U - 1 phase nodes correspond to the U - 1 phase bits, respectively.

At the receiver, after multicarrier demultiplexing and demapping, the extended LDPC codeword  $\mathbf{A}_{EX}$  is recovered by a standard LDPC decoder. The decoder employs  $\mathbf{H}_{EX}$  as the parity-check matrix and takes the received  $\widetilde{\mathbf{A}}$  as input (note that  $\widetilde{\mathbf{A}}$  is a punctured version of  $\mathbf{A}_{EX}$ ). Then, the decoded phase bits are extracted from the decoded  $\mathbf{A}_{EX}$ , and the original LDPC codeword  $\mathbf{A}$  is obtained after the flipping operation controlled by the decoded phase bits.

However, the degrees of phase nodes, i.e., the hamming weights of  $\mathbf{g}_u$  ( $1 \le u \le U - 1$ ), are very high, when the pseudo-random partition of  $S_1, S_2, \dots, S_U$  is employed [1]. As shown in Table I of [1], the phase node degrees are about M/2, where M denotes the number of check nodes in the Tanner graph. A great number of four cycles are introduced with the high-degree phase nodes, which leads to slow convergence of decoding and error-correcting performance degradation, especially when the group number U is large.

### **III. PARTITION OPTIMIZATION**

In this section, we formulate the optimization problem to improve the error-correcting performance of joint decoding by group partitioning, and propose two greedy-based algorithms to solve the problem.

## A. Optimization Objective

Since high-degree phase nodes are the direct cause of performance degradation, a natural and straightforward optimization objective is to minimize the degrees of phase nodes. Therefore, our optimization aims to minimize the highest degree of the phase nodes by choosing elements for  $S_u$ . In other words, we aim to minimize the maximum hamming weight of the columns  $\{\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_{U-1}\}$ . According to (4), we have

$$\Gamma\left(\mathbf{g}_{u}\right) = \Gamma\left(\sum_{i_{u}^{k} \in \mathcal{S}_{u}} \mathbf{h}(i_{u}^{k})\right), \text{ for } u = 1, 2, \cdots, U-1, \quad (5)$$

where  $\Gamma(\cdot)$  represents the Hamming weight of a vector. Thus, the optimization problem is formulated as

$$\min\left\{\max_{u=1,2,\ldots,U-1}\left[\Gamma\left(\sum_{i_{u}^{k}\in\mathcal{S}_{u}}\mathbf{h}(i_{u}^{k})\right)\right]\right\},\quad(6a)$$

subject to  $S_1 \cup S_2 \cup ... \cup S_U = S$ , (6b)

$$\mathcal{S}_u \cap \mathcal{S}_{u'} = \varnothing, \text{if } u \neq u',$$
 (6c)

$$|\mathcal{S}_u| = N_u,\tag{6d}$$

where  $|\cdot|$  denotes the number of elements in a set.

It is worth noting that other constraints on groups could be included in the formulation. For example, since the phase rotation of a Gray-mapped PSK/QAM symbol only flips its sign bits and does not change the other bits [1], it is highly recommended that only the bits affected by the phase rotation be chosen for  $S_u(1 \le u \le U-1)$ , and the other bits be kept in  $S_U$ . In this case, the bit flipping operation is exactly the same as the phase rotation operation of PTS. Otherwise, the flipping operation can not be accomplished by simply rotating the symbols, which will increase the implementation complexity. Anyway, we assume that any coded bits can be included in any  $S_u$  for simplicity of presentation in this paper.

Obviously, exhaustive search for the optimal solution is unacceptable in terms of complexity, when the codeword length or group number is large. In the followings, we propose two algorithms to obtain the sub-optimal solutions based on greedy criterion, which significantly reduce the computational complexity and achieve satisfactory results.

## B. Greedy Partition (GP)

In this subsection, we propose a low-complexity partition algorithm to obtain a sub-optimal solution, called as greedy partition (GP).

The key idea of the GP algorithm is explained as follows: For initialization, we set  $S_u = \emptyset$  for  $u = 1, 2, \dots, U - 1$ , and  $S_U = \{1, 2, \dots, N\}$ ; For each step of the algorithm, we choose an element from  $S_U$ , denoted as *i*, and move it to  $S_u$  $(u \in \{1, 2, \dots, U - 1\})$  according to the following criterion in GF(2):

$$\min_{i \in \mathcal{S}_U} \left\{ \Gamma\left(\sum_{\substack{i_u^k \in \mathcal{S}_u}} \mathbf{h}(i_u^k) + \mathbf{h}(i)\right) \right\}.$$
 (7)

Eq. (7) means that the Hamming weight  $\Gamma\left(\sum_{i_u^k \in S_u} \mathbf{h}(i_u^k)\right)$  is always minimized after adding a new element to  $S_u$ . Due to this criterion, we call it greedy partition. The detailed GP algorithm is presented in Algorithm 1. In Algorithm 1,  $L_u$ denotes the number of elements to be assigned to  $S_u$ , and  $L_u = N_u$  ( $u = 1, 2, \dots, U - 1$ ) if  $S_1, S_2, \dots, S_{U-1}$  are initialized to  $\varnothing$ .

Algorithm 1 GP Algorithm
for $j = 1$ to $\max(L_u)$ do
for $u = 1$ to $U - 1$ do
if $j \leq L_u$ then
Choose i in $S_U$ to satisfy Eq. (7).
Update $S_u$ and $S_U$ as: $S_u = S_u \cup \{i\}, S_U = S_U \setminus i$
end if
end for
end for

### C. Iterative Greedy Partition (IGP)

In this subsection, we propose an iterative greedy partition (IGP) algorithm to iteratively improve the partition. In each iteration of the IGP algorithm, with the partition result of the last iteration, we execute the GP-Reverse (GP-R) algorithm, then execute GP algorithm again to obtain a new partition result.

The GP-R algorithm is the reverse operation of the GP algorithm: for each step, an element is moved from  $S_u$   $(u = 1, 2, \dots, U - 1)$  back to  $S_U$ . The criterion of choosing the element is that the Hamming weight  $\Gamma\left(\sum_{i_u^k \in S_u} \mathbf{h}(i_u^k)\right)$  is minimized after deleting the element (denoted as i), i.e.,

$$\min_{i \in \mathcal{S}_u} \left\{ \Gamma\left(\sum_{\substack{i_u^k \in \mathcal{S}_u}} \mathbf{h}(i_u^k) - \mathbf{h}(i)\right) \right\}.$$
 (8)

The number of elements to be deleted is set to be  $d_r N_u$  for  $S_u$ ,  $u = 1, 2, \dots, U-1$ , where  $0 < d_r < 1$ . The detailed GP-R algorithm is summarized in Algorithm 2.

Algorithm 2 GP-R Algorithm
for $j = 1$ to $\max(d_r N_u)$ do
for $u = 1$ to $U - 1$ do
if $j \leq d_r N_u$ then
Choose <i>i</i> from $S_u$ to satisfy Eq. (8).
Update $S_u$ and $S_U$ as: $S_u = S_u \setminus i$ , $S_U = S_U \cup \{i\}$
end if
end for
end for

As shown in Fig. 2, the proposed IGP algorithm iteratively executes the GP and GP-R algorithm until the iteration number reaches the predefined maximum  $n_{max}$ . Note that, except for



Fig. 2. Flow chart of the proposed IGP Algorithm.

the first iteration,  $L_u$  in the GP algorithm equals to  $d_r N_u$ , i.e.,

$$L_u = \begin{cases} N_u, & n = 1 \\ d_r N_u, & 1 < n \le n_{max} \end{cases}.$$
 (9)

Since one partition result, i.e.,  $S_1, S_2, \dots, S_U$ , is obtained in each iteration, there are  $n_{max}$  partition results when the iteration stops. Finally, we choose the partition result that minimizes the highest degree of the phase nodes as the final partition result.

### **IV. SIMULATION RESULTS**

In this section, we demonstrate the error-correcting and PAPR performance of the proposed partition method by computer simulation. The OFDM system employs quasi-cyclic LDPC codes (QC-LDPC) of length 1152, QPSK or 16-QAM modulation, and cyclic prefix of 1/4 the symbol duration. The subcarrier number is 576 for OPSK modulation, and 288 for 16-QAM. For the LDPC codes of all rates, we employ QC-LDPC codes in the IEEE 802.16e standard [12]. Each bit group consists of N/U bits, i.e.,  $N_u = N/U(1 \le u \le U)$ . Without sacrificing the PAPR performance, group  $A_U$  are excluded from the flipping operation. The iteration number of the proposed IGP method is  $n_{max} = 100$ . In all simulations, the phase bits are not transmitted by the transmitter. The receiver employs a log-likelihood ratio BP (LLR-BP) decoder of 40 iterations, which is far less than the 500 iterations employed in [1].

Table I shows the degrees of phase nodes with different U, different partition methods,  $d_r = 1/4$  and code rate of 1/2. It is observed that the phase node degrees are very high when employing the pseudo-random partition, and they are significantly reduced with the proposed GP and IGP methods. Since the number of four cycles decreases with the phase node degrees, the decoding convergence speed and error-correcting performance will be improved with the proposed GP and IGP method. Moreover, the decoder complexity is also reduced with lower phase node degrees. It is also observed that IGP

TABLE IDegrees of Phase Nodes with Different U, Code Length of 1152AND Code Rate of 1/2.  $n_{max} = 100$  and  $d_r = 1/4$  for IGP.

	Degrees of	Average degree
	$b_1, \cdots, b_{U-1}$	
Pseudo-Random, $U = 4$	270, 290, 263	274
GP, $U = 4$	58, 59, 56	58
IGP, $U = 4$	30, 27, 27	28
Pseudo-Random, $U = 8$	249, 237, 250, 232,	245
	251, 256, 237	
GP, U = 8	56, 62, 60, 56,	58
	55, 56, 63	
IGP, $U = 8$	37, 36, 34, 37,	35
	34, 35, 32	
Pseudo-Random, $U = 16$	-	168
GP, $U = 16$	-	47
IGP, $U = 16$	-	40
Pseudo-Random, $U = 24$	-	123
GP, $U = 24$	-	38
IGP, $U = 24$	-	35



Fig. 3. FER performance with different partition methods, different U, code rate of 1/2, QPSK modulation, over AWGN channel.

results in lower phase node degrees than GP, therefore it is anticipated that better error-correcting performance would be achieved with IGP. For the selection of  $d_r$ , we have tested with different  $d_r$ , and found that the average phase node degrees are satisfactory for a wide range of U when  $d_r = 1/4$ . Therefore, we only present the results of  $d_r = 1/4$  in this paper.

Fig. 3 presents the frame error rate (FER) performance of the joint decoding scheme with different partition methods, different U, code rate of 1/2 and QPSK modulation, over AWGN channel. FER performance of decoding with ideal phase factor information is also presented for comparison. It is observed from Fig. 3 that the joint decoding scheme with GP and IGP achieves almost the same FER performance as the ideal case, for U = 8 and 16. In contrast, the scheme with the pseudo-random partition suffers significant performance loss when  $U \ge 8$ . The performance loss with the pseudo-random



Fig. 4. BER (dashed lines) and FER (solid lines) performance with IGP, different U, different code rates, 40 decoding iterations, QPSK modulation, over AWGN channel.



Fig. 5. FER performance with IGP, different U, different code rates, 40 decoding iterations, QPSK and 16QAM modulation, over uncorrelated Rayleigh fading channel.

partition is not acceptable for U = 16 and 24, even when 500 decoding iterations are employed as [1]. It is also observed that the performance of GP is slightly inferior to that of the IGP. Therefore, we only present simulation results with the IGP algorithm in the followings.

The above simulations demonstrate that the proposed partition method not only improves the error-correcting performance of the joint decoding scheme, but also improves the convergence speed so that less iterations are required in decoding.

The FER and bit error rate (BER) curves of the joint decoding scheme with the proposed IGP algorithm, different U, different code rates, and QPSK modulation, over AWGN channel are plotted in Fig. 4. Compared with the ideal case, the FER performance degradation with the proposed IGP algorithm is less than 0.2dB at FER of  $10^{-3}$  when  $U \leq 16$ , for both the code rate of 1/2 and 5/6. It is also observed that, the BER performance loss is greater than its corresponding



Fig. 6. PAPR performance of the OFDM system with different partition methods, different U, and QPSK modulation.

FER performance loss, due to the error propagation that occurs with fail-decoded phase bits and wrong flipping of their information bits after decoding. Nevertheless, FER is a measure of performance that is more important than BER for wireless communication systems with automatic repeat request (ARQ) mechanisms. Fig. 5 presents FER performance of the joint decoding scheme with the IGP algorithm, different U, different code rates, QPSK and 16QAM modulation, over uncorrelated Rayleigh fading channel. Compared with the ideal case, the performance degradation with the proposed IGP algorithm is less than 0.2dB (0.5dB) at FER of  $10^{-3}$  for QPSK (16QAM) modulation, when  $U \leq 16$ .

Fig. 6 plots the complementary cumulative distribution functions (CCDFs) of PAPR for the OFDM signal with PTS and different partition methods. PAPR of the original OFDM signal is also plotted for comparison. QPSK modulation is employed in the simulation. The time domain OFDM signals are four times oversampled to approximate the PAPR of continuoustime OFDM signals. It is observed from Fig. 6 that the OFDM signals with the IGP and pseudo-random partition possess almost the same PAPR performance, and they both achieve about 2.9, and 4.1dB PAPR reductions at  $CCDF = 10^{-4}$  when U = 4 and 8, respectively.

## V. CONCLUSION AND FUTURE WORKS

In this paper, we have proposed algorithms to improve the joint decoding performance by optimizing the group partition, for OFDM systems with LDPC coding and PTS PAPR reduction. Compared to the joint decoding with the pseudo-random partition in [1], the proposed method is more practical and attractive in the following two aspects: 1) it provides nearly perfect error-correcting performance for a larger number of PTS groups; 2) it possesses faster decoding convergence and lower decoder complexity. With the improved performance, better PAPR performance can be supported.

As future works, the following two aspects could be considered to further improve the decoding performance: 1) try other optimization objectives, such as reducing the number of shortlength cycles; 2) design better algorithms so that the degrees of phase nodes are further reduced, especially for large U.

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