A Robust Frequency Synchronization Method for Non-Contiguous OFDM-Based Cognitive Radio Systems

Jie DING*, Daiming QU†, Li LI†
*Department of Electronic Engineering, Macquarie University, Australia
Email: Jie.Ding3@students.mq.edu.au
†Department of Electronics and Information Engineering
Huazhong University of Science and Technology, China
Email: {qudaiming, tweedlely}@gmail.com

***Abstract*—For non-contiguous OFDM (NC-OFDM) based cognitive radio systems, one of the significant challenges for a secondary receiver is to establish frequency synchronization without spectrum synchronization information (SSI). In this paper, we propose a robust carrier frequency offset (CFO) estimation method for NC-OFDM receiver when the interference from primary users is considered. The key idea of the proposed method is first to make a decision on which subchannels are active by employing two consecutive and identical training symbols, and then to estimate the CFO effectively by using a maximum likelihood algorithm based on the information on selected active subchannels. Simulation results show that the proposed method can provide a satisfactory estimation accuracy, which is close to the corresponding Cramér-Rao lower bound (CRB) with the ideal SSI over an additive white Gaussian noise (AWGN) channel.**

I. Introduction

For wideband CR, OFDM is an attractive candidate physical layer technology due to its capability of transmitting over non-contiguous frequency bands [1] [2] [3]. However, one of the key challenges for non-contiguous OFDM-based CR (NC-OFDM-based CR) systems is to establish frequency synchronization without the spectrum synchronization information (SSI).

In [4] and [5], novel schemes are proposed to obtain the SSI for NC-OFDM-based CR systems by neglecting carrier frequency offset (CFO). In practical, however, NC-OFDM is very sensitive and vulnerable to the CFO which causes inter-carrier interference (ICI) occur, thus making the spectrum synchronization performance proposed in [4] degrade severely. Consequently, for NC-OFDM-based CR systems, the CFO must be considered and estimated at the secondary receiver before setting up the SSI.

CFO estimation in OFDM systems has been extensively investigated in the past and some good approaches can be found in [6]- [8]. In [6], Moose gave the maximum likelihood estimator (MLE) for the CFO based on the observation of two consecutive and identical symbols. However, this method works well only when the CFO is small. A two-symbol training sequence was also employed by Schmidl and Cox [7]. Compared to the method in [6], the scheme proposed in [7] could provide a wider acquisition range for the CFO. In [8], Morelli *et al.* extended the algorithm in [7] by considering a training symbol composed of $L > 2$ identical parts. This makes the estimation range as large as desired without the need of a second training symbol. All these above methods can provide accurate estimation results when the received signal is free form the interference, whereas significant degradations may occur in the presence of interference. In [9], the CFO and narrow band interference (NBI) power were jointly estimated for NBI-based OFDM systems by using maximum likelihood (ML) methods and assuming that the NBI is Gaussian distributed across the signal spectrum. Simulations indicated that the proposed methods are capable of removing most of the degradation that NBI imposes on the synchronization process and can achieve a satisfactory accuracy. In [10], based on [8], a novel joint frequency synchronization and spectrum occupancy characterization method for OFDM-based CR systems under a fractional bandwidth (FBW) mode was proposed. However, the fractional CFO estimation could not perform well considering the interference on inactive sub-bands.

In this paper, we extend the ML algorithm proposed in [9] to solve the CFO estimation problem for NC-OFDM-based CR systems. Due to the unknown SSI, the interference on those inactive subchannels will degrade the estimation performance severely. Thereby, a natural way for mitigating the interference is to operate in the frequency domain. First, a hard decision-based detection (HDD) scheme is used to detect whether a subchannel is active or inactive by employing two consecutive and identical training symbols, and then based on the information on the selected active subchannels, CFO is estimated effectively by using the ML algorithm. Simulation results show that the proposed method can achieve a satisfactory estimation accuracy, which is close to the Cramér-Rao lower bound (CRB) with the ideal SSI over an additive white Gaussian noise (AWGN) channel.

The remainder of this paper is organized as follows. In Section II, the dynamic spectrum model and the NC-OFDM-based CR system model are briefly described. In Section III, the extended ML estimation algorithm based on [9] is

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introduced. Simulation results are presented in Section IV. Conclusion is given in Section V.

II. System Model

For wideband CR systems, OFDM is a natural choice for non-contiguous spectrum transmission. As shown in Fig. 1, the secondary transmitter may have to use some non-contiguous sub-bands for data transmissions in NC-OFDM-based CR system, due to the primary users activities, where each subband consists of some contiguous subchannels.

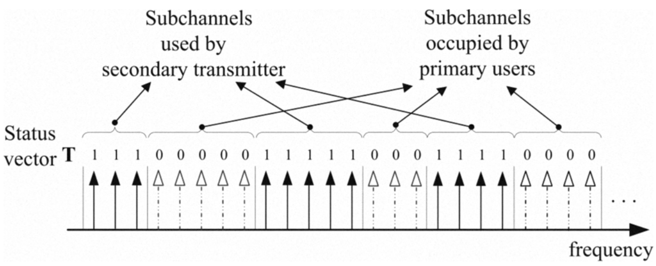

Fig. 1. Status of subchannels in NC-OFDM-based CR system.

Denoting $\mathbf{T} = [T(0), T(1), \ldots, T(N-1)]$ as the status of subchannels at the transmitter, we have

$$T(k) = \begin{cases} 1, & k \in \{a_1, a_2, \cdots, a_{N_T}\} \\ 0, & \text{else where} \end{cases}, \tag{1}$$

where N is the number of subchannels. $T(k) = 1$ means that the kth subchannel is used by the secondary transmitter, and $T(k) = 0$ means that the kth subchannel is not used by the secondary transmitter. $\{a_1, a_2, \cdots, a_{N_T}\}$ is the index set of the N_T active subchannels used by the secondary transmitter.

After obtaining $\mathbf{T}$ by spectrum sensing, the secondary transmitter only uses the active subchannels for its data transmissions. Fig. 2 shows the configurations of the NC-OFDM-based CR system considered in this paper. After a serial to parallel converter (S/P), the frequency domain symbols are generated as vector $\mathbf{X}'$ with length N

$$\mathbf{X}' = [X'(0), X'(1), \ldots, X'(N-1)]. \tag{2}$$

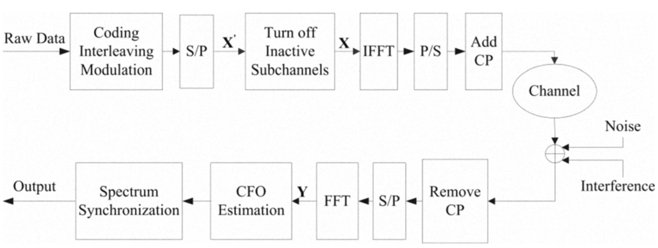

Fig. 2. NC-OFDM-based CR system.

To guarantee the protection of primary users from interference by secondary users, the secondary transmitter turns off the subchannels that overlap with the primary users's spectrum. Thus, the actual symbol sequence fed into inverse fast Fourier transform (IFFT) module is written as

$$\mathbf{X} = [X(0), X(1), \ldots, X(N-1)], \tag{3}$$

where

$$X(k) = X'(k)T(k) = \begin{cases} X'(k), & k \in \{a_1, a_2, \cdots, a_{N_T}\} \\ 0, & \text{else where} \end{cases}. \tag{4}$$

Then, $\mathbf{X}$ is transformed by the IFFT module, and the complex-valued outputs are converted back to serial data for transmission. Every OFDM symbol is added by a cycle prefix (CP) to prevent inter-symbol interference (ISI).

Similarly, the interference signal from primary users in the frequency domain can be also represented by a vector

$$\mathbf{I} = [I(0), I(1), \ldots, I(N-1)], \tag{5}$$

where $I(k) = 0$ for $k \in \{a_1, a_2, \cdots, a_{N_T}\}$. Without loss of generality, the interference from primary users is assumed to be approximated as a complex Gaussian random variable with mean zero and variance σ_p^2 per dimension [4].

At the receiver, considering the frequency mismatch between transmitter and receiver oscillator, let ν denote the CFO normalized to the subchannel spacing. In this paper, we assume the frequency acquisition procedure has been completed so that the CFO is within one half of an interval of the subchannel spacing, i.e., $|\nu| \leq 1/2$.

After discarding the CP and performing the FFT at the receiver, we can write the received frequency-domain signal at the kth sample as

$$Y(k) = S(k) + Q(k), \quad 0 \leq k \leq N-1 \tag{6}$$

where $S(k)$ is the useful signal component and $Q(k)$ accounts for the interference component plus background noise.

$$\begin{aligned} S(k) = &\frac{\sin \pi\nu}{N \sin \frac{\pi\nu}{N}} X(k) H(k) \exp \frac{j\pi(N-1)\nu}{N} \\ &+ ICI_X(k), \quad 0 \leq k \leq N-1 \end{aligned} \tag{7}$$

where $H(m)$ is the channel frequency response. Equation (7) shows that the CFO degrades the amplitude of the received signal on each subchannel and cause loss of orthogonality among subchannels [11].

$$\begin{aligned} Q(k) = &\frac{\sin \pi\nu}{N \sin \frac{\pi\nu}{N}} I(k) \exp \frac{j\pi(N-1)\nu}{N} \\ &+ ICI_I(k) + W(k), \quad 0 \leq k \leq N-1 \end{aligned} \tag{8}$$

where the first two terms in (8) are the interference component and $W(k)$ is the complex Gaussian white noise on the kth subchannel, with mean zero and variance σ_n^2 per dimension. Equation (8) shows that the useful signal on active subchannels would be inevitably affected by the ICI of the interference signal.

Due to the lack of SSI, the secondary receiver cannot identify the set $\mathbf{T}$ temporally employed by the secondary transmitter. In this case, the interference from primary users on inactive subchannels will be introduced at the receiver, which will complicate the problem of CFO estimation. Our goal is to design a CFO estimation scheme for NC-OFDM-based CR systems which may work without the SSI. For this purpose, we propose a novel estimation method to recover the CFO in Section III.

III. Proposed ML Method for CFO Estimation

In this Section, we extend the ML algorithm proposed in [9] to solve the CFO estimation in NC-OFDM-based CR systems.

We assume that one training symbol block is appended in the front of each data frame, which is shown in Fig. 3. The training symbol block consists of $L = 2$ identical training symbols, each containing N subchannels.

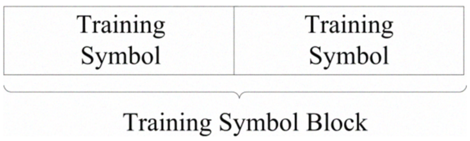

Fig. 3. Training symbol block for CFO estimation.

We denote by $Y_l(k)$ the kth FFT output corresponding to the lth training symbol, where $0 \leq l \leq 1$. Since the repetitive parts of the training block remain identical after passing through the channel except for a phase shift induced by the CFO [6], we may rewrite the equation (6) as

$$Y_l(k) = S(k)e^{j2\pi l\nu} + Q_l(k), \quad 0 \leq l \leq 1 \tag{9}$$

where $Q_l(k)$ is the interference plus background noise on the kth subchannel corresponding to the lth training symbol. From Equation (8), it is easy to derive that the $Q_l(k)$ can be also modeled as a complex Gaussian random variable with zero mean and unknown variance $\sigma(k)^2$.

For given k, we arrange the outputs of the training symbol block in the frequency-domain into a vector $\mathbf{Y}(k) = [Y_0(k), Y_1(k)]^T$, where the superscript $\{\}^T$ denotes the transpose operator. Hence, we have

$$\mathbf{Y}(k) = S(k)\mathbf{u}(\nu) + \mathbf{Q}(k), \quad 0 \leq k \leq N-1 \tag{10}$$

where $\mathbf{u}(\nu) = [1, e^{j2\pi\nu}]^T$ and $\mathbf{Q}(k) = [Q_0(k), Q_1(k)]^T$. By exploiting vectors $\{\mathbf{Y}(k); 0 \leq k \leq N-1\}$, we design a novel scheme to estimate ν based on ML algorithm.

A. A Hard-Decision-Based Active Subchannel Detection

As mentioned in Section II, the interference from primary users on inactive subchannels will be introduced at the receiver due to the lack of SSI, which has a negative effect on estimating CFO. To mitigate the effect of interference on those inactive subchannels, we derive a correlation coefficient of each subchannels being active based on two consecutive training symbols. Then, a hard-decision-based active subchannel detection (HDD) scheme is proposed to detect which subchannels are active.

For any given k, with the $Y_0(k)$ and $Y_1(k)$, we define the correlation coefficient of the kth subchannel as

$$\rho(k) = \frac{\left|\sum\limits_{m=k-J/2}^{k+J/2} Y_0(m)Y_1(m)^*\right|}{\sqrt{\sum\limits_{m=k-J/2}^{k+J/2} Y_0(m)Y_0(m)^*}\sqrt{\sum\limits_{m=k-J/2}^{k+J/2} Y_1(m)Y_1(m)^*}}. \quad 0 \leq k \leq N-1 \tag{11}$$

Since the adjacent subchannels tend to have the same status, equation (11) uses the adjacent subchannels to improve the accuracy of correlation coefficients, where J represents the number of adjacent ones of the kth symbol and the superscript $\{\}^*$ stands for conjugate.

From equation (11), it can be observed that the correlation coefficients on active subchannels tend to be bigger than those on inactive subchannels because the useful signal component on active subchannels makes a larger proportion in the received signal than that on inactive subchannels.

Based on the conclusion above, a HDD scheme of active subchannels can be made with a proper threshold after calculating all correlation coefficients,

$$R(k) = \begin{cases} 1, & \text{if} \quad \rho(k) \geq \mathrm{TH}_\rho \\ 0, & \text{if} \quad \rho(k) < \mathrm{TH}_\rho \end{cases}, \tag{12}$$

where $R(k)$ denotes the status of the kth subchannel at the receiver.

We assume that N_L subchannels with larger values of ρ are decided as active subchannels in the receiver. Considering the probability of ambiguity decision on the edge subchannels of sub-bands, N_L is chosen to be a little smaller than N_T.

B. ML Algorithm Based on HDD Scheme

After the HDD scheme, we can use the information on selected active suchannels (i. e., $R(k)$=1) to employ the ML algorithm. Since the vectors $\mathbf{Y}(k)$ are statistically independent and Gaussian distributed, the log-likelihood function can be given accordingly, after neglecting constants, as

$$\begin{aligned}\Lambda(\tilde{\mathbf{S}}, \tilde{\boldsymbol{\sigma}}^2, \tilde{\nu}) = &-2\sum_{k\in\Phi} \ln[\tilde{\sigma}^2(k)] \\ &- \sum_{k\in\Phi} \frac{1}{\tilde{\sigma}^2(k)} \|\mathbf{Y}(k) - \tilde{S}(k)\mathbf{u}(\tilde{\nu})\|^2,\end{aligned} \tag{13}$$

where Φ is the index set of selected active subchannels with length of N_L. $\tilde{\mathbf{S}}$ is the set of $\tilde{S}(k)$ for $k \in \Phi$. $\tilde{\boldsymbol{\sigma}}^2$ is the set of $\tilde{\sigma}^2(k)$ for $k \in \Phi$. $\tilde{\mathbf{S}}$, $\tilde{\boldsymbol{\sigma}}^2$ and $\tilde{\nu}$ are the trial values of unknown parameters.

To obtain the optimal CFO estimation, we must find out the $\tilde{\nu}$ that makes $\Lambda(\tilde{\mathbf{S}}, \tilde{\boldsymbol{\sigma}}^2, \tilde{\nu})$ achieve its global maximum. In this paper, the derivation of the maximum likelihood estimator is the same as that in [9]. Firstly, we fix $\tilde{\nu}$ and $\tilde{\mathbf{S}}$ but vary $\tilde{\boldsymbol{\sigma}}^2$, leading to the maximum of Λ with

$$\hat{\sigma}^2(k; \tilde{\nu}, \tilde{\mathbf{S}}) = \frac{1}{2}\|\mathbf{Y}(k) - \tilde{S}(k)\mathbf{u}(\tilde{\nu})\|^2. \quad k \in \Phi \tag{14}$$

Substituting (14) into (13), we have the log-likelihood function about $(\tilde{\nu}, \tilde{\mathbf{S}})$

$$\Lambda(\tilde{\nu}, \tilde{\mathbf{S}}) = -\sum_{k \in \Phi} \ln[\|\mathbf{Y}(k) - \tilde{S}(k)\mathbf{u}(\tilde{\nu})\|^2]. \quad (15)$$

Then, we fix $\tilde{\nu}$ and let $\tilde{\mathbf{S}}$ vary, in the same way it yields that

$$\hat{S}(k; \tilde{\nu}) = \frac{1}{2}\mathbf{u}^H(\tilde{\nu})\mathbf{Y}(k). \quad k \in \Phi \quad (16)$$

Substituting (16) into (15), it has the likelihood function of $\tilde{\nu}$

$$\Lambda(\tilde{\nu}) = -\sum_{k \in \Phi} \ln[\|\mathbf{Y}(k)\|^2 - \Omega(k, \tilde{\nu})], \quad (17)$$

where $\Omega(k, \tilde{\nu})$ is the periodogram of $\mathbf{Y}(k)$,

$$\Omega(k, \tilde{\nu}) = \frac{1}{2}|\sum_{l=0}^{1} Y_l(k) \exp(-j2\pi l\tilde{\nu})|^2. \quad (18)$$

Finally, we can get the optimal $\hat{\nu}$ to make the $\Lambda(\tilde{\nu})$ maximum, i. e.,

$$\hat{\nu} = \arg\max_{\tilde{\nu}}\{\Lambda(\tilde{\nu})\}. \quad (19)$$

All of the above is an extended method based on the ML algorithm mentioned in [9]. Different from the algorithm in [9], we first develop a HDD scheme to mitigate the negative effect of the interference on those inactive subchannels for NC-OFDM systems. Then with the useful information on selected active subchannels, ML algorithm is used to estimate the CFO. Although the proposed method could remove the most of the interference on inactive subchannels, it loses the useful information on those removed subchannels, which may make the conventional MLE outperform the proposed method when the interference power is small enough. This will be verified in Section IV.

IV. Simulation Results

In this section, we demonstrate the performance of the proposed method by simulations. For all the simulations, it is assumed that timing synchronization is already set up while the CR channel is considered as an AWGN channel. The NC-OFDM system considered consists of $N = 256$ subchannels, the length of the CP is $1/4$ duration of the OFDM symbol. The spectrum of the secondary user consists of three non-contiguous sub-bands and each sub-band consists of a number of contiguous subchannels. For each sub-band, the number of active subchannels and offset are generated randomly. The total number of active subchannels $N_T = 128$. At the receiver, the number of adjacent subchannels J in equation (11) is set to be 6 and N_L employed for HDD scheme is set to be 110.

The accuracy of the CFO estimator is measured in terms of mean square estimation error (MSEE), which is defined as $E\{|\hat{\nu} - \nu|^2\}$. To verify the accuracy of the proposed ML method, comparisons are made with the CRB with the ideal SSI and with the conventional MLE proposed in [6]. For the CRB with ideal SSI in this paper, it takes the form

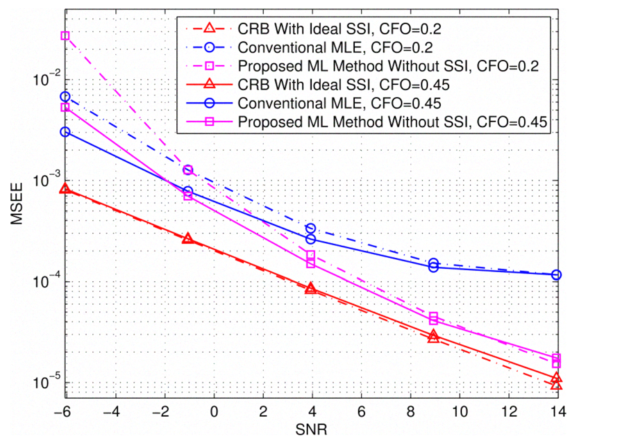

Fig. 4. MSEE versus SNR for $\nu = 0.45$ and $\nu = 0.2$, SIR = 0 dB, over AWGN channel.

$$CRB(\nu) = \frac{(\overline{SINR})^{-1}}{4\pi^2 N_T}, \quad (20)$$

where

$$\overline{SINR} = \frac{1}{N_T}\sum_{k \in \Psi}\frac{|S(k)|^2}{\sigma^2(k)}$$

is the average signal-to-interference-plus-noise ratio (SINR) on active subchannels at the receiver, where $\Psi = \{a_1, a_2, \cdots, a_{N_T}\}$.

Fig. 4 depicts MSEE versus signal-to noise ratio (SNR) for $\nu = 0.45$ and $\nu = 0.2$ at SIR $= 0$ dB (SIR is defined as the ratio of the average signal power on each active subchannel over the average interference power on each inactive subchannel at the transmitter). As shown in this figure, when $\nu = 0.45$, the conventional MLE cannot work well in NC-OFDM systems since all subchannels are assumed to be active and too much interference is introduced. Compared with the conventional MLE, the performance of the proposed ML method is only about 2 dB inferior to that of the CRB with ideal SSI at large SNR values, which is quite acceptable considering that the secondary receiver does not have SSI. Similar results are observed from the curves with $\nu = 0.2$. But it is noticed that the curves with $\nu = 0.2$ outperform that with $\nu = 0.45$ when the SNR value is large. This could be explained by the fact that the interference dominates the system performance at large SNR values. Since the interference power on active subchannels is smaller when $\nu = 0.2$, the system performance is better.

To further explore the performance of the proposed method, we consider SIR $= -6$ dB in Fig. 5. With the increased interference power, significant degradations are observed with the conventional MLE, which exhibits a large error floor at MMSE of 10^{-3}. For the proposed method, it degrades slightly and keeps close to the CRB with the gap of about 3 dB.

Fig. 6 illustrates MSEE versus SNR for $\nu = 0.45$ at SIR= $+\infty$ (i.e., the interference power from primary users is so low that it could be neglected). As expected, it is shown that the conventional MLE outperforms the proposed ML method by approximately 1 dB and achieves the accuracy the same as the CRB. It also could be concluded that for low interference power from primary users, the frequency synchronization is not a critical issue for the NC-OFDM-based CR systems and even the conventional MLE has a satisfactory performance.

In summary, our proposed method can achieve an acceptable accuracy, which is close to the CRB with the ideal SSI when a broad range of SIR is considered. It is also indicated that the proposed method is capable of removing most of the interference imposed on those inactive subchannels and can be effectively used for estimating CFO before setting up the spectrum synchronization.

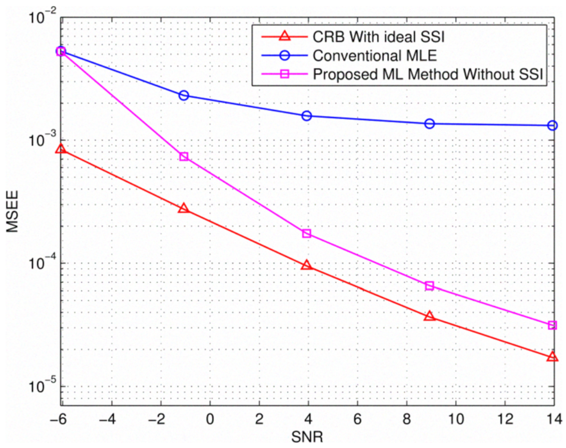

Fig. 5. MSEE versus SNR for $\nu = 0.45$ and SIR $= -6$ dB, over AWGN channel.

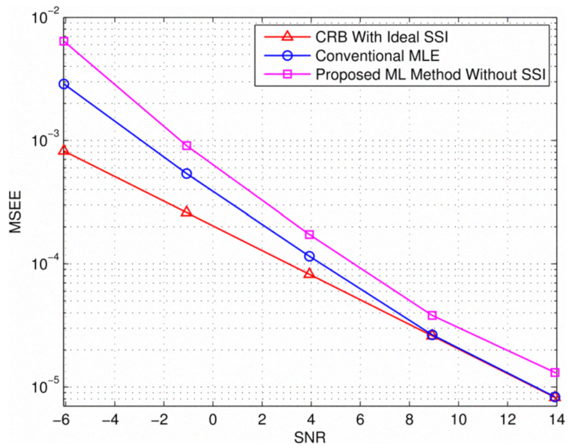

Fig. 6. MSEE versus SNR for $\nu = 0.45$ and SIR $= +\infty$, over AWGN channel.

V. Conclusion

In this paper, we proposed a robust method to estimate the CFO for NC-OFDM-based CR systems. For this method, a HDD scheme is used to detect whether a subchannel is active or inactive by employing two consecutive and identical training symbols, and then based on the information on selected active subchannels, the CFO is estimated by using a ML algorithm. Simulation results show that the proposed method can obtain a satisfactory accuracy, which is close to the CRB with the ideal SSI when a broad range of SIR is considered. With the proposed method, the CFO can be recovered effectively. Then, we can use the schemes proposed in [4] to establish the spectrum synchronization.

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