Design of LDPC codes for Non-Contiguous OFDM-Based Communication Systems

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***Abstract*—For non-contiguous OFDM (NC-OFDM) based communication systems, especially NC-OFDM-based cognitive ratio (CR) systems, one of critical challenges is to establish spectrum synchronization between the transmitter and receiver before data transmission. In [5], an *a posterior* probability (APP) detection algorithm has been proposed for the receiver to detect the subchannels occupied by the transmitter (active subchannels). However, the encoding scheme adopted in [5] is a concatenation of a convolutional coder and a low rate repetition coder so that the system code rate is only $1/4$ when half of the subcarriers are active. In this paper, we consider using low density parity-check (LDPC) codes to improve the system data transmission rate. Furthermore, with the channel model a of NC-OFDM-based communication system, we employ density evolution algorithm to obtain good degree distribution pairs and adopt a modified progressive edge-growth (PEG) algorithm to construct the parity-check matrix for the LDPC code. Simulation results show that our optimized LDPC code for NC-OFDM-based communication systems could obtain satisfactory error performance at a high data transmission rate (system code rate of $1/2$).**

Index Terms — **LDPC, NC-OFDM, density evolution, progressive edge-growth.**

I. Introduction

Recently, wideband cognitive radio (CR) has attracted considerable attention owing to its capability of improving the utilization of the radio spectrum [1]. A variant of OFDM known as non-contiguous OFDM (NC-OFDM), which only employs non-contiguous subbands to keep primary users from interference, has been proposed for wideband CR networks. However, due to non-contiguous spectrum usage, one of key challenges is to establish spectrum synchronization between the transmitter and receiver for NC-OFDM-based communication systems, especially NC-OFDM-based CR systems. In [2], a dedicated out-of-band control channel was assigned for transmitting spectrum synchronization information from the transmitter to its receiver. However, the licensed spectrum resource required for the dedicated channel might suffer from high cost and control congestion. Without the aid of a dedicated out-of-band control channel, a fractional bandwidth mode detection algorithm was developed with a specially designed PN preamble sequence. However, this in-band synchronization algorithm is very sensitive to the interference from primary users. In [5], an *a posterior* probability (APP) detection algorithm was proposed to detect active subchannels, where the interference from primary users has been considered. However, the encoding scheme adopted in [5] is a concatenation of a convolutional coder and a low rate repetition coder so that the system code rate is only $1/4$ when half of the subcarriers are active.

Low-density parity-check (LDPC) codes, one of the most powerful and versatile codes, have been shown to have extraordinary performance for various types of communication channels [6]. In this paper, we adopt LDPC codes instead of the concatenated codes adopted in [5] to improve the system code rate of NC-OFDM-based communication systems.

This paper designs LDPC codes for NC-OFDM-based communication systems using the density evolution algorithm [6] and the PEG algorithm [7]. Firstly, we regard the channel model of a NC-OFDM-based communication system as a fading channel with burst erasures. Then, we employ the density evolution algorithm in this specific channel to obtain good degree distribution pairs for LDPC codes. Furthermore, we construct the low density parity-check matrix by a modified PEG algorithm to further improve the error performance in this specific channel. Simulation results show that, at the system code rate of $1/2$, the symbol error rate (SER) performance of the optimized LDPC code for NC-OFDM-based communication systems only suffers about 0.8dB loss in SER performance compared with that of the ideal spectrum synchronization.

II. NC-OFDM-Based CR Systems and APP Detection Algorithm

A. Spectrum Model in NC-OFDM-Based CR Systems

In NC-OFDM-based CR systems, due to primary users' activities, the secondary transmitter may have to use some non-contiguous subbands for data transmissions, where each subband consists of some contiguous subchannels. As shown in Fig.1, each subchannel in a NC-OFDM-based CR system could be modeled as a two-switch channel model [4], in which the switch is open when a primary user is detected, and vice versa. Since the secondary transmitter and receiver may experience different wireless environments and have different sensing outcomes, there is no guarantee that the subbands occupied at the transmitter and receiver could keep synchronized with each other.

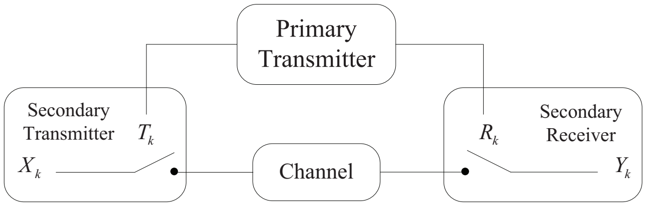

Fig. 1. Two-switch model for dynamic and distributed spectrum allocation.

Let $\mathbf{T} = [T_0, T_1, \cdots, T_{N-1}]$ and $\mathbf{R} = [R_0, R_1, \ldots, R_{N-1}]$ denote the status of subchannels at the transmitter and receiver, respectively. We have

$$T_k = \begin{cases} 1, & k \in \{a_1, a_2, \ldots, a_{N_T}\} \\ 0, & \text{else} \end{cases}, \tag{1}$$

and

$$R_k = \begin{cases} 1, & k \in \{b_1, b_2, \cdots, b_{N_R}\} \\ 0, & \text{else} \end{cases}. \tag{2}$$

where N is the number of subchannels. $T_k = 1$ means that the kth subchannel is used by the secondary transmitter, and $T_k = 0$ means that the kth subchannel is not used by the secondary transmitter. $\{a_1, a_2, \cdots, a_{N_T}\}$ is the order index of the N_T active subchannels used by the secondary transmitter. Similarly, $R_k = 1$ means that the kth subchannel is considered being active at the secondary receiver, and $R_k = 0$ means that the kth subchannel is considered being inactive at the secondary receiver. $\{b_1, b_2, \cdots, b_{N_R}\}$ is the order index of the N_R active subchannels used by the secondary receiver.

To correctly recover the information, a conventional receiver is required to be perfectly synchronized with the transmitter, i.e., $\mathbf{R} = \mathbf{T}$ ($R_k = T_k$ for all k). Obviously, when $\mathbf{R} \neq \mathbf{T}$, the system performance will be seriously degraded due to the interference from primary users or loss of data symbols. Thus, the active subchannels should be detected accurately at the receiver when a dedicated out-of-band control channel is not available.

B. System Model and APP Detection Algorithm

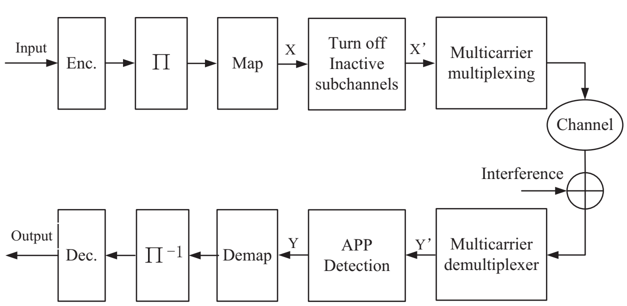

Fig. 2. System diagram of a NC-OFDM-based CR system. (Π and Π^{-1} represent interleavers and de-interleavers.)

The system diagram of a NC-OFDM-based CR system is shown in Fig.2. At the secondary transmitter, the inactive subchannels that overlap with the primary users' spectrum should be turned off to guarantee the protection of primary users from interference by secondary users. Thus,

$$X'_k = X_k T_k = \begin{cases} X_k, & k \in \{a_1, a_2, \cdots, a_{N_T}\} \\ 0, & \text{else} \end{cases}. \tag{3}$$

where X_k and X'_k are data symbol sequence before and after the "switch" at the transmitter.

Similarly, at the secondary receiver, the active subchannels occupied by the secondary transmitter should be detected to avoid interference from primary users. Thus,

$$Y_k = Y'_k R_k = \begin{cases} Y'_k, & k \in \{b_1, b_2, \cdots, b_{N_R}\} \\ 0, & \text{else} \end{cases}. \tag{4}$$

where Y'_k and Y_k are data symbol sequence before and after the "switch" at the receiver.

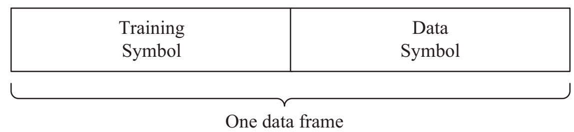

Fig. 3. Frame structure in a NC-OFDM-based CR system.

Fig. 3 shows a frame structure in a NC-OFDM-based CR system, which consists of a training symbol and a data symbol. The key idea of APP detection algorithm [5] was to employ the received training symbol to calculate the *a posterior* probability (APP) of each subchannel being active. Then, a decision of active subchannels could be made with a proper threshold. Therefore, the spectrum information R_k could be obtained at the secondary receiver, which is almost the same as T_k. However, the encoding scheme adopted in [5] has a very low data rate: the input data bits are coded by a 1/2 convolutional encoder and then repeated by a four-fold repetition coder. In this paper and [5], it is assumed that half of the subchannels overlap with the primary users' spectrum are turned off, thus the actual system code rate of the encoding scheme in [5] is only 1/4. Since LDPC codes have shown excellent performance on many communication channels, we consider using LDPC codes to improve the system code rate of NC-OFDM-based CR systems in Section III.

III. Design of LDPC Codes for NC-OFDM-based Communication Systems

In this section, we design LDPC codes to achieve a good error performance at a high data transmission rate for NC-OFDM-based communication systems.

A. Channel Model for NC-OFDM-based Communication Systems

As mentioned in Section II, the secondary transmitter turns off the inactive subchannels to keep primary users from interference, which could be regarded as a burst-erasure channel. In addition, a random interleaver is employed (shown in Fig. 2) to break up the contiguous erasures as well as the correlation of the data symbols. Consequently, we could approach the channel model of a NC-OFDM-based communication system as Fig. 4, consisting of a binary erasure channel (BEC) concatenated with an uncorrelated fading channel and AWGN channel, where $\varepsilon^{(0)}$ is the random erasure probability, a is the fading factor and σ^2 represents the noise variance. Thus, we could optimize and design LDPC codes for this specific channel.

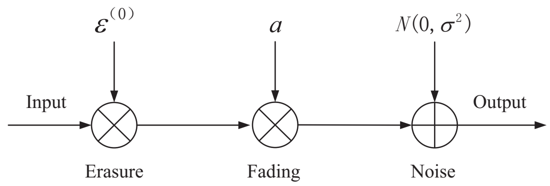

Fig. 4. The channel model of a NC-OFDM-based communication system.

B. Density Evolution for Fading Channel with Erasures

In general, LDPC codes could be characterized by degree distribution pairs. Let $d_{v_{max}}$ ($d_{c_{max}}$) denote the maximum variable (check) node degree, and let λ_i (ρ_i) represent the fraction of edges emanating from variable (check) nodes of degree i. Then we could define

$$\lambda(x) = \sum_{i=2}^{d_{v_{max}}} \lambda_i x^{i-1}, \quad \text{and} \quad \rho(x) = \sum_{i=2}^{d_{c_{max}}} \rho_i x^{i-1} \tag{5}$$

as the variable node and check node degree distribution, respectively.

Based on an assumption that the code's underlying graph is cycle-free, Richardson *et al.* [6] has developed an efficient and effective approach, called density evolution, to obtain good irregular degree distribution pairs for LDPC codes. Moreover, optimizing a degree distribution pair for a specific channel would generally give better results. In [9], LDPC codes have been designed for AWGN channel with erasures. Assuming the fading channel to be Rayleigh fading channel, we employ density evolution algorithm in the uncorrelated Rayleigh fading channel with random erasures (BEC-R), which is the equivalent channel model of a NC-OFDM-based communication system.

In [8], LDPC codes have been optimized for the uncorrelated Rayleigh fading channel, with the assumption that all zero codes are transmitted, the conditional *probability density function* (PDF) of the channel output y is

$$P(y|a) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-a)^2}{2\sigma^2}\right). \tag{6}$$

where a is the normalized Rayleigh fading factor with $E[a^2] = 1$ and density function $p(a) = 2a \exp(-a^2)$.

Let q_r denote the initial log-likelihood ratio (LLR) message over the uncorrelated Rayleigh fading channel with ideal side information (SI), then

$$q_r = \log \frac{P(x=0|y,a)}{P(x=1|y,a)} = \frac{2}{\sigma^2} y \cdot a. \tag{7}$$

Then, the PDF of q_r is

$$P_r(q_r|a) = \frac{\sigma}{2a\sqrt{2\pi}} \exp\left(-\frac{(q_r - 2a^2/\sigma^2)^2}{8a^2/\sigma^2}\right). \tag{8}$$

Thus, the unconditional PDF of q_r over the density of a is

$$\begin{aligned} P_r(q_r) &= \int_0^\infty \frac{\sigma}{\sqrt{2\pi}} \exp\left(-\frac{\left(q_r - 2a^2/\sigma^2\right)^2}{8a^2/\sigma^2}\right) \exp(-a^2)\,\mathrm{d}a \\ &= \frac{\sigma}{\sqrt{2\pi}} \exp\left(\frac{-q_r(\sqrt{2\sigma^2+1}-1)}{2}\right) \\ &\quad \times \int_0^\infty \exp\left(-\frac{\left(q_r \cdot \sigma^2/2a - a\sqrt{2\sigma^2+1}\right)^2}{2\sigma^2}\right) \mathrm{d}a. \end{aligned} \tag{9}$$

Let q_b denote the initial LLR message over the BEC-R channel, then the unconditional PDF of q_b would be

$$P_b(q_b) = (1-\varepsilon^{(0)}) P_r(q_b) + \varepsilon^{(0)} \Delta_0(q_b). \tag{10}$$

where $\varepsilon^{(0)}$ is the random erasure probability, and $\Delta_x(q_b) = \delta(q_b - x)$ is a shifted delta function. In this paper, we assume the primary transmitter and secondary transmitter occupy half of subchannels, respectively, i.e., $\varepsilon^{(0)} = 0.5$. Therefore, assigning the initial PDF of q_b in density evolution algorithm as (10), we could obtain the optimized degree distribution pairs as in Tab. I, where "1/2-Rayleigh" refers to density evolution for the uncorrelated Rayleigh fading channel at the code rate $1/2$, "1/4-Rayleigh" refers to the uncorrelated Rayleigh fading channel at the code rate $1/4$, and "1/4-BEC-R" refers to the BEC-R channel at the code rate $1/4$. Note that, as to "1/4-Rayleigh", the noise thresholds σ^* are computed for the channel with erasures.

TABLE I
GOOD DEGREE DISTRIBUTION PAIRS WITH MAXIMUM VARIABLE NODE DEGREES $d_{v_{max}} = 15$.

	1/2-Rayleigh	1/4-Rayleigh	1/4-BEC-R
$d_{v_{max}}$	15	15	15
λ_2	0.27051	0.34859	0.37640
λ_3	0.19945	0.18300	0.20252
λ_4		0.08589	0.03435
λ_5	0.14979	0.05526	0.07842
λ_6		0.01762	0.05960
λ_7	0.02481	0.03204	
λ_9		0.02462	
λ_{12}	0.01967		0.07875
λ_{14}	0.18145		0.02721
λ_{15}	0.15432	0.25298	0.14275
ρ_4		0.42396	0.59988
ρ_5		0.57604	0.40012
ρ_7	0.28366		
ρ_8	0.71634		
σ^*	0.792	0.762	0.78

C. Modified Progressive Edge-Growth Algorithm

For finite-length LDPC codes, one of the most important specifications is the length of shortest cycle in the Tanner graph, which is called "Girth". The PEG algorithm [7], in which the local girth is maximized during construction, has been proved to be a powerful algorithm to construct the low density parity-check matrix with the variable-node degree distribution optimized by density evolution. However, it does not take into account the check-node degree distribution, which may lead to a performance loss when the check-node degree distribution of PEG codes is much different from that optimized by density evolution. In [10], the free check-node degree (FCD) criterion was proposed to modify the PEG algorithm, which follows both the variable-node and check-node degree distributions during the construction processing. In this paper, we employ the FCD criterion to modified the PEG algorithm to construct the parity-check matrix. We found that using the FCD criterion obtains a considerable improvement in SER performance over the original PEG algorithm in BEC-R channel.

Let $d(v_j)$ and $d(c_i)$ denote the assigned degree of variable-node v_j and check-node c_i, respectively. And, let $d_c(c_i)$ denote the current degree of check node c_i in the original PEG construction processing. When there are multiple check-node candidates to connect to variable node v_j (all such candidates result in the same local girth), one chooses the check node which has the minimum $d_c(c_i)$ among these candidates to connect to variable node v_j. FCD of check node c_i, denoted as $f(c_i)$, is defined as the difference between the assigned check-node degree and the current check-node degree, i.e., $f(c_i) = d(c_i) - d_c(c_i)$. In our modified PEG algorithm, we choose the check node which has the maximum $f(c_i)$ instead of which has the minimum $d_c(c_i)$ among the check-node candidates in the construction processing. Similar as the original PEG algorithm in [7], our modified PEG algorithm is described as follows.

Algorithm 1 Modified PEG algorithm with FCD criterion

Require: $d(v_{j_1}) \le d(v_{j_2}) \quad \forall j_1 < j_2$
Require: $d(c_{i_1}) \ge d(c_{i_2}) \quad \forall i_1 < i_2$
  for $j = 0$ to $n-1$ **do**
    for $k = 0$ to $d(v_j) - 1$ **do**
      if $k = 0$ **then**
        $E^0_{v_j} \leftarrow$ edge (c_i, v_j), where $E^0_{v_j}$ is the first edge incident to v_j and c_i is a check node such that it has the highest FCD under the current graph setting $E_{v_0} \cup E_{v_1} \cup \cdots \cup E_{v_{j-1}}$.
      else
        Expand a subgraph from variable node v_j up to under the current graph setting such that the cardinality of $\mathcal{N}^l_{v_j}$ stops increasing but is less than m, or $\overline{\mathcal{N}}^l_{v_j} \neq \emptyset$ but $\overline{\mathcal{N}}^{l+1}_{v_j} = \emptyset$, then $E^k_{v_j} \leftarrow$ edge (c_i, v_j), where $E^k_{v_j}$ is the kth edge incident to v_j and c_i is a check node picked from the set $\overline{\mathcal{N}}^l_{v_j}$ having the highest FCD.
      end if
    end for
  end for

In the case that there are still multiple choices to connect to variable node v_j (all such candidates result in the same FCD), we choose the one with the smallest index. And, in the other case that the FCDs of all check-node candidates are zero for v_j, check nodes are searched in the previous candidate ensemble $\overline{\mathcal{N}}^{(l-1)}_{v_j}$.

IV. Simulation Results

In this section, we demonstrate the SER performance of LDPC codes in NC-OFDM-based communication systems. For all the simulations, it is assumed that the data symbol consists of $N = 2048$ subchannels, where primary users and secondary users occupy half of the subchannels, respectively, i.e., $N_P = N_S = 1024$. Moreover, the spectrum of the secondary user consists of three non-contiguous subbands, each of which consists of several contiguous subchannels. For each subband, the number of active subchannels and offset are generated randomly. At the transmitter, the input data bits are coded by a LDPC encoder, interleaved by a random interleaver and then mapped into BPSK symbols that are modulated onto OFDM subchannels. Note that, half of the subchannels (inactive subchannels) would be turned off to guarantee the protection of primary users from interference by secondary users. At the receiver, a belief-propagation (BP) decoder is employed for the LDPC code, with a maximum of 80 iterations.

With perfect spectrum synchronization, the BPSK symbols could be modulated onto only active subchannels, the performance of which is supposed to be ideal in this paper. However, in this ideal case, a perfect spectrum synchronization is necessary for the reason that any error in the synchronization information would lead to a serious mismatch between the spectrum detected at the receiver and actual spectrum used at the transmitter. Fig. 5 compares the SER performance for the following four different LDPC codes with the assumption that APP detection module offers perfect spectrum synchronization information:

1) The input data bits are coded by a $1/2$ LDPC encoder and the BPSK symbols are modulated only onto active subchannels. The degree distribution pair refers to "1/2-Rayleigh" in Tab. I and the parity-check matrix is constructed by the original PEG algorithm. Obviously, the system code rate equals to $1/2$ in this ideal case. ("1/2-Ray-Ideal" in Fig. 5)
2) Different from the ideal case in 1), the input data bits are coded by a $1/4$ LDPC encoder and the BPSK symbols are modulated onto all subchannels. The degree distribution pair refers to "1/4-Rayleigh" in Tab. I. Note that, the actual system code rate also equals to $1/2$ for the reason that half of subchannels would be turned off. ("1/4-Ray, PEG" in Fig. 5)
3) Different from 2), the degree distribution pair refers to "1/4-BEC-R"in Tab. I. ("1/4-BEC-R, PEG" in Fig. 5)
4) Different from 3), the parity-check matrix is constructed with the modified PEG algorithm using FCD criterion. ("1/4-BEC-R, Modified PEG" in Fig. 5)

Since the actual system code rates of the above four LDPC codes are all equal to $1/2$, the system code rate of a LDPC-coded NC-OFDM-based communication system is twice as much as the concatenated coded system [5]. Furthermore, as shown in Fig. 5, at SER $= 10^{-3}$, we could obtain about 0.3dB gain by optimizing degree distribution pairs and about 0.5dB gain by our modified PEG construction algorithm. Therefore, the optimized LDPC code could obtain about 0.8dB gain in total. Thus, the performance gap between the optimized LDPC code and the ideal one is decreased to just about 0.8dB.

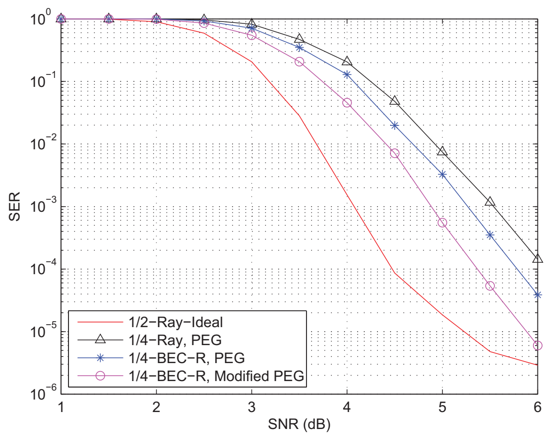

Fig. 5. Comparison of the SER performance for four different codes with perfect synchronization. Note that, the actual code rates are all equal to $1/2$.

Fig. 6 presents the SER performance of the optimized LDPC code in NC-OFDM-based communication systems when APP detection module offers imperfect synchronization information. In this simulation, we set the error probability to be 0, 0.5%, 1% and 1.5%, respectively. Note that, the error probability e means that both the miss detection probability P_m and false alarm probability P_s equal to e. As shown in Fig. 6, when the error probability e increase about 0.5%, the NC-OFDM-based communication system would suffer about 0.2dB loss in SER performance. Considering that the ideal case would suffer a terrible decrease in SER performance with imperfect synchronization, the performance here is quite acceptable. Thus, the optimized LDPC code is robust to imperfect synchronization in NC-OFDM-based communication systems.

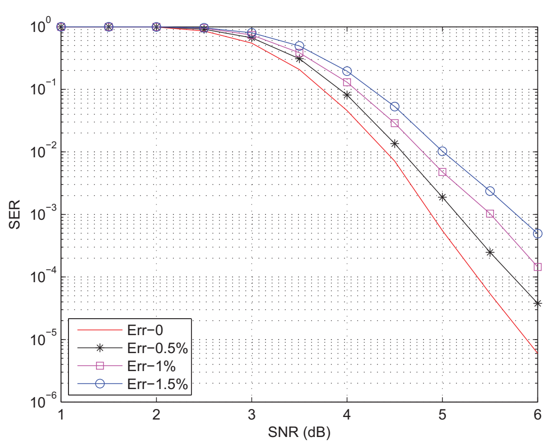

Fig. 6. SER performance of the optimized LDPC code with imperfect synchronization.

V. Conclusion

In this paper, we have designed LDPC codes for NC-OFDM-based communication systems. With the channel model of a NC-OFDM-based communication system, we obtained good degree distribution pairs of LDPC codes using density evolution. Moreover, a modified PEG construction algorithm was adopted to construct the low density parity-check matrix. Simulation results showed that our optimized LDPC code was robust to imperfect spectrum synchronization and could obtain satisfactory SER performance at a high data transmission rate (system code rate of $1/2$).

Acknowledgements

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