ELEG 867 HW 8 due 21 Nov '01

HW 8-1

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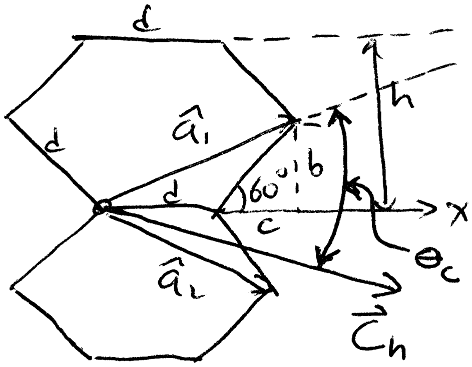


$b = d\sin 60° = d\frac{\sqrt{3}}{2}$

$c = d\cos 60° = d/2$

$\hat{a}_1 = \frac{1}{2}(\sqrt{3}\,\hat{x} + \hat{y})$ $\quad |\hat{a}_1|^2 = 1 = |\hat{a}_2|^2$

$\hat{a}_2 = \frac{1}{2}(\sqrt{3}\,\hat{x} - \hat{y})$ $\rightarrow$ unit vectors.

$$\hat{a}_1 \cdot \hat{a}_2 = \frac{1}{4}\left[3|\hat{x}|^2 - |\hat{y}|^2\right] = \frac{1}{2}$$

$\vec{C}_h = n\hat{a}_1 + m\hat{a}_2 \rightarrow$ (arbitrary direction with θ_c: 0 to 60° then repeats)

$$|\vec{C}_h|^2 = \left[n^2\hat{a}_1^2 + nm\hat{a}_1\cdot\hat{a}_2 + mn\hat{a}_1\cdot\hat{a}_2 + m^2|\hat{a}_2|^2\right] = n^2 + m^2 + nm$$

$$\hat{a}_1 \cdot \vec{C}_h = |\hat{a}_1||\vec{C}_h|\cos\theta_c$$

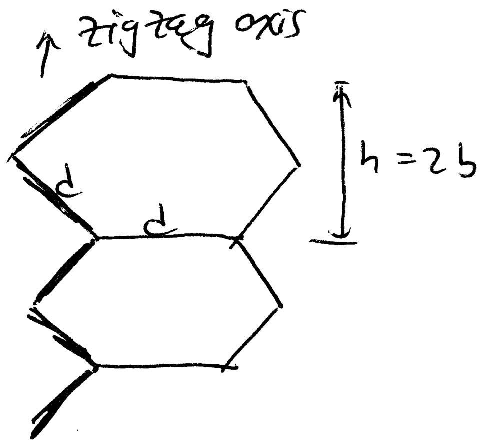
$$\theta_c = \cos^{-1}\left[\frac{\hat{a}_1\cdot\vec{C}_h}{|\hat{a}_1||\vec{C}_h|}\right] = \cos^{-1}\left[\frac{n + m/2}{\sqrt{n^2+m^2+nm}}\right]$$

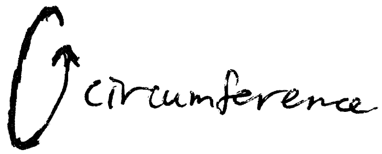
$$\cos\theta_c = \frac{n + m/2}{\sqrt{n^2+m^2+nm}}$$ identify $\cos^2\theta_c + \sin^2\theta_c = 1$

so $\cos^2\theta_c = \frac{(n + m/2)^2}{n^2+m^2+nm} \rightarrow \sin^2\theta_c = \frac{\frac{3}{4}m^2 \cancel{+nm}}{n^2+m^2+nm}$

so $\tan\theta_c = \frac{\sin\theta_c}{\cos\theta_c} = \cancel{\frac{\sqrt{m^2+nm}}{n}} = \frac{\frac{\sqrt{3}}{2}m}{n + m/2} = \frac{\sqrt{3}\,m}{2n+m}$

$$\theta_c = \tan^{-1}\left[\frac{\sqrt{3}\,m}{2n+m}\right]$$





$d = a_{cc} = 0.144\,nm$

for N_h hexagons of length $h = 2b$,
= circumference.

$$C = N_h \times h = N_h \sqrt{3}\, a_{cc}$$

$$= \pi\, dia$$

$$dia = N_h \frac{\sqrt{3}}{\pi} a_{cc} = N_h \times 0.0794\ nm$$

$$= N_h \times 0.0794\ nm$$

where N_h = integer

the minimum diameter is 0.71 nm →
corresponding to 5 hexagons