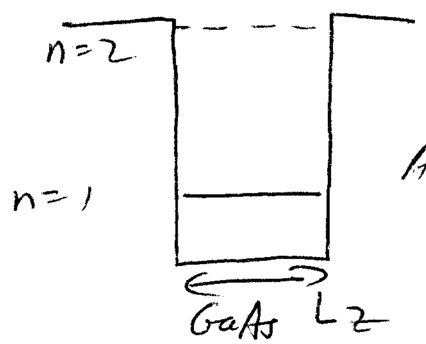
12 Nov '01 ELEG 867 - Hmwk 7

HW (7.1)



$$E_n = \frac{\hbar^2 \pi^2}{2m^*}\left(\frac{n_z}{L_z}\right)^2$$

let $\lambda = 1.55\,\mu m$

with $E = \dfrac{1.24\ eV\mu m}{1.55\,\mu m} = 0.8\,eV$

[note for GaAs/AlGaAs, the $\Delta E \lll 0.8\,eV$ so this is not a good material system for near infrared QWIPs → instead GaN/AlN

$$\Delta E_{2-1} = \frac{\hbar^2\pi^2}{2m^*}\frac{1}{L^2}\underbrace{(2^2-1^2)}_{3}$$

$$= \frac{(1.055\times10^{-34}\,J\text{-}s)^2\pi^2}{2\times0.07\times9.11\times10^{-31}\,kg}\times 3\ \frac{1}{L^2} = \frac{2.58\times10^{-36}\ J\text{-}m^2}{L^2}$$

$$L^2 = \frac{2.58\times10^{-36}\,J\text{-}m^2}{0.8\,eV\times1.6\times10^{-19}\,J/eV} = 2.017\times10^{-17}\,m^2$$

$$L = 4.49\times10^{-9}\,m = 4.49\,nm$$ → width of well for ∞ well approximation

$\Gamma = 10\%\,\Delta E = 0.08\,eV = 80\,meV$

For a detector, the first level must be occupied and the upper state empty → so E_F is ~ mid way

estimate the carrier concentration:

$$n \sim \frac{m^*}{\pi \hbar^2} \times \left(E_F = \frac{E_2 - E_1}{2}\right)$$

2D-DOS

$$= \frac{0.07 \times 9.11 \times 10^{-31}\,\text{kg}}{\pi \times (1.054 \times 10^{-34}\,\text{J-s})^2} \times (0.4\,\text{eV} \times 1.6 \times 10^{-19}\,\text{J/eV})$$

$$= 1.827 \times 10^{36} \times 0.4 \times 1.6 \times 10^{-19}\,\text{J} = 1.17 \times 10^{17}\,\text{m}^{-2}$$

$n_- = 1.17 \times 10^{+13}\,\text{cm}^{-2}$ → this is a very large density
→ hard to achieve in practice

assume $n = 1 \times 10^{12}\,\text{cm}^{-2}$ as more realistic

from class notes for GaAs well:

$$\alpha_{2D} = 0.15 \times n_s[10^{12}\,\text{cm}^{-2}] \times \frac{f_{12}}{\Gamma}$$

find $f_{12} = \dfrac{64 n^2 m^2}{\pi^2 (n^2 - m^2)^3} = \dfrac{256}{27\pi^2} = 0.96$

$$\alpha_{2D} = 0.15 \times 1 \times \frac{0.96}{0.08} = 1.8 \text{ per well}$$

so the absorption per well is $e^{-1.8}$

the spatial matrix element is:

$$|\langle 1|z|2\rangle| = \frac{16 L}{9\pi^2} = \frac{16 \times 4.5\,\text{nm}}{9\pi^2} = 0.81\,\text{nm}$$

The momentum matrix element is

$$|\langle 1|P_z|2\rangle| = \frac{8\hbar}{3L} = \frac{8\times 1.055\times10^{-34}\,Js}{3\times 4.5\times10^{-9}\,m} = 6.22\times10^{-26}\,\frac{Js}{m}$$

each well absorbs: $e^{-1.8} = I(\text{well})/I_o$ = transmitted intensity

To absorb 90% of the incident intensity

so the transmitted intensity is $100-90 = 10\%$

$$0.1 = e^{-1.8N}$$ where N = number of wells

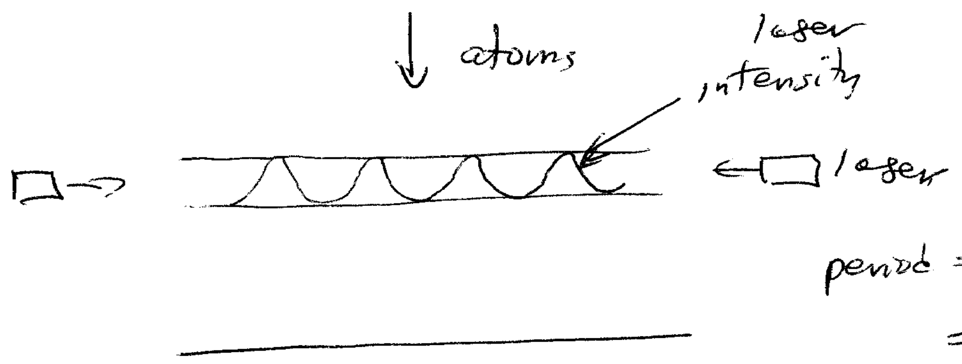
$$\ln 0.1 = -1.8N \rightarrow N = \frac{\ln 10}{1.8} = \frac{2.3026}{1.8} = 1.28$$

we Need $N = 1.28$ wells or 1–2 wells,

2 wells absorbs to give a transmitted intensity

of: $e^{-3.6} = 0.027 = 97.3\%$ absorption

7. Atom Optics Lithography



$$\text{period} = \frac{\lambda}{2} = 200\,nm$$

$$\Rightarrow \lambda = 400\,nm$$

$$E(eV) = \frac{1.24\,eV\mu m}{0.4\,\mu m} = 3.1\,eV$$

$$\hbar\omega = 3.1\,eV \qquad \omega = \frac{3.1\,eV \times 1.6\times10^{-19}\,J/eV}{1.054\times10^{-34}\,J\text{-}s} = 4.7\times10^{15}\ \text{rad/sec}$$

atomic polarizability $\alpha = \dfrac{e^2}{m\omega_0^2} = \dfrac{(1.6\times10^{-19}\,C)^2}{9.11\times10^{-31}\,kg \times (4.7\times10^{15}\,r/s)^2}$

$$= 1.27\times10^{-39}\ C^2 s^2/kg$$

the stimulated dipole force on an atom is:

$$F = \alpha_{pol}\ \eta\ \underbrace{\cos\phi}_{\approx 1}\ \frac{\partial I}{\partial x}$$

$\eta \rightarrow 377\,\Omega$

$\dfrac{\partial I}{\partial x} \rightarrow \dfrac{1\ \text{MegaWatt}/cm^2}{100\,nm} = \dfrac{10^{10}\,W/m^2}{10^{-7}\,m}$

$$= 10^{17}\,W/m^3$$

$$F = 1.27\times10^{-39}\,\frac{Coul^2 s^2}{kg} \times 377\,\frac{V}{A} \times \frac{10^{17}\,W}{m^3} = 4.8\times10^{-20}\,\underbrace{\frac{C^2 s^2\,V \cdot VA}{kg\,A\,m^3}}$$

$= \cancel{4.8}$

$$\frac{Joule^2\,s^2}{kg\ m^3} = \frac{Joule}{m} = Newton$$

$$F_{light} = 4.8\times10^{-20}\ Newton$$

compare force of gravity on atom (Si)

$$F_{grav} = mg = 28 \times 1.67\times10^{-27}\,kg \times 9.8\,m/sec^2 = 4.6\times10^{-25}\,Nt$$

$F_{light} \sim 10^5 \times$ force of gravity