Homework #2 - due Friday, 18 September 2009, in class

Photodetector calculations

1. Problem 1.2, from Rieke, p. 28.

2. Problem 1.6, from Rieke, p. 28. (Hint, use the result given in Problem 1.4).

3. Problem 2.2, from Rieke, p. 54.

4. Problem 2.3, from Rieke, p. 54.

Homework assignments will appear on the web at:
http://www.ece.udel.edu/~kolodzey/courses/eleg867f09.html.

Note: On each homework and report submission, please give your name, the due date, assignment number and the course number.
1.2 Consider a detector with an optical receiver of entrance aperture 2 mm diameter, optical transmittance (excluding bandpass filter) of 0.8, and field of view 1° in diameter. This system views a blackbody source of 1000 K with an exit aperture of diameter 1 mm and at a distance of 2 m. The signal out of the blackbody is interrupted by a shutter at a temperature of 300 K. The receiver system is equipped with two bandpass filters, one with $\lambda_0 = 20 \mu m$ and $\Delta \lambda = 1 \mu m$ and the other with $\lambda_0 = 2 \mu m$ and $\Delta \lambda = 0.1 \mu m$; both have transmittances of 0.8. The transmittance of the air between the source and receiver is 1 at both wavelengths. Compute the net signal at the detector, that is compute the change in power incident on the detector as the shutter is opened and closed.

1.3 Show that for $h\nu/kT \ll 1$ (setting $\epsilon = n = 1$),

$$L_\nu = 2kT \nu^2/c^2. $$

This expression is the Rayleigh–Jeans law and is a useful approximation at long wavelengths. For a source temperature of 100 K, compute the shortest wavelength for which the Rayleigh–Jeans law is within 20% of the result given by equation (1.4). Compare with $\lambda_{\text{max}}$ from Problem 1.4.

1.4 For blackbodies, the wavelength of the maximum spectral irradiance times the temperature is a constant, or

$$\lambda_{\text{max}} T = C. $$

This expression is known as the Wien displacement law; derive it. For wavelength units, show that $C \approx 0.3 \text{ cm K}$.

1.5 Derive equation (1.9). Note the particularly simple form for small $\theta$.

1.6 From the Wien displacement law (Problem 1.4), suggest suitable semiconductors for detectors matched to the peak irradiance from

(a) stars like the Sun ($T = 5800$ K)
(b) Mercury ($T = 600$ K)
(c) Jupiter ($T = 140$ K).

1.7 Consider a bandpass filter that has a transmittance of zero outside the passband $\Delta \lambda$ and a transmittance that is the same for all wavelengths within the passband. Compare the estimate of the signal passing through this filter when the signal is determined by integrating the source spectrum over the filter passband with that where only the effective wavelength and FWHM bandpass are used to characterize the filter. Assume a source radiating in the Rayleigh–Jeans regime. Show that the error introduced by the simple effective wavelength approximation is a factor of

$$1 + \frac{5}{6} \left( \frac{\Delta \lambda}{\lambda_0} \right)^2 $$

plus terms of order $(\Delta \lambda/\lambda_0)^4$ and higher. Evaluate the statement in the text that the approximate method usually gives acceptable accuracy for $\Delta \lambda/\lambda_0 \leq 0.2$. 
2.98 \times 10^{-6} (\Omega \text{ cm})^{-1}. Substituting into equation (2.1), we find that the detector dark resistance is 4.2 \times 10^7 \Omega.

(d) The time response of the detector is determined by the longest of three time constants. The first is the recombination time, which in this case is 10^{-4} \text{s}. The second is the $RC$ time constant. The capacitance can be calculated from equation (2.19) with the dielectric constant from Table 2.1; the result is $C = 8.4 \times 10^{-15} \text{F}$, leading to $\tau_{RC} = 3.5 \times 10^{-7} \text{s}$. Since we have not specified the circuit in which the detector is operated, the dielectric relaxation time constant will be the same as $\tau_{RC}$. Comparing these three time-response limitations, we find that the recombination time is the important limitation.

(e) The Johnson noise current is given by equation (2.33) with $T = 300 \text{K}$ and $R = 4.2 \times 10^7 \Omega$. It is $1.99 \times 10^{-14} df^{1/2} \text{A}$.

(f) The performance of the detector can be characterized by calculating the signal-to-noise ratio it would achieve in a given integration time on a given signal power. For example, in a 0.5 s integration, from equation (2.30), $df = 1 \text{Hz}$ and the Johnson noise current $I_j = 1.99 \times 10^{-14} \text{A}$. An input photon signal of $1.05 \times 10^{-13} \text{W}$ gives a signal current of $2 \times 10^{-14} \text{A}$, since the detector responsivity is $0.19 \text{A W}^{-1}$. Therefore, the ratio of signal to noise on this strength of signal is unity in 0.5 s of integration. By definition, the detector $NEP = 1.05 \times 10^{-13} \text{W Hz}^{-1/2}$, the detectivity $D = 9.5 \times 10^{12} \text{Hz}^{1/2} \text{W}^{-1}$, and $D^* = 0.1 \text{cm} \times D = 9.5 \times 10^{11} \text{cm Hz}^{1/2} \text{W}^{-1}$.

2.5 Problems

2.1 Use equations (1.38) and (2.28) to prove equation (2.29).

2.2 A circular photoconductor of diameter 0.5 mm operating at a wavelength of 1 \mu m has a spectral bandwidth of 1% (0.01 \mu m). Suppose it views a circular blackbody source of diameter 1 mm at a distance of 1 m and at a temperature of 1500 \text{K}. If the detector puts out a signal of $5 \times 10^{-13} \text{A}$, what is its responsivity and $\eta G$ product?

2.3 Suppose the output of the detector in Problem 2.2 is sampled in a series of well-defined one-second intervals. With the view of the source (and other photons) blocked off, it is found that the rms noise current is $2.00 \times 10^{-16} \text{A}$, which we take to be excess electronics noise. Viewing the source, the rms noise is $4.58 \times 10^{-16} \text{A}$. Determine the detective quantum efficiency (in the absence of electronics noise) and use it to estimate the photoconductive gain.

2.4 Show that the quantum efficiency of a photoconductor that has reflectivity $R$ at both the front and back faces, absorption coefficient $a$, and length $\ell$ from the front to the back is

$$
\eta = \frac{(1 - R)(1 - e^{a\ell})}{1 - Re^{a\ell}}.
$$